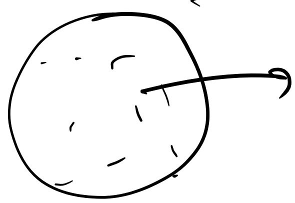
ADVANCED PROBABILITY 30 /11/2023

Thursday, November 30, 2023 6:38 PM

Advanced Probability

Ind vs china



Possibility Zone

$P \to 0$

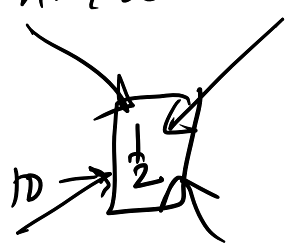
$p(H) = \frac{1}{2}$

$n \to \infty$

①

Trials are close to ∞

Possibility is close to $\frac{1}{2}$



HHT

HHTHTH TTTTH

H, H, H, T, H, T - - - - ∞ times

$n(A \cup B \cup C \cup D)$

$= n(A) + n(B) + C + D - AB - AC - AD - BC - BD - CD$

$+ ABC + ABD + CDA + CDB - ABCD$

$2^4 - 1 = 15$

$n(A \cup B \cup C \cup D \cup E)$

$2^5 - 1 = 31$

Stat 6.30

Z P Q $\boxed{AB}$

Z $\boxed{AB}$ L M

1. (b) Total Case → 12!

then regarding A, B are as tied together so, we have 11 total selection

2 × 11!

$P = \frac{2 \times 11!}{12!} = \frac{1}{6}$

1. *The first 12 letters of the English alphabet are written down at random. Find the probability that :*

(a) There are 4 letters between A and B ; (b) A and B are written down side by side.

$^{26}C_{12}$ ~~X~~

12! ways

AKCL - - -

KABP - -

9062395123

A $\quad ^{10}C_4$ ways $\quad$ B

Total possible letters

4 letters → 4! ways arrangement..

Case I $\quad A^{1st} \quad X X X X \quad B^{6th}$

Case II $\quad A^{2nd} \quad X X X X \quad B^{7th}$

7 possible Cases

A & B

(VII) $\quad A^{7th} \quad X X X X \quad B^{12th}$

$\frac{10! \, ^{10}C_4 \times 1?}{6! \, 4!}$

$P = \frac{\text{Favor ways}}{\text{Total case}} \to \frac{2 \times 7 \times {}^{10}C_4 \times 4! \times 6!}{12!} = \frac{14 \times 10!}{12!} = \frac{14}{12 \times 11} = \frac{7}{66}$

2. *n persons are seated on n chairs at a round table. Find the chance that two specified persons are sitting next to each other.*

Total → (n-1)! ways

1, 2, 3, 4, 5 $\quad$ (n-1)!

Total → $(n-1)!$ ways

$\boxed{n_1 n_2}$ $2!$ Rest $(n-2)$ ~~are~~ persons

We are dealing with $\underline{(n-1)}$ people

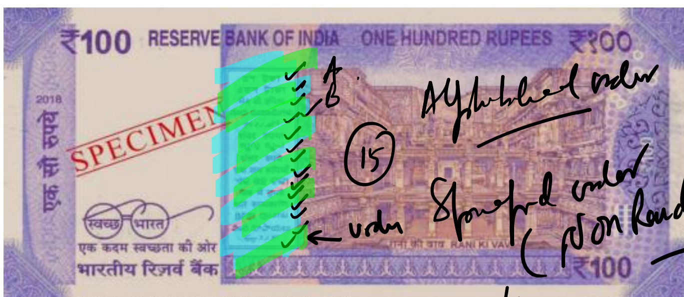
Circle → $(n-2)! \times 2$

Favourable Case → $2(n-2)!$

Total Case → $(n-1)!$

$$\Rightarrow \frac{2(n-2)!}{(n-1)(n-2)!}$$

$$= \frac{2}{n-1}$$



15 Alphabetical order

order Sequential order (Non Random)

40

15 22

RBD

HP UK 15:20

Assamese Bengali Bodo Dogri Malayalam Manipuri Marathi Maithili | Gujarati Hindi Kannada Konkani (Goa) Kashmiri Nepali (Sik) Punjabi Odia Sanskrit Santhali Sindhi Tamil Telugu Urdu

39th $\frac{1}{44}$ 39 → 39

5. An elevator starts with 10 passengers and stops at 100 floors. Assuming that any of the passengers may get down at any floor with equal probability, find the probability that no two or more of them get down at the same floor.

Total → 100^{10}

1 → 100 ~~9 Pelt~~ 99 Remaining 99

$100 \times 99 \times 98 \times \dots \times 91$

100×100

12345 95

$y = \alpha + \beta x_i$

$= \alpha_1 + \alpha_2 + \beta x_i$

29 32

94 Urbana

All @ diff floor → $\dfrac{100 \times 99 \times \dots \times 91}{100^{10}}$

P[No two or more on same floor]

= P[All @ different floor]

$= \left(1-\frac{1}{100}\right)\left(1-\frac{2}{100}\right)\left(1-\frac{3}{100}\right)\cdots\left(1-\frac{9}{100}\right)$

$= 1 - \dfrac{1+2+\dots+9}{100}$

$= 1 - \dfrac{9 \cdot 10}{\dots} = \dfrac{11}{20} = 0.55$

$= 1 - \frac{9 \cdot 10}{2 \cdot 100} = \frac{11}{20} = 0.55$

1st Game of probability

10. *Five digited numbers are formed from the digits 1, 2, 3, 4, 5. Find the chance that the number formed is greater than 23,000.*

$5! \Rightarrow 120$

first
2/3/4/5

Case I Start 2 3/4/5 Remain 1/4/5 or 1/3/5 or 1/3/4
$3 \times 3!$

Case II Start 3 1/2/4/5 $1 \times 4!$ ways
$4!$ ways

Case III
Case IV
Start 4/5

$3 \times 3! + 3 \times (4!)$
$\Rightarrow 3(6 + 24) \Rightarrow 90$

$P \rightarrow \frac{90}{120} = 0.75$

9062395723

11. *The chance of one event happening is the square of the chance of a second event but the odds against the first are the cubes of the odds against the second. Find the chances of these events.*

p_1, p_2
$p_1 = p_2^2$

$\frac{1-p_1}{p_1} = \left(\frac{1-p_2}{p_2}\right)^3$ — (1)

2

$p_1 = p_2^2$ $\quad p_1$

$$\frac{1-p_2^2}{p_2^2} = \frac{(1-p_2)^3}{p_2^3}$$

$$p_2(1-p_2^2) = (1-p_2)^3$$

$$p_2 - p_2^3 = 1 - 3p_2 + 3p_2^2 - p_2^3$$

$$3p_2^2 - 4p_2 + 1 = 0$$

$$p_2 = 1, \ 1/3$$

$$p_2 = 1/3, \quad p_1 = (1/3)^2 = 1/9$$

Google Read

Repeate

$2+3=5$

HW

12. *In a game called 'odd man out', m, (m ≥ 2) persons toss a coin to determine a person who will pay for the cold drink (Coca-cola) for the entire group. A person is termed as 'odd man out' if his outcome is different from all other members of the group. Show that the probability for the loser in any game is* $m / 2^{m-1}$.

Assorted (series)

18. *A bag contains an assortment of blue and red balls. If two balls are drawn at random, the probability of drawing two red balls is five times the probability of drawing two blue balls. Furthermore, the probability of drawing one ball of each colour is six times the probability of drawing two blue balls. How many red and blue balls are there in the bag ?*

$\frac{r!}{(r-2)2!}$

$p_1 = (2 \text{ Red}) = \frac{{}^rC_2}{{}^{b+r}C_2} = \frac{r(r-1)}{(b+r)(b+r-1)}$

$p_2 = 2 \text{ Blue} \quad \frac{{}^bC_2}{{}^{b+r}C_2} = \frac{b(b-1)}{(b+r)(b+r-1)}$

$p_3 = 1 \text{ each} \quad \frac{{}^bC_1 \cdot {}^rC_1}{{}^{b+r}C_2} = \frac{2br}{(b+r)(b+r-1)}$

$p_1 = 5p_2$

$p_3 = 6p_2$

$r(r-1) = 5b(b-1)$ — (1)

$2br = 6b(b-1)$

$r^2 - r = 5b^2 - 5b$

$r = 3(b-1)$ — (2)

Put (2) in (1)

$9(b-1)^2 - 3(b-1) = 5b^2 - 5b$

$4b^2 - 16b + 12 = 0$

$b^2 - 4b + 3 = 0$

$b = 3, 1$

$r = 6, \cancel{0}$

$b = 3$

21. *From a pack of 52 cards an even number of cards is drawn. Show that the probability of half of these cards being red is $[52!/(26!)^2 - 1] / (2^{51} - 1)$.*

25. *(a)* $A_1, A_2, \ldots, A_n$ *are n independent events with* $P(A_i) = 1 - \frac{1}{\alpha^i}$; $i = 1, 2, \ldots, n$. *Find the value of* $P(A_1 \cup A_2 \ldots \cup A_n)$.

(b) Suppose the events $A_1, A_2, \ldots, A_n$ *are independent and that* $P(A_i) = \frac{1}{i+1}$ *for* $1 \le i \le n$. *Find the probability that none of the n events occurs, justifying each step in your calculation.*

25(*c*). *What is the probability that at least two out of n people will have the same birth date in a non-leap year?*

26. *A total of n shells are fired at a target. The probability of the ith shell hitting the target is* $p_i = 1, 2, 3, \ldots, n$. *Assuming that the n firings are n mutually independent events, find the probability that at least two shells out of n hit the target.*

33. *A certain drug manufactured by a company is tested chemically for its toxic effect. Let us define the events :*

A : *"the drug is toxic" and* B : *"The chemical test reveals that the drug is toxic."*

Let $P(A) = \theta$ *and* $P(B \mid A) = P(\bar{B} \mid \bar{A}) = 1 - \theta$ ...(*)

Show that the probability that the drug is not toxic given that the chemical test reveals that it is toxic, is independent of θ.

42. *An urn contains n white and m black balls, a second urn contains N white and M black balls. A ball is randomly transferred from the first to the second urn and then from the second to the first urn. If a ball is now selected randomly from the first urn, prove that the probability that it is white is :*

$$\frac{n}{n+m} + \frac{mN - nM}{(n+m)^2 (N+M+1)}.$$

50. *The sample space consists of {1, 2, 3, 4} and p_i is the probability assigned to i, $i = 1, 2, 3, 4$. Given that :*

$$p_1 = \frac{1}{\sqrt{2}} - \frac{1}{4}, \quad p_2 = \frac{1}{4}, \quad p_3 = \frac{3}{4} - \frac{1}{\sqrt{2}}, \quad p_4 = \frac{1}{4},$$

Examine the independence of the events :

$$E_1 = \{1, 3\}, \quad E_2 = \{2, 3\}, \quad E_3 = \{3, 4\}.$$