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1. Number System.
2. Functions
3. Calculus.
4. Vectors.
5. Matrices.

Number System

2nd last digit.

Examples: find the last digit/ in 207^{309}

find the no of zeroes before the unit digit in $(401)^{50}$.

Remainders

1. Number free
2. Cyclicity of powers
3. divisibility rules.
4. AM ≥ GM

$(a-b)^2 + (b-c)^2 + (bc-8)^2 = 0$

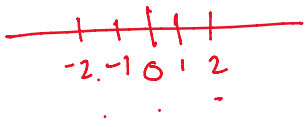
find a, b, c.

$a=b$ $b=4$ $bc=8$

$a=4$ $\therefore$ $c=2$.

Real Number.

$R^2 \geq 0$



- RATIONAL ($\frac{p}{q}$ where $q \neq 0$, HCF(p,q)=1) — p, q → integers. no common factor between p & q.
  - INTEGER. $\frac{p}{q}$, $q=1$
    - -ve
    - 0
    - +ve — Natural Nos. (Nature) — Whole Nos.
  - FRACTIONS / DECIMALS. $\frac{p}{q}$, $q \neq 1$
    - Proper $\frac{p}{q}$, $p<q$
    - Improper ↔ Mixed. $\frac{p}{q}$, $p>q$
      - $\frac{5}{3} = 1 + \frac{2}{3}$ (Integer + Proper fr.)
- IRRATIONAL.
  1. $\neq \frac{p}{q}$
  2. non-terminating, non-recurring decimal.

5+0=5.

10.

Decimals → division by 10 or powers of 10.

$10 = 2 \times 5$

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$10 = 2 \times 5$

- TERMINATING.
- RECURRING.

$\frac{1}{2} = 0.5$

$\frac{1}{4} = 0.25$

$\frac{1}{5} = 0.2$

$\frac{1}{8} = 0.125$

$\frac{1}{20} = \frac{1 \times 5}{2 \times 5 \times 5} = \frac{5}{100} = 0.05$

$\frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5$

$\frac{1 \times 5^2}{2^2 \times 5^2} = \frac{25}{100} = 0.25$

$\frac{1 \times 2}{5 \times 2} = \frac{2}{10} = 0.2$

$\frac{1 \times 5^3}{2^3 \times 5^3} = \frac{125}{1000} = 0.125$

57.1% of 77

$\frac{4}{7} \times 77$

If the denominator consists of powers of 2 and 5 then it will be a terminating decimal.

The no of digits after the decimal = the highest power of 2 or 5 in the denominator.

$\frac{1}{3} = 0.333\ldots = 0.\dot{3}$

$\frac{1}{6} = \frac{1}{2 \times 3} = 0.1\dot{6}$

$\frac{1}{7} = 0.\overline{142857}$

$\frac{2}{7} = 0.\overline{285714}$

$\frac{3}{7} = 0.\overline{428571}$

$\frac{4}{7} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{714285}$

$\frac{6}{7} = 0.\overline{857142}$

$14.3\% = \frac{1}{7}$

$28.5\% = \frac{2}{7}$

$42.8\% = \frac{3}{7}$

14.5% of 35

$= \frac{1}{7} \times 35$

$\frac{1}{12} = \frac{1}{2^2 \times 3} = 0.08\dot{3}$

No of non-recurring digits in a recurring decimal = the highest power of 2 or 5 in the denominator.

Non-terminating & Non-recurring decimal.

$1.234957618359124\ldots$

$1.101001000100000001\ldots$

} irrational.

π: 3.14159265358979323846264338327 95

$\pi = \frac{22}{7} = 3.14$

0.142857

$\frac{22}{7} = \frac{2 \times 11}{7} = 0.285714 \times 11$

$\frac{22}{7} = 3 + \frac{1}{7} = 3 + 0.142857$

AM ≥ GM

a, b → real numbers.

$\therefore (a-b) \rightarrow$ " "

$\frac{}{2}$

Arithmetic Mean ≥ Geometric Mean.

a, b → 2 real numbers.

Arithmetic Mean $(a, b) = \frac{a+b}{2}$.

$\therefore (a-b) \to$ " "

$\therefore (a-b)^2 \geq 0$.

$2ab + a^2 - 2ab + b^2 \geq 0 + 2ab$.

$a^2 + b^2 \geq 2ab$

$\frac{a^2+b^2}{2} \geq ab$.

$\left(\frac{a^2+b^2}{2}\right) \geq \sqrt{a^2b^2}$

VVImp $\boxed{AM(a^2, b^2) \geq GM(a^2, b^2)}$

Arithmetic Mean $(a,b) = \frac{a+b}{2}$.

Geometric Mean $(a,b) = \sqrt{ab}$

$-2 + x + 2 = 3 - 2$.

$x = 1$

$\frac{a^2+b^2}{2} \geq \frac{2ab}{2}$

$\rightarrow$ AM $\geq$ ( ) ( ) $\geq$ GM $\leftarrow$

$\boxed{(AM)_{min} = ( ) \qquad ( ) = (GM)_{max}}$

$\boxed{(AM)_{min} = (GM)_{max}}$

$\frac{a^2+b^2}{2} = ab \Rightarrow a^2 + b^2 = 2ab \Rightarrow a^2 - 2ab + b^2 = 0$

$\Rightarrow (a-b)^2 = 0 \Rightarrow \boxed{a = b}$

$\boxed{a+b=10}$ find $(ab)_{max}$

$AM(a,b) \geq GM(a,b)$

$\frac{a+b}{2} \geq \sqrt{ab}$.

$\frac{10}{2} \geq \sqrt{ab}$.

$5 \geq \sqrt{ab}$

$\boxed{25 \geq ab}$.

$(ab)_{max} = 25$ → this will happen when $a=b$

$\boxed{a=b=5}$

In general

$a_1, a_2, a_3, \ldots a_n$ are the real nos.

$$\frac{a_1 + a_2 + a_3 + \cdots + a_n}{n} \geq \sqrt[n]{a_1 a_2 a_3 \cdots a_n}$$

$$(a_1 a_2 a_3 \cdots a_n)^{\frac{1}{n}}$$