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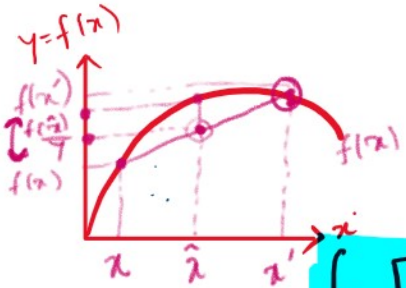
① Concave Function:

Let $f: S \to R$ be a function defined on the convex set $S \subset R^n$.

Then f is concave on the set S if for all $x' \in S$ and $\lambda \in [0,1]$ we have

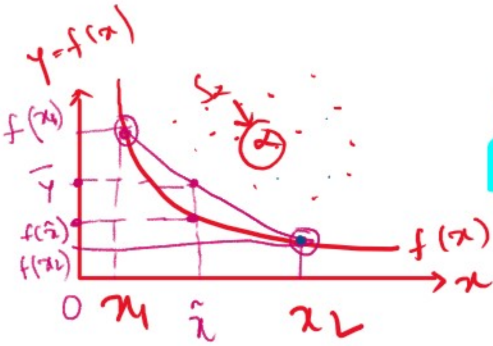


$$f[\lambda x + (1-\lambda)x'] \geq \lambda f(x) + (1-\lambda) f(x')$$

② Convex function:

A function $f: S \to R$ defined on convex set $S \subset R^n$; for all x_1, x_2 in it's domain and $\lambda \in [0,1]$ we have

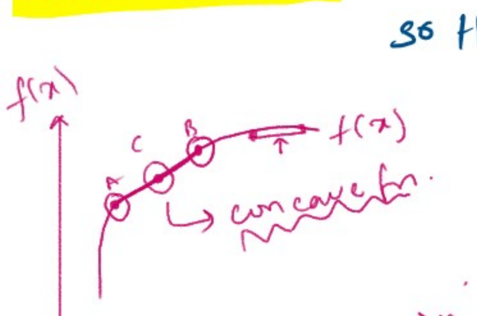
$$f[\lambda x_1 + (1-\lambda) x_2] \leq \lambda f(x_1) + (1-\lambda) f(x_2).$$

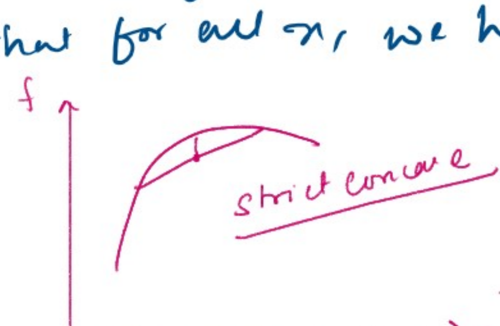


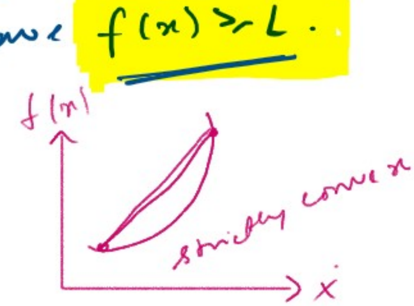
Upper bound:

Upper bound for a function f is a number 'U' so that for all x, we have $f(x) \leq U$

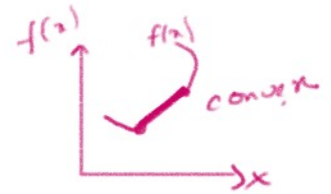
Lower bound: Lower bound for a function f is a number 'L' so that for all x, we have $f(x) \geq L$.







$f(x)$ ↑ $f(x)$



③ Quasiconcave and Quasiconvex fn

Quasiconcave Functions:

Let $f: S \to R$ be a fn defined on the convex set $S \subset R^n$. Then f is Quasiconcave if every upper contour set of f is convex i.e,

$$\underline{\underline{P_a}} = \{ x \in S : f(x) \geq a \} \text{ is a convex set } \forall a \in R$$

In other words: f(x) is quasi concave if

$$f(\hat{x}) \geq \min [f(x_0), f(x_1)]$$

where $\boxed{\hat{x} = \lambda x_0 + (1-\lambda) x_1}$ and $\lambda \in [0,1]$

Rough explaination: $x \in a, b$

$y = f(x)$

$f(\lambda a + (1-\lambda) b) \geq \min [f(a), f(b)]$

Concave → quasi-concave
Convex → quasi-concave

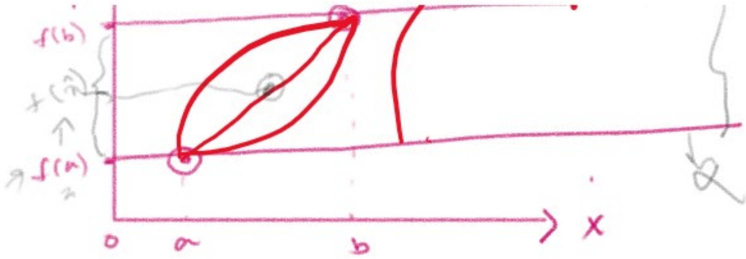
Case (i) $f(a) < f(b)$

Case (i) $f(a) < f(b)$

$\min(f(a), f(b))$

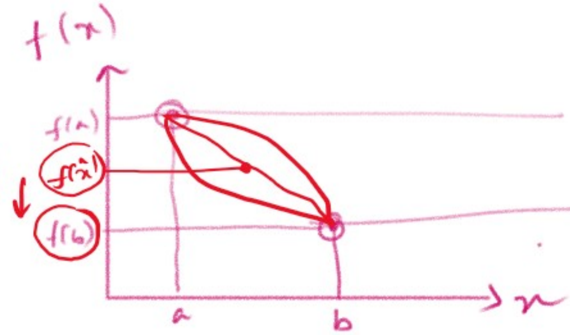
$\uparrow$

$f(a)$



Case (ii) $f(b) < f(a)$

$\boxed{f(\hat{x}) > f(b)}$



What is quasi convex fn: Let $f : S \to R$ where $S \subset R^n$ is a nonempty convex set. The fn f is said to be quasiconvex if for each $x_1, x_2 \in S$, we have

$$f(\lambda x_1 + (1-\lambda) x_2) \leq \max[f(x_1), f(x_2)]$$

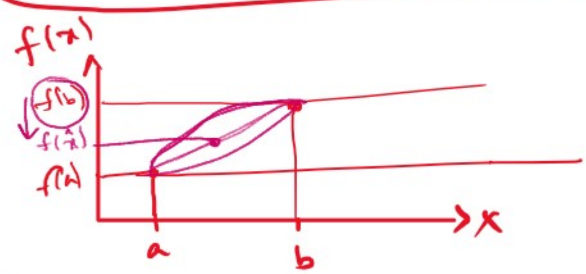
i.e., iff $S_\alpha = \{x \in S : f(x) \leq \alpha\}$ is convex for each real number.

Rough $x \in a, b$; $f(x) = y$

Quasiconvex $\Rightarrow$ $\boxed{f[(1-\lambda) a + \lambda b] \leq \max[f(a), f(b)]}$

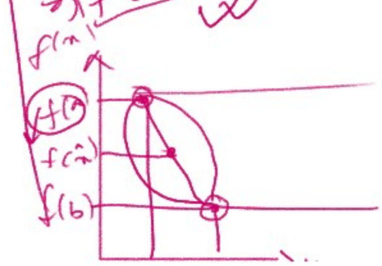
Case 1: $f(a) < f(b)$

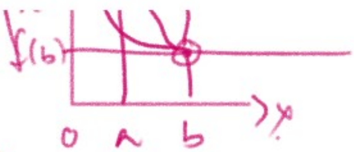
$\boxed{f(1-\lambda) a + \lambda b \leq f(b)}$



Case (ii) $f(a) > f(b)$

$\Rightarrow \boxed{f(\hat{x}) \leq f(a)}$





Theorem 1: A convex function is always quasiconvex, but the converse is not true.
[it mean a quasi convex can be concave]

Theorem 2: A concave function is always quasiconcave, but the converse is not true.
[it means a quasiconcave can be convex].

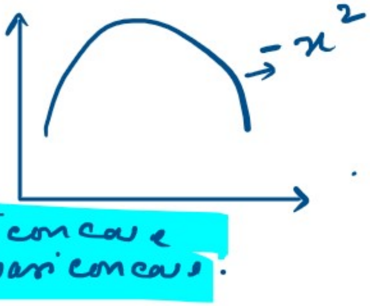
Theorem 3: $F(g(x))$ is quasiconcave iff $g(x)$ is quasiconcave and F is strictly increasing (ex: exponential fn)

Theorem 4: $f(g(x))$ is quasiconvex iff $g(x)$ is quasiconvex and f is strictly increasing.

Ex: $z = e^{-x^2}$. Prove whether quasiconcave or not.

Let $g(x) = -x^2 \rightarrow g'(x) = -2x$, $g''(x) = -2 < 0$ ∴ thus $g(x)$ is concave

~~from diagram that~~ $g(x)$ is concave.



we know a concave fn is also quasiconcave
∴ $g(x)$ is quasiconcave.

Now $F(g(x)) = F(u) = e^u$
exponential fn is strictly ↑sing ie F is strictly ↑sing.
Hence $z = F(g(x)) = e^{-x^2}$

is quasi- concave.

(Proved).