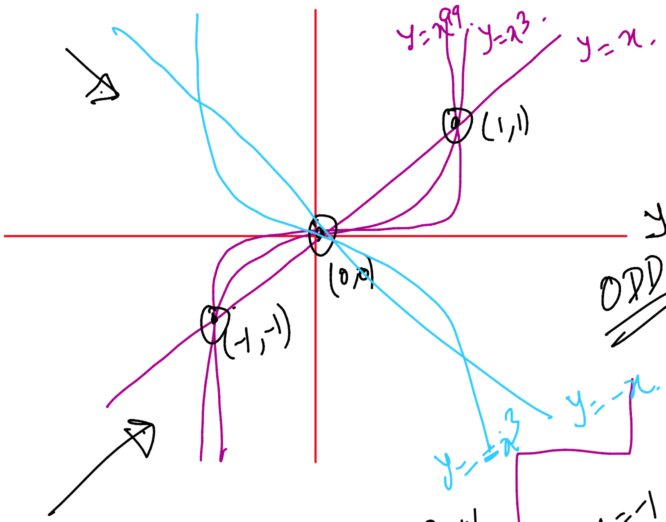
Functions 4 types.

1. Polynomials.
2. Trigonometric.
3. Exponential
4. logarithmic.



$y=\pm x^{2n+1}$

ODD

Polynomials

odd

$y=x^{2n+1}$

$y=x$
$y=x^3$.

increasing or decreasing

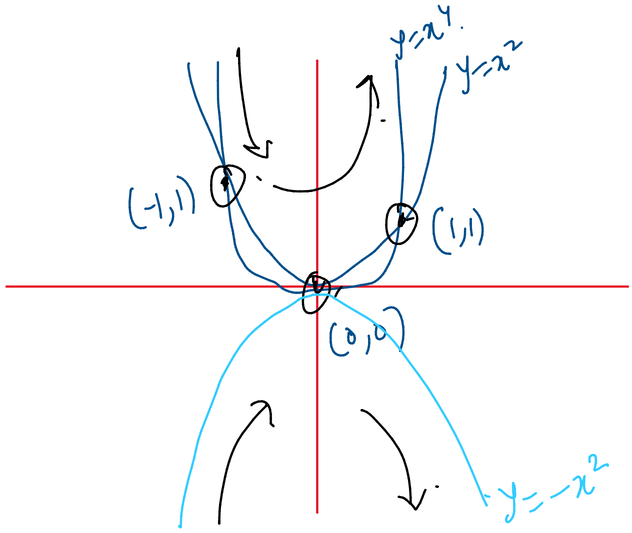
even.

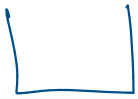
$y=x^{2n}$

$y=x^2$
$y=x^4$.

$y=x^{2n+1}$

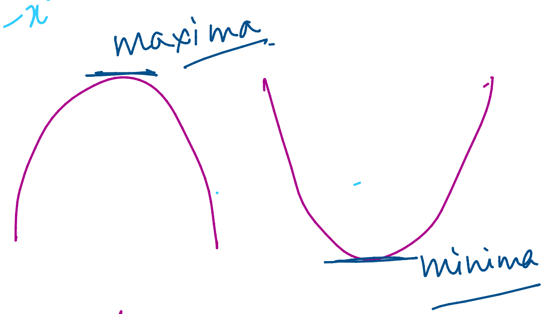
$x=0 \quad y=0$
$x=1 \quad y=1$
$x=-1 \quad y=-1$





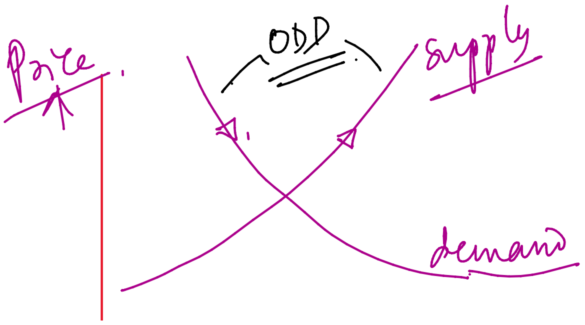
$y=x^{2n}$.

$x=0 \quad y=0$
$x=1 \quad y=1$
$x=-1 \quad y=1$

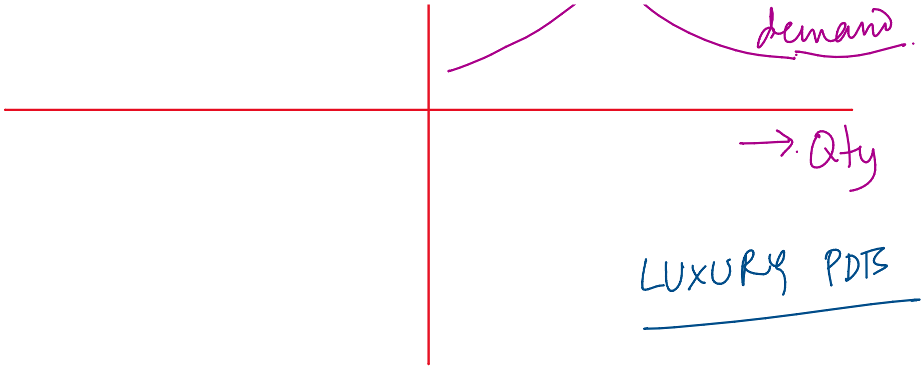


If $f(x)$ has no maxima/minima ⟹ ODD.

If $f(x)$ has at least 1 maxima/minima ⟹ EVEN.



Price inelastic demand
⇓
MEDICINES

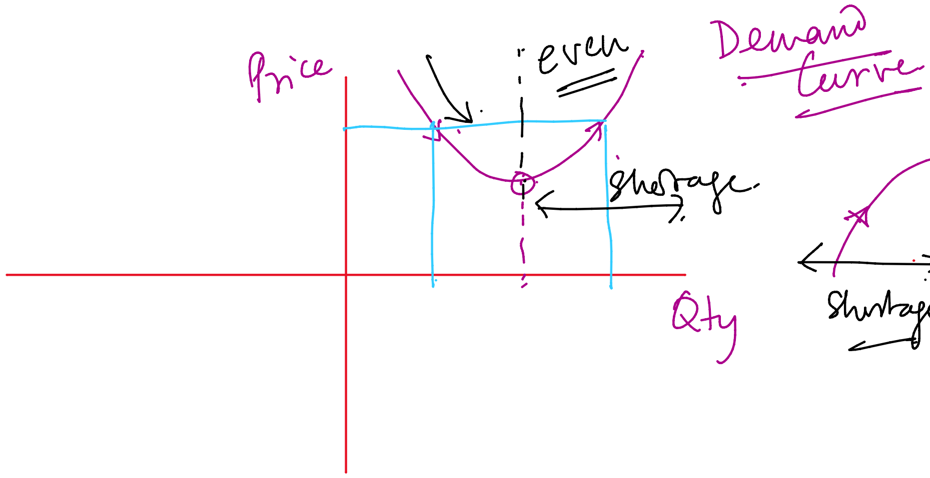


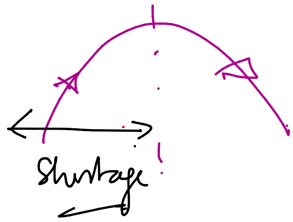
MEDICINES

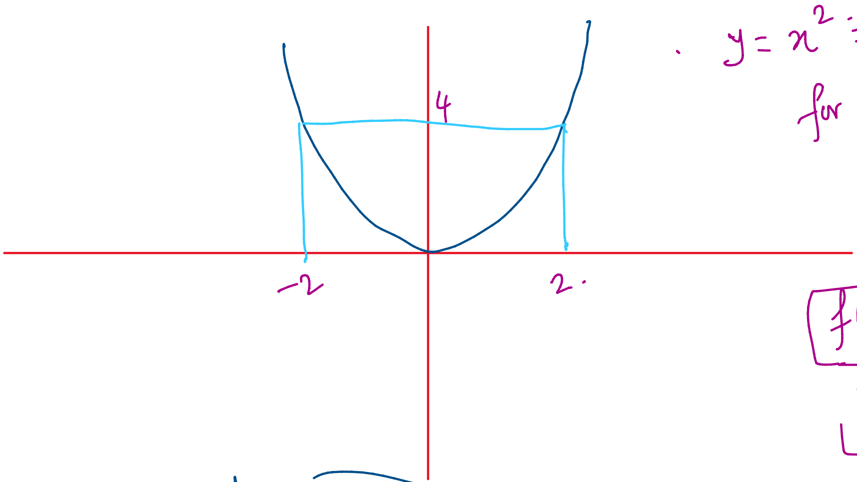
ADDICTION PDTS

STOCK MARKET

LUXURY PDTS







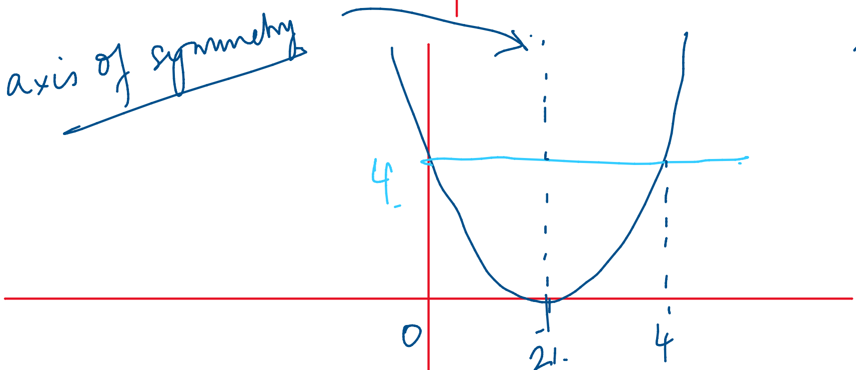
$y = x^2 = f(x)$

for $x = 2 \quad y = 4$

$x = -2 \quad y = 4$

$\boxed{f(-2) = f(2)}$

$\boxed{f(-x) = f(x)}$ even



$f(x) = (x-2)^2$

$x = 4 \quad f(x) = 4$

$x = 0 \quad f(x) = 4$

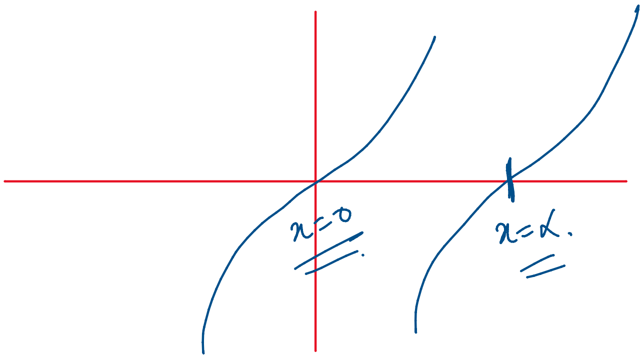
$f(0) = f(4)$

$f(2-2) = f(2+2)$

$f[-(x-2)] = f[x+2]$

$x = \alpha$ is the axis of symmetry

$f[-(x-\alpha)] = f[x-\alpha]$



$f(-x) = -f(x)$

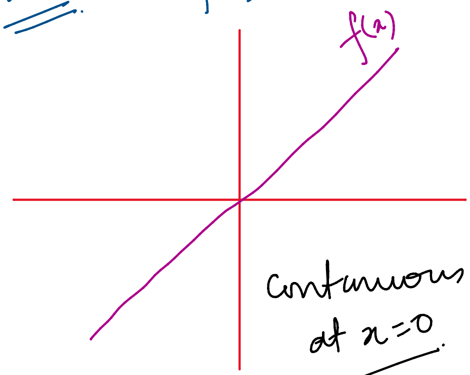
$\downarrow$

$f[-(x-\alpha)] = -f[x-\alpha]$

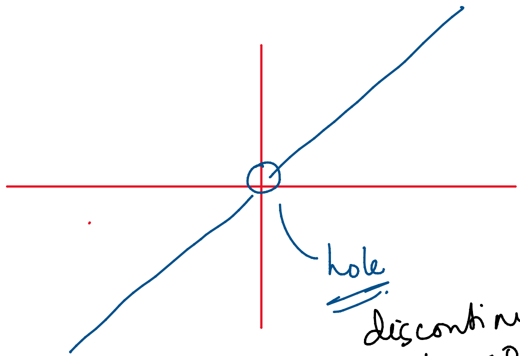
Limits.

$f(x) = x$. $g(x) = \frac{x^2}{x}$.

for $x=0$ $f(x) = 0$ $g(x) \to$ undefined (doesn't exist)



continuous at $x=0$



discontinuous at $x=0$

discontinuity.

$\lim_{x\to 0_-} g(x) = \lim_{x\to 0_+} g(x) = 0 \neq g(x=0)$

limit of $g(x)$ exists when $x \to 0$.

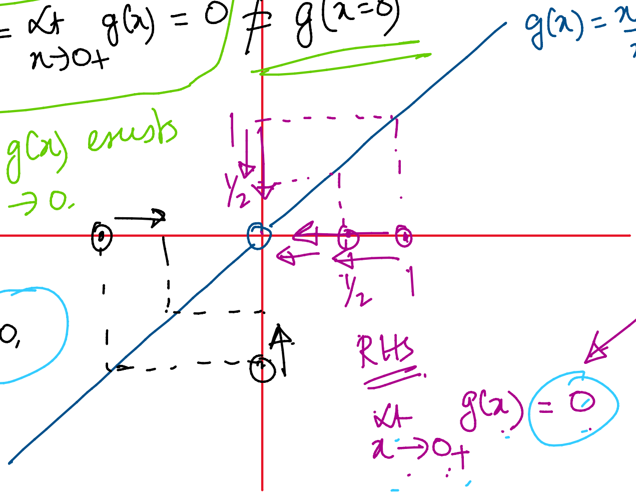
$g(x) = \frac{x^2}{x}$. $g(x) = x \ \forall x \neq 0$

$=$ undefined for $x=0$

what happens when $\boxed{x \to 0}$

x approaches 0 but $x \neq 0$.

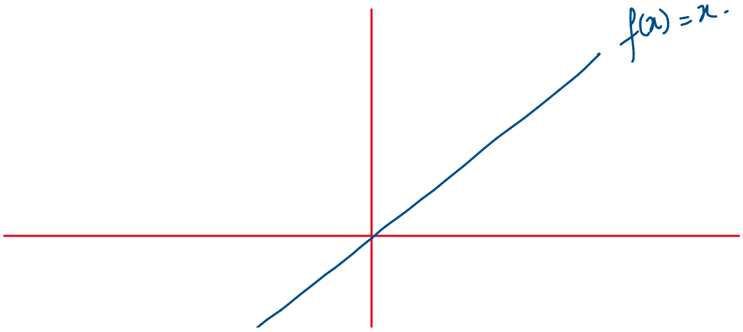
$\boxed{g(x) \to 0}$



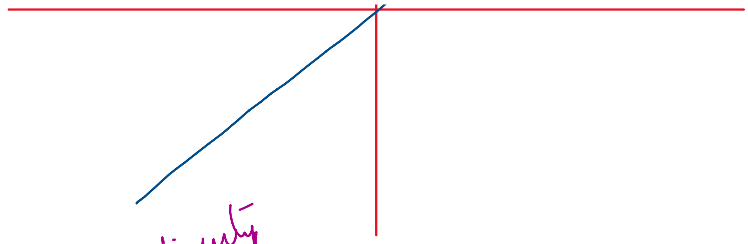
$\lim_{x\to 0_-} g(x) = 0$,

RHS

$\lim_{x\to 0_+} g(x) = 0$



$\lim_{x\to 0_+} f(x) = \lim_{x\to 0_-} f(x) = 0 = f(x=0)$



Condition of continuity

If $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(x=a)$

then $f(x)$ is continuous at $x=a$

If $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) \neq f(x=a)$

then $f(x)$ is discontinuous at $x=a$.

If $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$ then $\lim_{x\to a} f(x)$ EXISTS

$f(x) = \frac{x^2}{x}$.

$$f(x) = \frac{(x+1)(x-1)}{(x+1)(x+2)}$$

What happens at $x=-1$ and $x=-2$.

$\Downarrow$

denominator $= 0$

$\Downarrow$

$f(x)$ is undefined

$\Downarrow$

$f(x)$ is discontinuous

rational functions

$\downarrow$

ratio $= \frac{p}{q}$.

$f(x) = \frac{x-1}{x+2}$

provided $x \neq -1$

discontinuity at $x=-1$ is removable.

$\Downarrow$

removable discontinuity $\to$ hole

$$f(x) = \frac{x-1}{x+2}.$$

when $x=-2$ $f(x) \to$ undefined

$\Downarrow$

irremovable discontinuity

as $x \to -2$ from the $-$ve side.

let $x = -2.001$

$$f(x) = \frac{-2.001-1}{-2.001+2} = -\frac{3.001}{-0.001} = 3001$$

$$f(x) \to +\infty.$$

as $x \to -2$ from the +ve side

let $x = -1.99999$.

$$f(x) = \frac{\overbrace{-1.99999-1}^{-ve}}{\underbrace{-1.99999+2}_{+ve}}$$

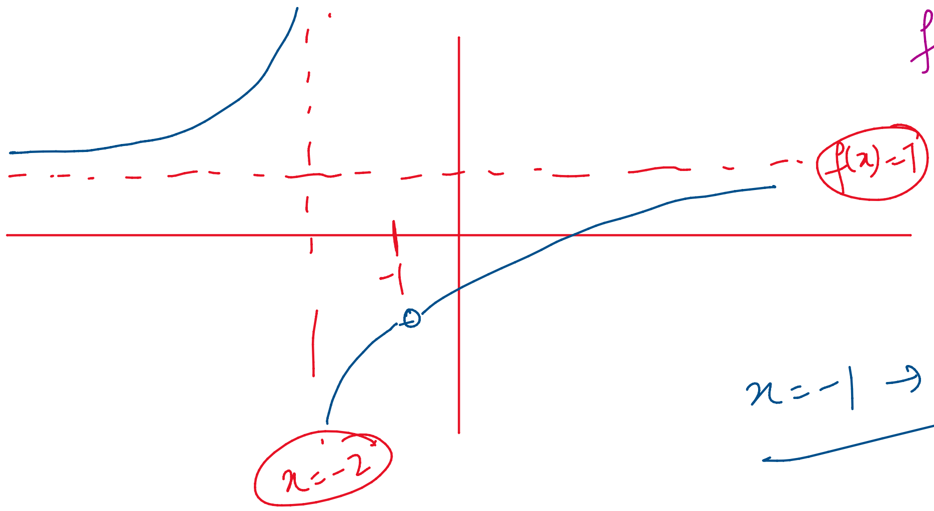
$$f(x) \to -\infty$$

$$f(x) = \frac{x-1}{x+2} = \frac{1-1/x}{1+2/x}.$$

as $x \to -\infty$.

$\frac{1}{x} \to 0$.

$f(x) \to 1$



$x = -1 \to$ hole