24 February 2024 08:19

$z^2\bar{z} + \bar{z}^2 z = 0$

$z\bar{z}(z+\bar{z}) = 0$

$|z|^2(z+\bar{z}) = 0$.

$\boxed{|z| = 0}$ or $\boxed{z+\bar{z} = 0}$

$\sqrt{x^2+y^2} = 0$

$x^2+y^2 = 0$

$x=0, y=0$.

$z = -\bar{z}$

$\bar{z} = \frac{|z|^2}{z}$ ...

$z = -\bar{z}$

$= -\frac{|z|^2}{z}$

$z^2 = -|z|^2 = i^2|z|^2$.

$z = i|z|$ ...

Re Im.

$z = x+iy$ $\bar{z} = x-iy$. $z+\bar{z} = 2x$

$|z| = \sqrt{x^2+y^2}$ $|\bar{z}| = \sqrt{x^2+y^2}$.

$|z| = |\bar{z}| = \sqrt{x^2+y^2}$.

$z\bar{z} = (x+iy)(x-iy) = x^2 - i^2y^2 = x^2+y^2$.

$\boxed{z\bar{z} = |z|^2}$

$x+iy = i\sqrt{x^2+y^2}$.

$y = \sqrt{x^2+y^2}$, $x=0$,

$y =$ any real no.

$z = x \pm i$

$z = \pm iy$

infinite

$z = x \rightarrow$ only real.

ROOTS OF A COMPLEX NUMBER

$z = r(\cos\theta + i\sin\theta)$

$$z^{1/n} = r^{1/n}\left[\cos\left(\frac{2k\pi+\theta}{n}\right) + i\sin\left(\frac{2k\pi+\theta}{n}\right)\right],$$

$z = re^{i\theta}$

$\alpha = z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\frac{i\theta}{n}} = r^{\frac{1}{n}}\left[\cos\frac{\theta}{n} + i\sin\frac{\theta}{n}\right]$

$\cos\theta = \cos x$ & $\sin\theta = \sin x$

$\theta = 2n\pi + x$.

Cube root $1+\omega+\omega^2 = 0$ $\omega^3 = 1 \leftarrow x^3 = 1$

fourth root $1+\alpha+\alpha^2+\alpha^3 = 0$. $\leftarrow x^4 = 1$

$\leftarrow x^4 - 1 = (x-1)(1+x+x^2+x^3)$

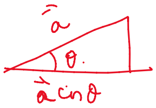
nth root:

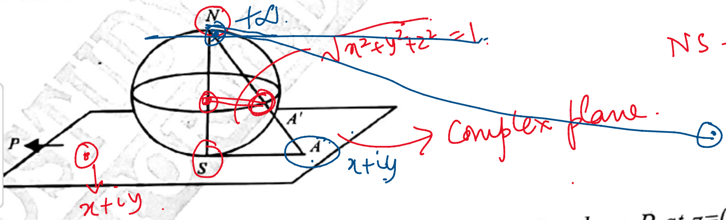
$x^n = 1 \Rightarrow x^n - 1 = 0 \Rightarrow (x-1)(1+x+x^2+\dots+x^{n-1}) = 0$

$\boxed{\text{Sum of roots} = 0}$

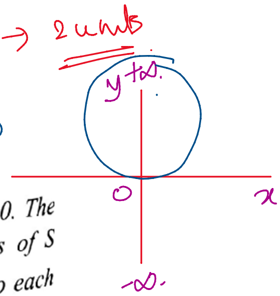
$1+\alpha+\alpha^2+\alpha^3+\dots+\alpha^{n-1} = 0$.

SPHERICAL REPRESENTATION OF COMPLEX NUMBERS (STEREOGRAPHIC PROJECTION)





NS → 2 units



Let P be the complex plane and consider a unit sphere S (radius one) tangent to plane P at z=0. The diameter NS is perpendicular to P and we call points N and S the north and south poles of S respectively. For any point A on P we can construct line NA intersecting S at point A′. Thus to each point of the complex plane P there corresponds one and only one point of the sphere S and we can represent any complex number by a point on the sphere. For completeness, we say that the point N itself corresponds to the "point at infinity" of the plane. The set of all points of the complex plane including the point at infinity is called the extended complex plane.

Conceptual

The above method for mapping the plane on to the sphere is called $\boxed{\text{stereographic projection.}}$

Imp. **Important Result:** Under the stereographic projection of points on the unit sphere $\boxed{x^2+y^2+z^2=1}$ in $\mathbb{R}^3$ to the extended complex plane, the point $z=x+iy$ corresponds to the point

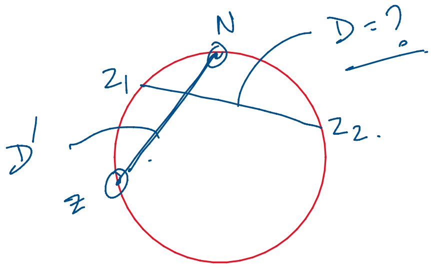
$A = x+iy$ ↓ A'

$$A' = \left(\frac{2x}{x^2+y^2+1}, \frac{2y}{x^2+y^2+1}, \frac{x^2+y^2-1}{x^2+y^2+1}\right)$$

on the sphere. Conversely, any point (X, Y, Z) on the sphere corresponds to the complex number $z=\dfrac{X+iY}{1-Z}$

A ← A'

CHORDAL DISTANCE



$A = 1+i$

$z = A' = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

$$D = \chi(z_1,z_2) = \frac{|z_1-z_2|}{\sqrt{1+|z_1|^2}\times\sqrt{1+|z_2|^2}}$$

$D' = \dfrac{1}{\sqrt{1+1}} = \dfrac{1}{\sqrt{2}}$

Chordal distance between z and ∞ is given by $\chi(z,\infty)=\dfrac{1}{\sqrt{1+|z|^2}}$

D'

$|z|^2 = \left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2 = 1$

$z_1 = 1+i \qquad z_2 = 1-i$

$z_1' = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \qquad z_2' = \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)$

$D = \dfrac{|z_1'-z_2'|}{\sqrt{1+|z_1'|^2}\sqrt{1+|z_2'|^2}}$

$D = \dfrac{\sqrt{0^2+\left(\frac{4}{3}\right)^2+0}}{\sqrt{1+1}\sqrt{1+1}} = \dfrac{\frac{4}{3}}{2} = \dfrac{2}{3}.$

Example 1. If z_1 and z_2 are distinct complex numbers such that $|z_1|=|z_2|=1$ and $z_1+z_2=1$, then the triangle in the complex plane with z_1, z_2 and -1 as vertices

(CSIR UGC NET JUNE-2013)

(a) must be equilateral

(b) must be right-angled

(c) must be isosceles, but not necessarily equilateral

(d) must be obtuse angled

$a+1 = 2-a$

$a = \frac{1}{2}$

$\frac{9}{4} + b^2 = 1$

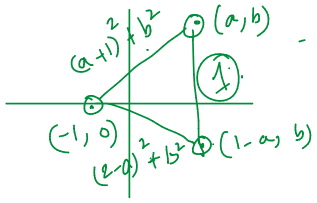
$b^2 = -$

$A = z_1 \quad B = z_2 \quad C = -1$

$AB = |z_2 - z_1|$

$BC = |-1 - z_2|$

$CA = |z_1 + 1|$



$z_1 = a + bi \qquad z_2 = c + di = (\pm 1 - a) - bi$

$a^2+b^2=1 \qquad c^2+d^2=1$

$z_2 - z_1 = (c-a) + (d-b)i$

$ac+bd$

$= a(\pm 1 - c) + b(-b)$

$|z_2 - z_1|^2 = (c-a)^2 + (d-b)^2 = c^2+a^2-2ac+b^2+d^2-2bd = 2-2(ac+bd)$

$z_1 + z_2 = 1 \Rightarrow (z_1+z_2)^2 = 1 \Rightarrow z_1^2 + z_2^2 + 2z_1z_2 = 1.$

$(a+c)^2 - (b+d)^2 = 1$

$z_1^2 = a^2 + b^2i^2 + 2abi = a^2 - b^2 + 2abi$

$\boxed{(a^2-b^2+c^2-d^2) + 2i(ab+cd)}$

$ab+cd + ad+bc = 0.$

$z_2^2 = c^2 - d^2 + 2cdi$

$z_1z_2 = (ac-bd) + (ad+bc)i$

$a(b+d) + c(b+d) = 0$

$\boxed{2z_1z_2 = 2ac - 2bd + 2adi + 2bci}$

$(a+c)(b+d) = 0.$

X $a+c=0 \qquad b+d=0.$ ✓

$(b+d)^2 = -1 \qquad \boxed{a+c = \pm 1}$

$b+d = \pm i$ X

$(1+i)^{10}+(1-i)^{10}=$

(a) -1 (b) 1 (c) 0 (d) 2

If $iz^3+z^2-z+i=0$, then find $|z|$.

If $z=\sqrt{2i}$, then z is equal to

(a) $\pm\frac{1}{\sqrt{2}}(1+i)$ (b) $\pm\frac{1}{\sqrt{2}}(1-i)$ (c) $\pm(1-i)$ (d) $\pm(1+i)$