

Inequalities

Sunday, May 14, 2023 10:00 AM

(1) $a > b, b > c$, then $a > c$

(2) $a > b$, then $a+c > b+c$, $c \geq 0$

(3) $a+b > c+d \Rightarrow a+b-c > d$ (can transpose signs)

(4) $a > b \Rightarrow b < a$

(5) $a > b \Rightarrow ac > bc$ & $a/c > b/c$, for $c > 0$

(6) $a > b \Rightarrow -b > -a$

(7) If $a > b$, then $1/a < 1/b$ for $a, b > 0$

(8) $a_1 > b_1, a_2 > b_2, \dots, a_n > b_n \Rightarrow \sum a_i > \sum b_i$

(9) $a_1 > b_1, a_2 > b_2, \dots, a_n > b_n \Rightarrow \prod a_i > \prod b_i$

(10) $a > b, a^n > b^n$ and $a^{1/n} > b^{1/n}$

} positive numbers

→ Which is one is greater? $(31)^{12}$ or $(17)^{17}$

$31 < 32, (31)^{12} < (32)^{12} = (2^5)^{12} = 2^{60} < 2^{68} [(a^m)^n = a^{mn}]$

$\therefore (31)^{12} < 2^{60} < 2^{68} = 16^{17} < 17^{17}$

$\hookrightarrow = 2^{4 \times 17} = (2^4)^{17} = (16)^{17}$

We know, $17 > 16, (17)^{17} > (16)^{17}$. Hence, $(17)^{17} > (31)^{12}$

→ Which is greater? $(30)^{100}$ or 2^{567}

$30 < 32 \Rightarrow 30^{100} < 32^{100} = (2^{10})^{50} = (1024)^{50} < (1024)^{54} = (2^{10})^{54} = (2^{20})^{27}$

$(2^{20})^{27} < (2^{21})^{27} = 2^{567}$

$\therefore 30^{100} < 2^{567}$

HW 1 7^{92} or 8^{91} ?

HW 2 150^{300} or $(20000)^{100} \times (100)^{100}$

→ Show $(1.01)^{1000} > 1000$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{n} b^n$

$\uparrow = 1 \cdot a^n + C_1 a^{n-1}b + C_2 a^{n-2}b^2 + \dots + C_n b^n$

Put $a=1, b=x : (1+x)^n = 1 + C_1 x + C_2 x^2 + \dots + C_n x^n$

$\Rightarrow (1+x)^n > 1 + C_1 x = 1 + nx$

} Pascal's triangle

→ Binomial theorem

positive, if $x > 0$

$(1+x)^n > 1+nx$

Inequalities

Sunday, May 14, 2023 10:00 AM

$$\begin{aligned} \text{LHS} &> (1.4)^{25} > (1.4)^{24} \\ &= (1.4^3)^8 \\ &> (2.7)^8 \\ &> (2.5)^8 \\ &> 5^8/2 > 1000 \end{aligned}$$

$$\begin{aligned} (1+0.01)^8 &> 1+8 \times 0.01 \\ &= 1.08 \\ [(1+0.01)^8]^{125} &> (1+0.08)^{125} \\ &= [(1+0.08)^5]^{25} \\ &> (1+0.4)^{25} \end{aligned}$$

Blank

Sunday, April 30, 2023 4:03 PM