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How to calculate A^{-1}?

$$A^{-1} = \frac{Adj(A)}{|A|}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2\times 2}$$

$1 - 4 = -3$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

→ A singular matrix

Determinant of A

$$|A| = 2\times(4-3) - 3(2-1) + 1(3-2)$$
$$= 2\times(1) - 3(1) + 1(1)$$

$$A = \begin{bmatrix} 5 & 1 & 6 \\ 2 & 3 & 2 \\ 1 & 7 & 9 \end{bmatrix}$$

$$A' = \begin{bmatrix} 5 & 2 & 1 \\ 1 & 3 & 7 \\ 6 & 2 & 9 \end{bmatrix}$$

~~= 11~~

$2 - 3 + 1 = 0$

$$|A| = 5(9\times 3 - 2\times 7) - 1(9\times 2 - 2\times 1) + 6(7\times 2 - 3\times 1)$$
$$= 5(27-14) - 1(18-2) + 6(14-3)$$
$$= 5(13) - 1(16) + 6(11)$$
$$= 65 - 16 + 66$$
$$= 115 \neq 0$$ (A is Non-singular matrix)

... the following is a singular

Show that whether the following is a singular matrix or not

$|A| = 0 \rightarrow$ singular matrix

$|A| \neq 0 \rightarrow$ Non-singular matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 3 & 5 \end{bmatrix}$$

$|A| = 11 \neq 0$ (Non singular matrix).

Cofactors of A

$$\begin{bmatrix} a_{11} = (10-3) = 7 & a_{12} = -4 & a_{13} = 1 \\ a_{21} = -12 & a_{22} = 11 & a_{23} = -3 \\ a_{31} = 1 & a_{32} = -1 & a_{33} = 1 \end{bmatrix}$$

$$\text{Adj}(A) = \begin{bmatrix} 7 & -12 & 1 \\ -4 & 11 & -1 \\ 1 & -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}(A)}{|A|} = \frac{1}{11}\begin{bmatrix} 7 & -12 & 1 \\ -4 & 11 & -1 \\ 1 & -3 & 1 \end{bmatrix}$$

② $A = \begin{bmatrix} 6 & 7 & -9 \\ 2 & -8 & -4 \\ 1 & 2 & 3 \end{bmatrix}$

(a) calculate $A' = \begin{bmatrix} 6 & 2 & 1 \\ 7 & 8 & 2 \\ 9 & 4 & 3 \end{bmatrix}$

(b) $\boxed{\text{calculate } A^{-1}}$

(b) $|A| = 6(24-8) - 7(6-4) + 9(4-8)$

$= 6(16) - 7(2) + 9(-4)$

$= 96 - 14 - 36$

$= 60 - 14$

$= 46 \neq 0$ (Non singular matrix)

$= 46 \neq 0$ (matrix) -

Cofactor of A -

$a_{11} = 16$ $\quad a_{12} = -2$ $\quad a_{13} = -4$

$a_{21} = -3$ $\quad a_{22} = 9$ $\quad a_{23} = -5$

$a_{31} = -44$ $\quad a_{32} = -6$ $\quad a_{33} = 34$

$$C = \begin{bmatrix} 16 & -2 & -4 \\ -3 & 9 & -5 \\ -44 & -6 & 34 \end{bmatrix}$$

$$Adj(A) = C' = \begin{bmatrix} 16 & -3 & -44 \\ -2 & 9 & -6 \\ -4 & -5 & 34 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{Adj(A)}{|A|} = \frac{1}{46}\begin{bmatrix} 16 & -3 & -44 \\ -2 & 9 & -6 \\ -4 & -5 & 34 \end{bmatrix}$$

<u>Q</u> Solve the following system of equation using matrix form or matrix inversion method.

$$\begin{cases} 4x_1 + x_2 - 5x_3 = 8 \\ -2x_1 + 3x_2 + x_3 = 12 \\ 3x_1 - x_2 + 4x_3 = 5 \end{cases}$$

$$\begin{cases} -2x_1 + 3x_2 + x_3 = 12 \\ 3x_1 - x_2 + 4x_3 = 5 \end{cases}$$

Let us write the system of equation in matrix form.

$$\begin{bmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 5 \end{bmatrix}$$

$$AX = B \Rightarrow \boxed{X = A^{-1}B}$$

Now $|A| = \begin{vmatrix} 4 & 1 & -5 \\ -2 & 3 & 1 \\ 3 & -1 & 4 \end{vmatrix} = 4(12+1) - 1(-8-3) - 5(2-9)$

$= 4 \times 13 + 11 + 35$

$= 52 + 11 + 35$

$|A| = 98 \neq 0$

$$Adj(A) = \begin{bmatrix} 13 & 1 & 16 \\ 11 & 31 & 6 \\ -7 & 7 & 14 \end{bmatrix}$$

$a_{11} = 13 \quad a_{12} = 11 \quad a_{13} = -7$

$a_{21} = 1 \quad a_{22} = 31 \quad a_{23} = 7$

$a_{31} = 16 \quad a_{32} = 6 \quad a_{33} = 14$

$$\therefore A^{-1} = \frac{1}{|A|} Adj(A)$$

$$A^{-1} = \frac{1}{98} \begin{bmatrix} 13 & 1 & 16 \\ 11 & 31 & 6 \\ -7 & 7 & 14 \end{bmatrix} = \begin{bmatrix} \frac{13}{98} & \frac{1}{98} & \frac{16}{98} \\ \frac{11}{98} & \frac{31}{98} & \frac{6}{98} \\ \frac{-1}{14} & \frac{1}{14} & \frac{1}{7} \end{bmatrix}$$

Now $X = A^{-1} B$

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$$X = \begin{bmatrix} 13/98 & 1/98 & 16/98 \\ 11/98 & 31/98 & 06/98 \\ -1/14 & 1/14 & 1/7 \end{bmatrix} \begin{bmatrix} 8 \\ 12 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{13}{98}\times 8 + \frac{1}{98}\times 12 + \frac{16}{98}\times 5 \\ \frac{11}{98}\times 8 + \frac{31}{98}\times 12 + \frac{06}{98}\times 5 \\ \frac{-1}{14}\times 8 + \frac{1}{14}\times 12 + \frac{1}{7}\times 5 \end{bmatrix} = \begin{bmatrix} \frac{196}{98} \\ \frac{490}{98} \\ \frac{14}{14} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

$\therefore x_1 = 2$, $x_2 = 5$ and $x_3 = 1$

$$4x_1 + x_2 - 5x_3 = 4\times 2 + 5 - 5\times 1$$
$$= 8 = RHS$$

(verified).

Cramer's Rule.

$$\begin{bmatrix} \boxed{\begin{matrix} 4 \\ -2 \\ 3 \end{matrix}} & \begin{matrix} 1 \\ 3 \\ -1 \end{matrix} & \begin{matrix} -5 \\ 1 \\ 4 \end{matrix} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 5 \end{bmatrix}$$

$\Delta = 98$

$$\Delta x_1 = \begin{vmatrix} 8 & 1 & -5 \\ 12 & 3 & 1 \\ 5 & -1 & 4 \end{vmatrix} = 196$$

$$\Delta x_2 = \begin{vmatrix} 4 & 8 & -5 \\ -2 & 12 & 1 \\ 3 & 5 & 4 \end{vmatrix} = 490$$

$$\Delta x_3 = \begin{vmatrix} 4 & 1 & 8 \\ -2 & 3 & 12 \\ 3 & -1 & 5 \end{vmatrix} = 98$$

$$\left.\begin{aligned} x_1 &= \frac{\Delta x_1}{\Delta} = \frac{196}{98} = 2 \\ x_2 &= \frac{\Delta x_2}{\Delta} = \frac{490}{98} = 5 \\ x_3 &= \frac{\Delta x_3}{\Delta} = \frac{98}{98} = 1 \end{aligned}\right\} \underline{\underline{\text{ans}}}$$