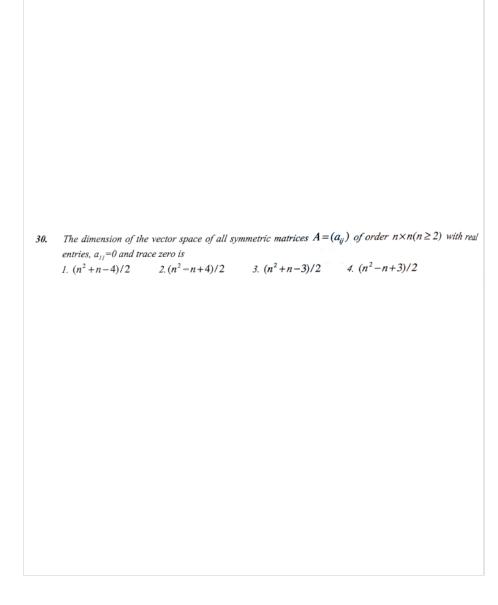
## 22 JUNE PG

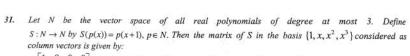
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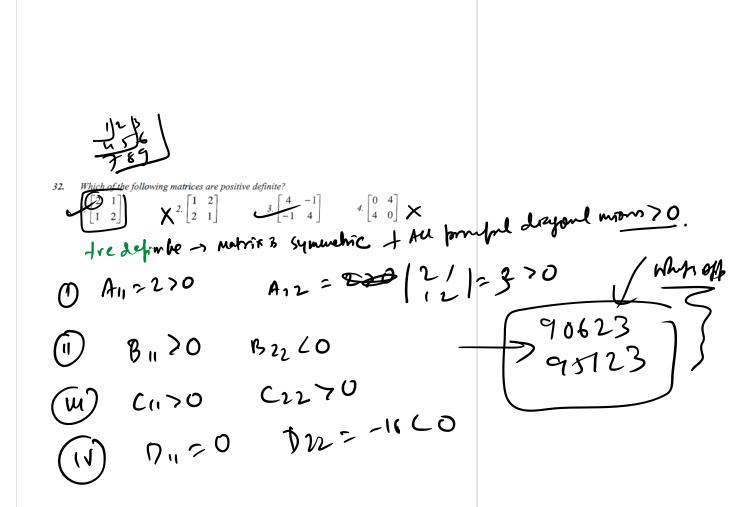


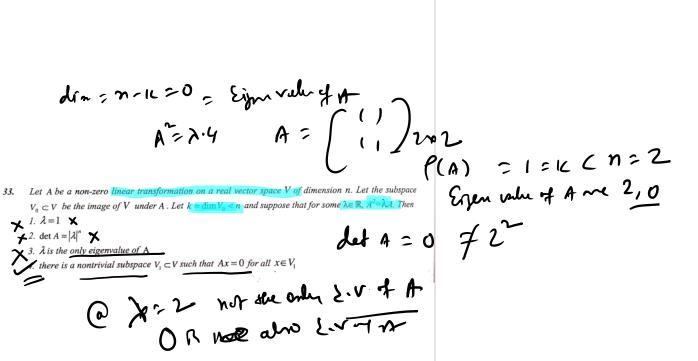
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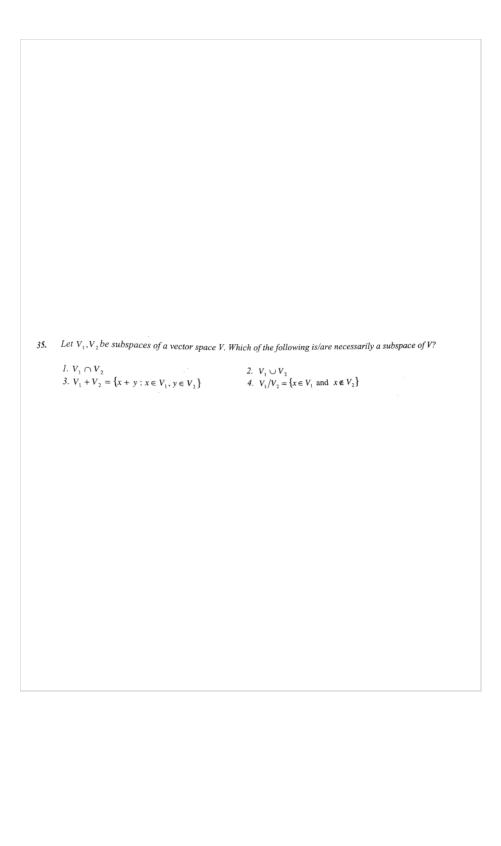


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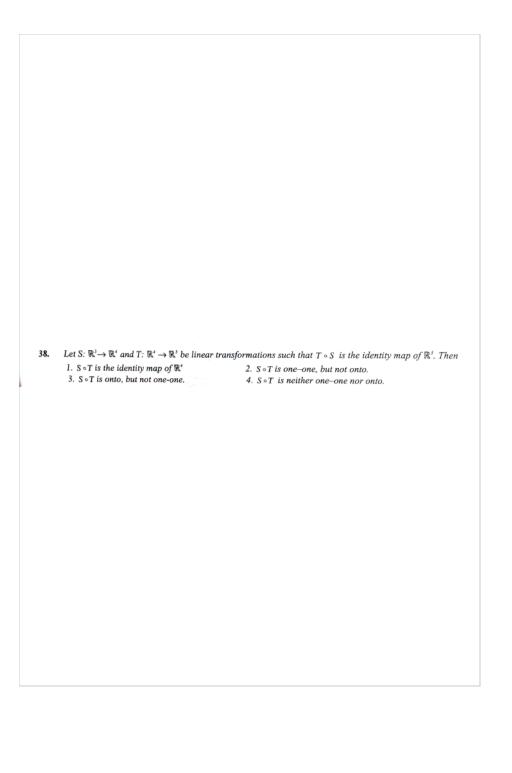


34.	Let C be an n×n re of the vector space W 1. 2n	al matrix. Let W be to 7 is 2. atmost n	the vector space span. $3. n^2$	ned by $\{I, C, C^2,, C^{2n}\}$ . The dit $A$ . atmost $2n$	imension



Enry we meeting the proper of the following inference of the following infe

27				ull vVv vod vrstnings 167	C. M (D) \ M (D)
37.	is a linear transforma the rank of T is	ger and let $M_n(\mathbb{R})$ denotion such that $T(A)=0$ .  2. $\frac{n(n-1)}{2}$	the space of a whenever $A \in M_n$ .  3. $n$	ull n×n real matrices. If 1 限) is symmetric or skew 4.0	: M <sub>n</sub> (K) -M <sub>n</sub> (K) p-symmetric, then



39.	Let V be a 3-di 1-dimensional 1. 13	mensional vector space subspaces of V is 2. 26	over the field $F_3=\mathbb{Z}/3\mathbb{Z}$ 3. 9	of 3 elements. The number 4. 15	of distinct

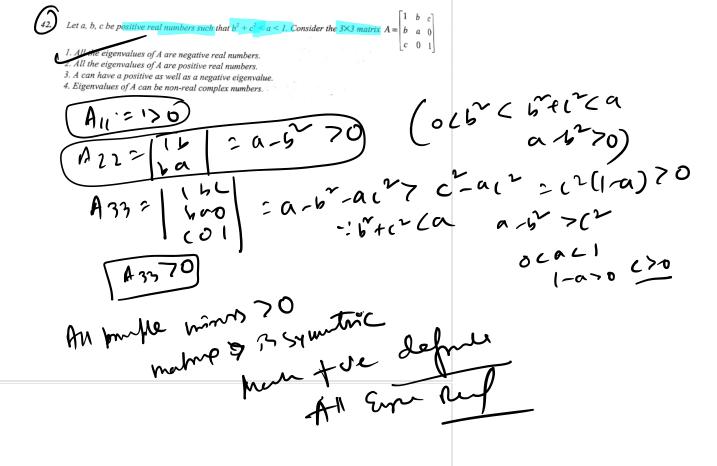
**40.** Let V be the inner product space consisting of linear polynomials,  $p:[0,1] \to \mathbb{R}$  (i.e., V consists of polynomials p of the form p(x) = ax + b; a,  $b \in \mathbb{R}$ ), with the inner product defined by  $\langle p,q \rangle = \int_{0}^{1} p(x) q(x) dx \text{ for } p,q \in V. \text{ An orthonormal basis of } V \text{ is}$ 1.  $\{1,x\}$ 2.  $\{1,x\sqrt{3}\}$ 3.  $\{1,(2x-1)\sqrt{3}\}$ 4.  $\{1,x-\frac{1}{2}\}$ 

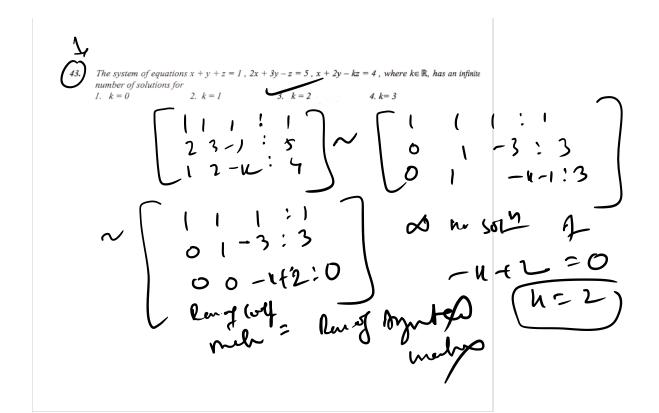
[0 0 0 1] 1 0 0 0 0 1 0 0 . Then the rank of the 4 $imes_4$ 41. Let f(x) be the minimal polynomial of the 4×4 matrix A =0 0 1 0 Every metry Sahrfres on missional follymounts

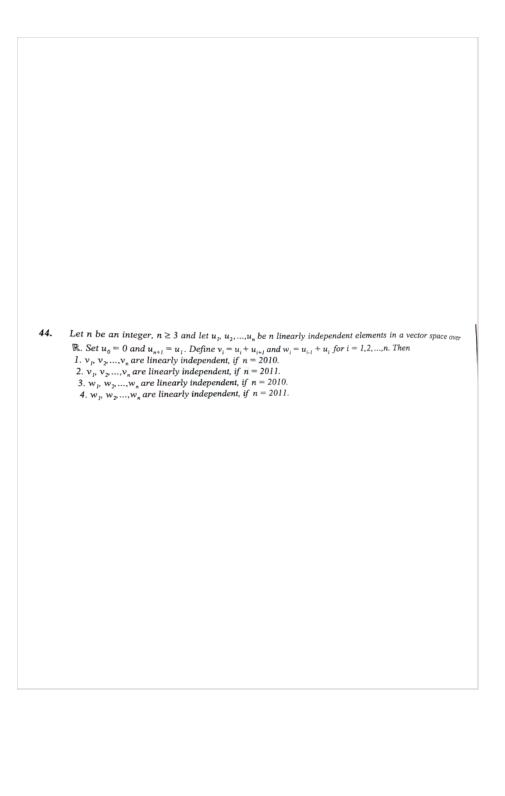
(A) = Zero Mahry >0

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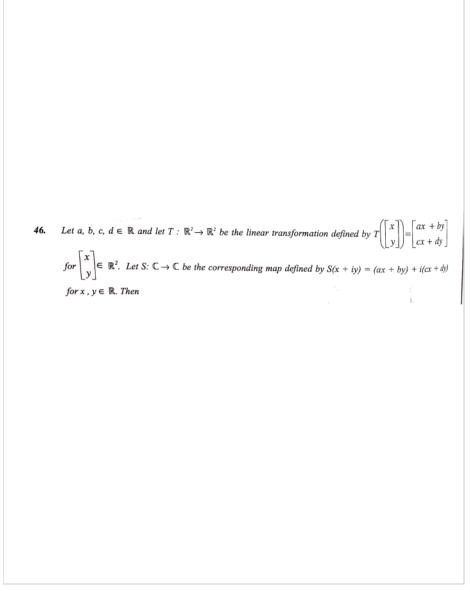
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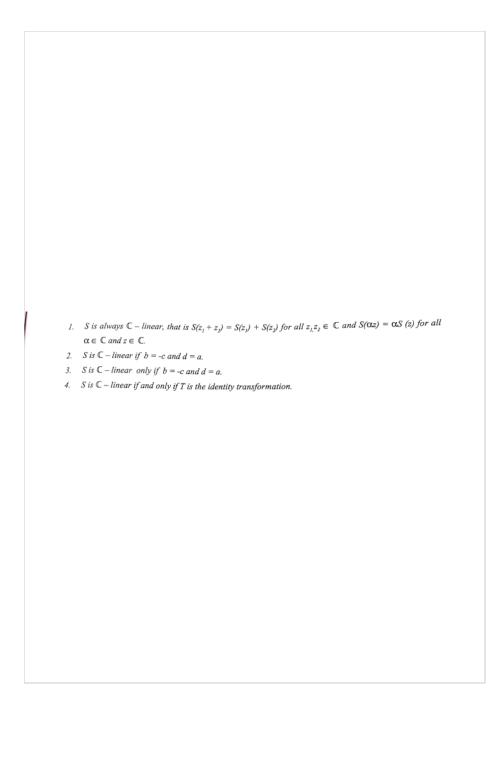






45.	Let $V$ and $W$ be finite-dimensional vector spaces over $\mathbb{R}$ and let $T_i : V \to V$ and $T_2 : W \to W$ be linear transformations whose minimal polynomials are given by $f_i(x) = x^3 + x^2 + x + 1$ and $f_2(x) = x^4 - x^2 - 2$ . Let $T: V \oplus W \to V \oplus W$ be the linear transformation defined by $T((v, w)) = (T_i(v), T_2(w))$ for $(v, w) \in V \oplus W$ and let $f(x)$ be the minimal polynomial of $T$ . Then $T: V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \oplus W \to V \oplus W$ and let $T: V \to V \to V \oplus W$ and let $T: V \to V \to V \oplus W$ and let $T: V \to V \to V \oplus W$ and let $T: V \to V \to V \to V \oplus W$ and let $T: V \to V $





<ul> <li>47. Let A = [a<sub>ij</sub>] be an n × n complex matrix and let A* denote the conjugate transpose of A. Which of following statements are necessarily true?</li> <li>1. If A is invertible, then tr(A*A) ≠ 0, i.e., the trace of A*A is non zero.</li> <li>2. If tr(A*A) ≠ 0, then A is invertible.</li> <li>3. If   tr(A*A)  &lt; n², then  a<sub>ij</sub>  &lt; 1 for some i.j.</li> <li>4. If tr(A*A) = 0, then A is the zero matrix.</li> </ul>	of the

48.	Let n be a positive integer and V be an $(n + 1)$ -dimensional vector space over $\mathbb{R}$ . If $\{e_1e_2,e_{n+l}\}$ is a basis of V and T: $V \rightarrow V$ is the linear transformation satisfying $T(e_i) = e_{i+1}$ for $i=1, 2,, n$ and $T(e_{n+1}) = 0$ . Then  1. trace of T is non-zero.  2. rank of T is n.  3. nullity of T is 1  4. $T^n = T \circ T \circ \circ T$ (n times) is the zero map.
	The second point of the se

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Let A and B be  $n \times n$  real matrices such that AB = BA = 0 and A + B is invertible. Which of the following are always true? 1. rank(A) = rank(B)3. pullity(A) + nullity(B) = nA=I B=O AB=BA PLB7=0 p(A)=1 d (mm) = de( A-5) P(ATB) (P(A)+P(B) d (m)70 P(AB) >, P(M) +P(B) ~ ()
A+B is smoother P(A+B)=1 de (A-B) \$0 07, P(A) +(P(B)-N7) P(A)+P(B) 7,~ ((a)+PLB) un NA9) 20 0(A) 1P(B) 2n P(B)+P(B)=N

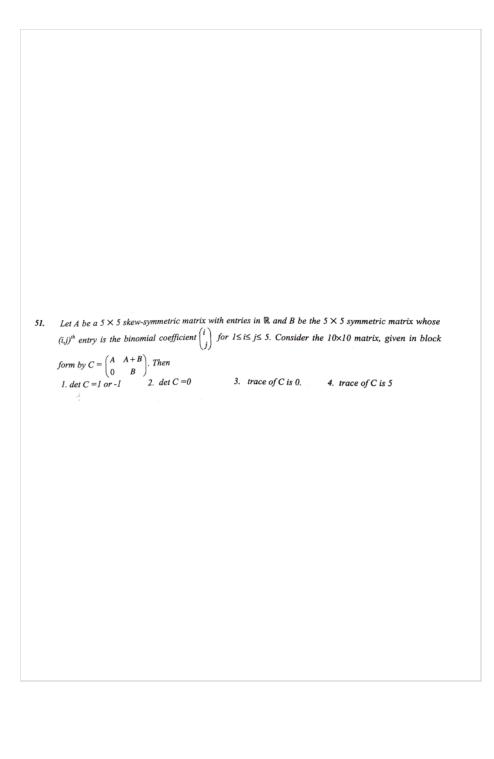
P(B)+P(B)=N

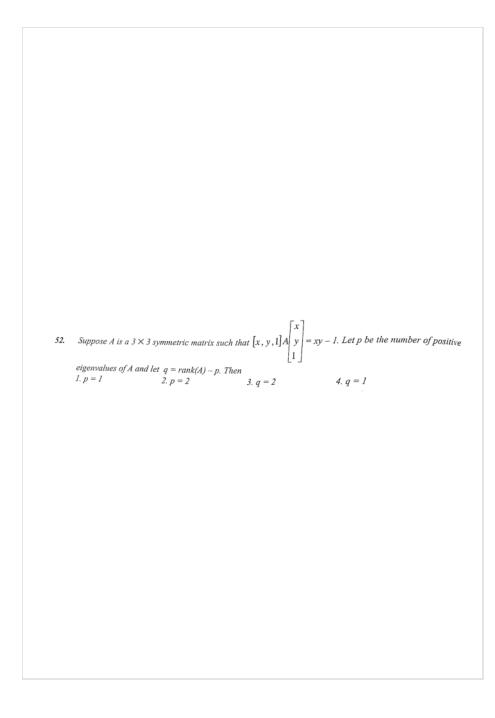
P(B)+N(B)=N

P(B)+N(B)=N planth(al=n) (n-N(A))(n-N(B)) = N (n-N(A))(n-N(B)) = N

<sup>50.</sup> Let n be an integer  $\geq 2$  and let  $M_n(\mathbb{R})$  denote the vector space of  $n \times n$  real matrices. Let  $B \in M_n(\mathbb{R})$  be an orthogonal matrix and let B' denote the transpose of B. Consider  $W_n = \{B' \mid AB : A \in M \mid \mathbb{R}\}\}$ . Which of

- 50. Let n be an integer  $\geq 2$  and let  $M_n(\mathbb{R})$  denote the vector space of  $n \times n$  real matrices. Let  $B \in M_n(\mathbb{R})$  be an orthogonal matrix and let B' denote the transpose of B. Consider  $W_B = \{B'AB : A \in M_n(\mathbb{R})\}$ . Which of the following are necessarily true?
  - 1.  $W_B$  is the subspace of  $M_n(\mathbb{R})$  and dim  $W_B \leq rank(B)$ .
  - 2.  $W_B$  is the subspace of  $M_n(\mathbb{R})$  and dim  $W_B = rank(B) rank(B')$ .
  - 3.  $W_B = M_n(\mathbb{R}).$
  - 4.  $W_B$  is not a subspace of  $M_n$  ( $\mathbb{R}$ ).

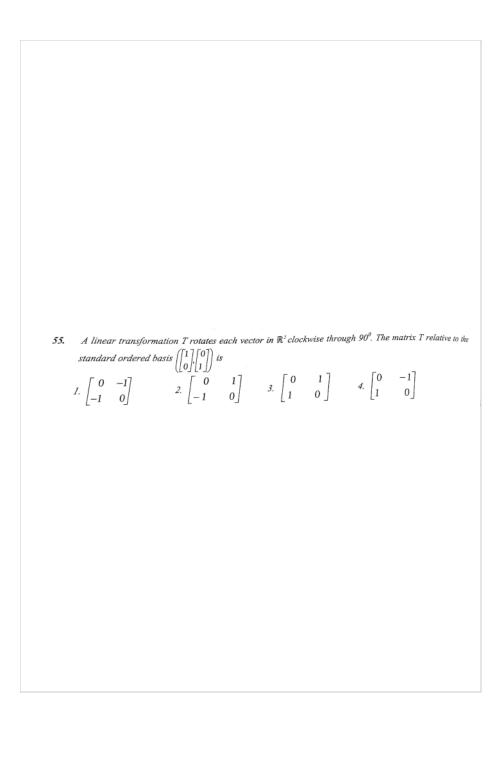


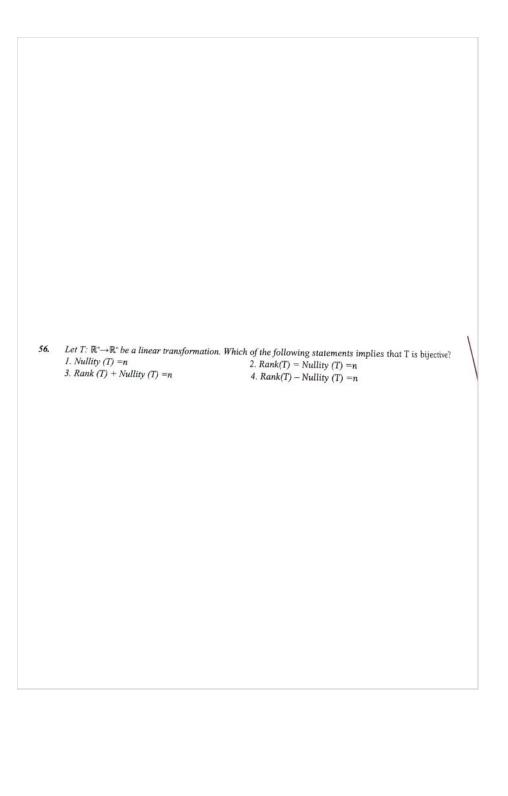


53. Let  $A = \begin{bmatrix} 1 & 3 & 5 & a & 13 \\ 0 & 1 & 7 & 9 & b \\ 0 & 0 & 1 & 11 & 15 \end{bmatrix}$ , where  $a, b \in \mathbb{R}$ . Choose the correct statement.

- There exist values of a and b for which the columns of A are linearly independent.
   There exist values of a and b for which Ax=0 has x=0 as the only solution.
   For all values of a and b, the rows of A span a 3-dimensional subspace of R<sup>2</sup>.
   There exist values of a and b for which rank (A)=2.

54.	Consider $\mathbb{R}^3$ with the standard inner product. Let $W$ be the subspace of $\mathbb{R}^3$ spanned by $(1,0,-1)$ . Which of the following is a basis for the orthogonal complement of $W$ ?  1. $\{(1,0,1),(0,1,0)\}$ 2. $\{(1,2,1),(0,1,1)\}$ 3. $\{(2,1,2),(4,2,4)\}$ 4. $\{(2,-1,2),(1,3,1),(-1,-1,-1)\}$



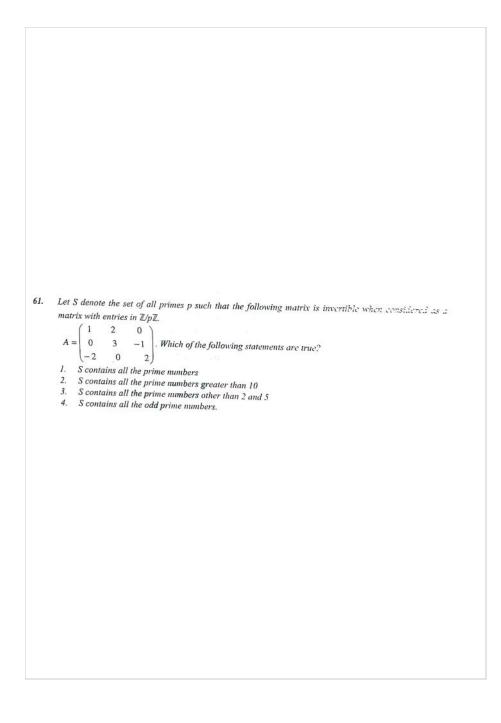


57.	Let $A \in M_{10}(\mathbb{C})$ , the vector space of $10 \times 10$ matrices with entries in $\mathbb{C}$ . Let $W_A$ be the subspace of $M_{10}(\mathbb{C})$ spanned by $\{A^n \mid n \geq 0\}$ . Choose the correct statements. 1. For any $A$ , $\dim(W_A) \leq 10$ 2. For any $A$ , $\dim(W_A) < 10$ 3. For some $A$ , $10 < \dim(W_A) < 100$ 4. For some $A$ , $\dim(W_A) = 100$	

58.	Let $A$ be a complex $3 \times 3$ matrix with $A^1 = -1$ . 1. A has three distinct eigenvalues 3. A is triangularizable over $\mathbb{C}$	Which of the following statements are correct?  2. A is diagonalizable over $\mathbb{C}$ 4. A is non-singular	

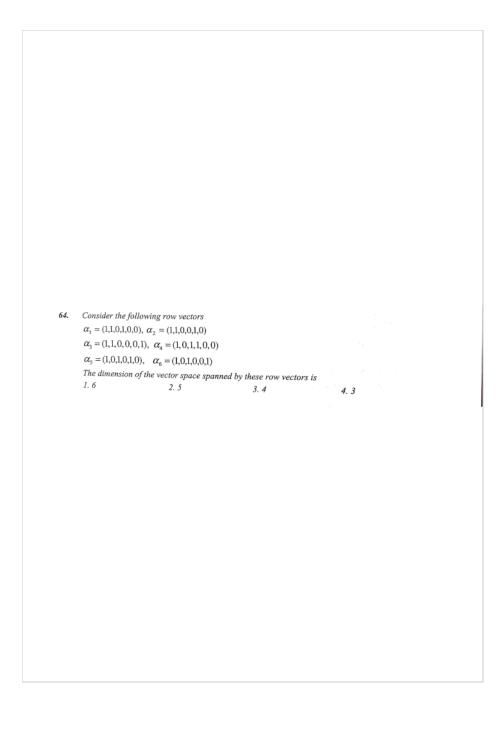


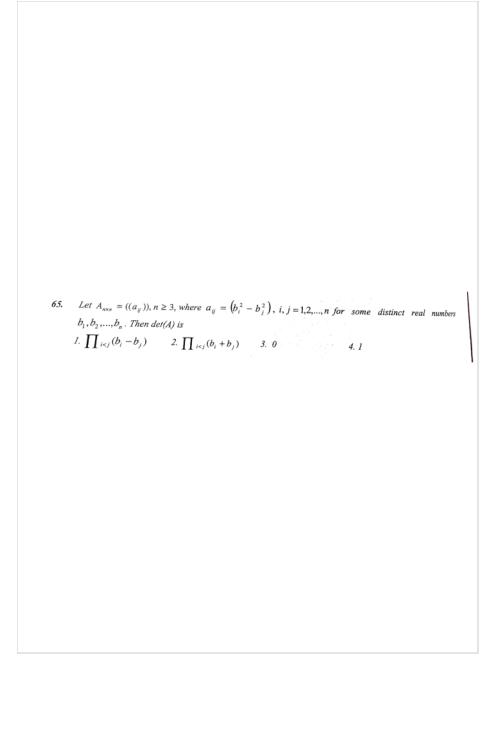
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60.	A tim polym 1. 2. 3. 4.	operator	se all correct option T is uniquely detern Jordan blocks in th ed by T on the quot	ns. nined by the given e Jordan decompo tient space V/Ker(1	information sition of T T-51) is nilpotent,	$x'(x - 5)^2$ and min. where I is the idenultiple of the iden	tity



62. For a fixed positive integer n > 3.	the second with the complete the second	
identity matrix and J is the n×n matrix NOT true?  1. A*=A for every positive integer k.  3. Rank(A)+Rank(I - A)=n.	et $A$ be the $n \times n$ matrix defined as $A = I - \frac{1}{n}J$ , where $I$ is the ix with all entries equal to $I$ . Which of the following statements is 2. Trace $(A)=n-1$ 4. $A$ is invertible.	
		1

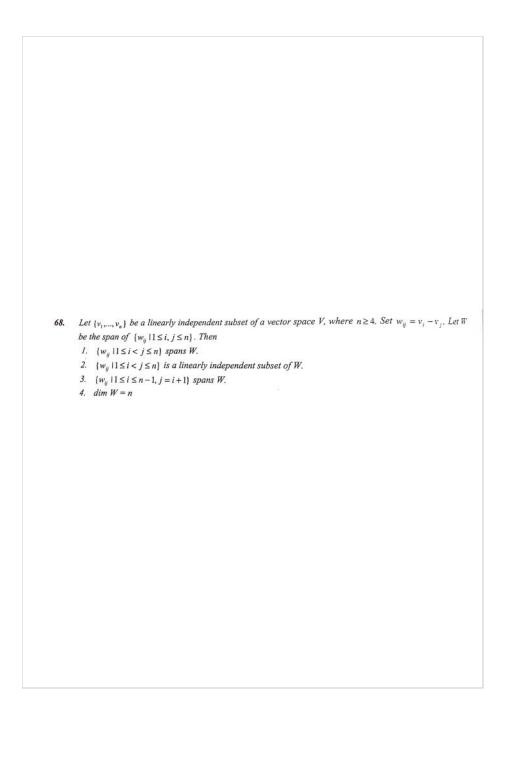
63.	Let A be a $5 \times 4$ matrix with real entries such that $A \underline{x} = \underline{0}$ if and only if $\underline{x} = \underline{0}$ , where $\underline{x}$ is a $4 \times 1$ vector
03.	and $\underline{0}$ is a null vector. Then, the rank of $A$ is  1. 4  2. 5  3. 2  4. 1





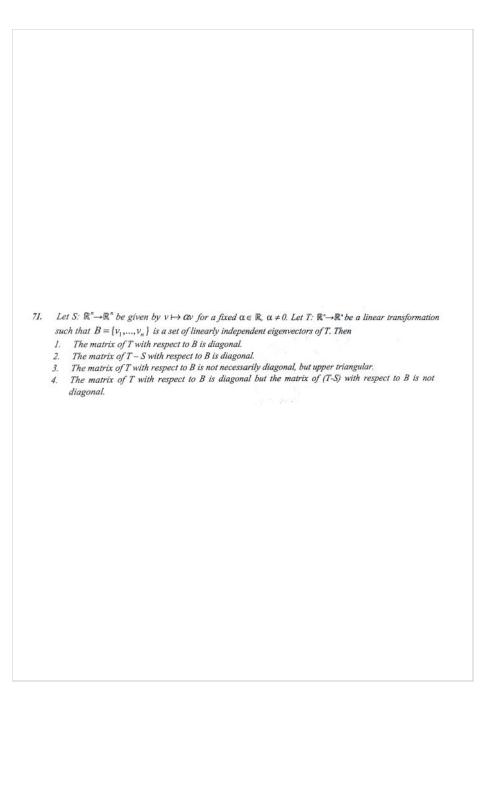
<ol> <li>Let A be an n×n matrix with real entries. Which of the following is correct?</li> <li>If A²=O, then A is diagonalizable over complex numbers.</li> <li>If A²=I, then A is diagonalizable over real numbers</li> <li>If A²=A, then A is diagonalizable only over complex numbers.</li> <li>The only matrix of size n satisfying the characteristic polynomial of A is A.</li> </ol>			
	66.	<ol> <li>If A²=O, then A is diagonalizable over complex numbers.</li> <li>If A²=I, then A is diagonalizable over real numbers</li> <li>If A²=A, then A is diagonalizable only over complex numbers</li> </ol>	

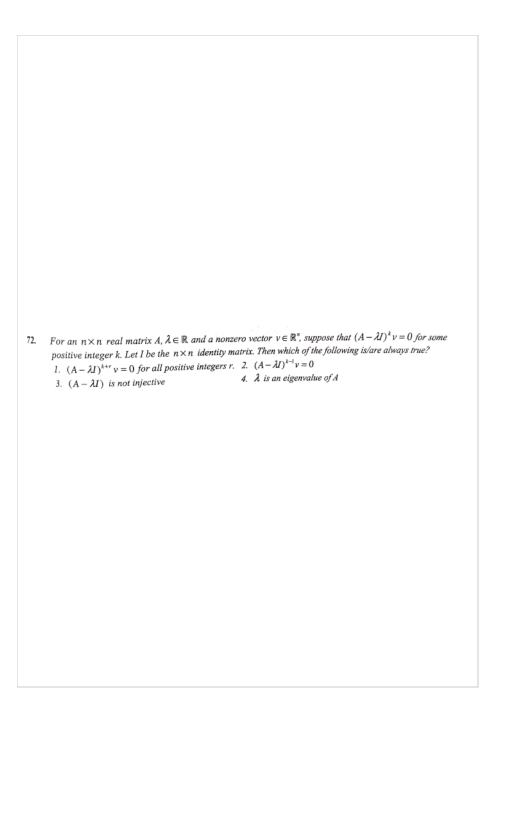
<ul> <li>67. Let A be a 4×4 invertible real matrix. Which of the following is NOT necessarily true?</li> <li>1. The rows of A form a basis of ℝ'.</li> <li>2. Null space of A contains only the 0 vector.</li> <li>3. A has 4 distinct eigenvalues.</li> <li>4. Image of the linear transformation x → Ax on ℝ' is ℝ'.</li> </ul>



<ul> <li>69. For any real square matrix M, let λ<sup>+</sup>(M) be the number of positive eigenvalues of M counting multiplicities. Let A be an n×n real symmetric matrix and Q be an n×n real invertible matrix. Then</li> <li>1. Rank A=Rank Q<sup>T</sup>AQ</li> <li>2. Rank A=Rank Q<sup>I</sup>AQ</li> <li>3. λ<sup>+</sup>(A) = λ<sup>+</sup>(Q<sup>T</sup>AQ)</li> <li>4. λ<sup>+</sup>(A) = λ<sup>+</sup>(Q<sup>-1</sup>AQ)</li> </ul>	

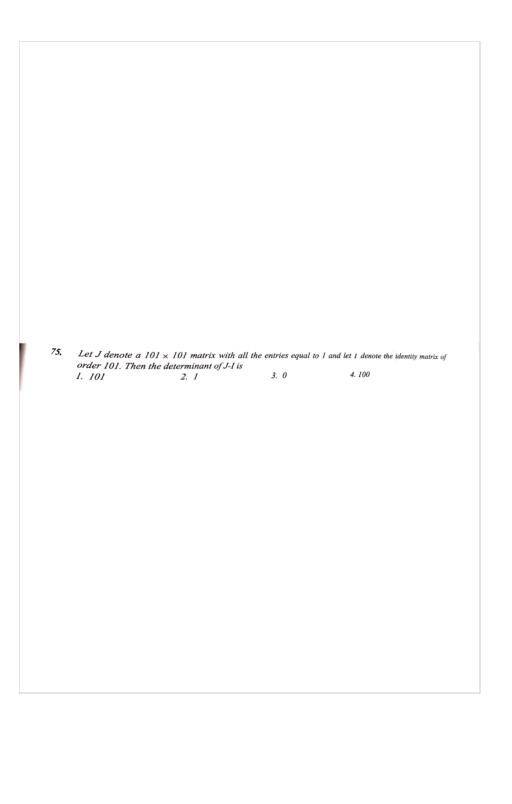


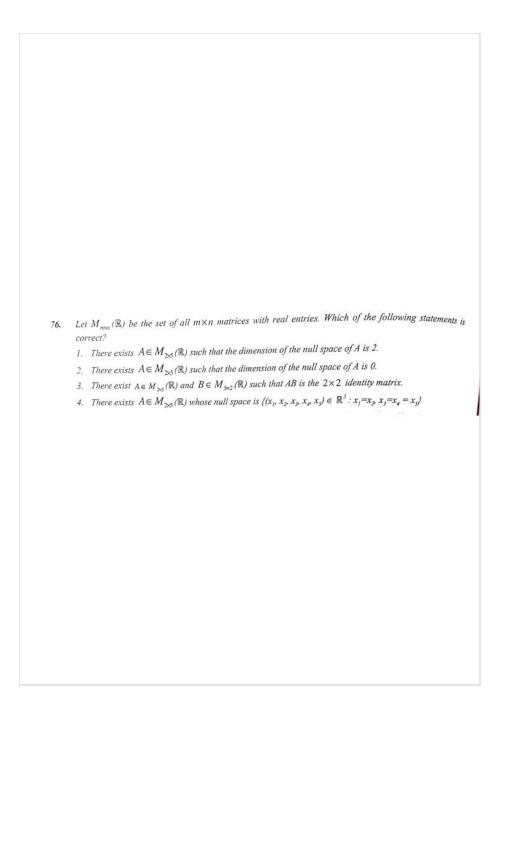




	to the second of the following and subangers of 1/2
73.	Let y be a non-zero vector in an inner product space V. Then which of the following are subspaces of V?  1. $\{x \in V \mid x, y >= 0\}$ .  2. $\{x \in V \mid x, y >= 1\}$ .  3. $\{x \in V \mid x, z >= 0 \text{ for all } z \text{ such that } \langle z, y \rangle = 0\}$ .  4. $\{x \in V \mid x, z \rangle = 1 \text{ for all } z \text{ such that } \langle z, y \rangle = 1\}$ .

74.	Let A be a 5×5 mat sum of all the entries 1. 3	rix with real entries so in A <sup>3</sup> is 2. 15	uch that the sum of the	entries in each row of $A$ is 1. Then the 4. 125	





77. For the matrix A as given below, which of them satisfy 
$$A^6=I$$
?

$$I. \ \ A = \begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} & 0\\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0\\ 0 & 0 & 1 \end{pmatrix}$$

For the matrix A as given below, which of them satisfy 
$$A^6 = I$$
?

1.  $A = \begin{pmatrix} \cos \frac{\pi}{4} & \sin \frac{\pi}{4} & 0 \\ -\sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 

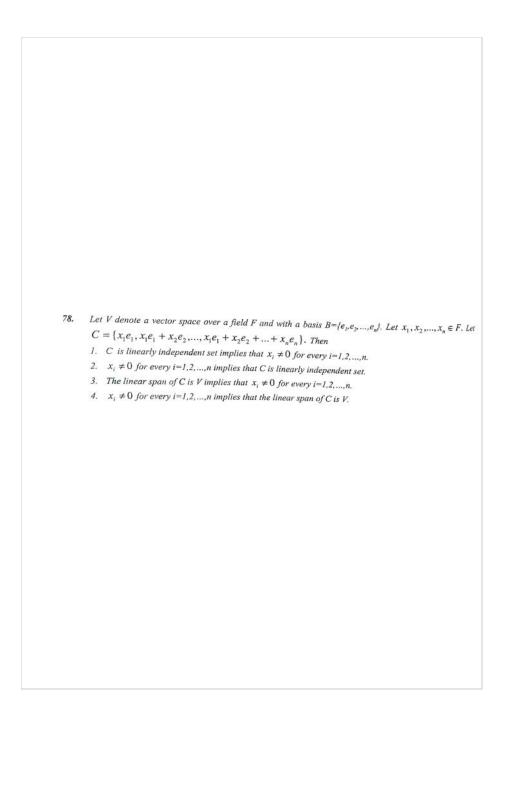
2.  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \frac{\pi}{3} & \sin \frac{\pi}{3} \\ 0 & -\sin \frac{\pi}{3} & \cos \frac{\pi}{3} \end{pmatrix}$ 

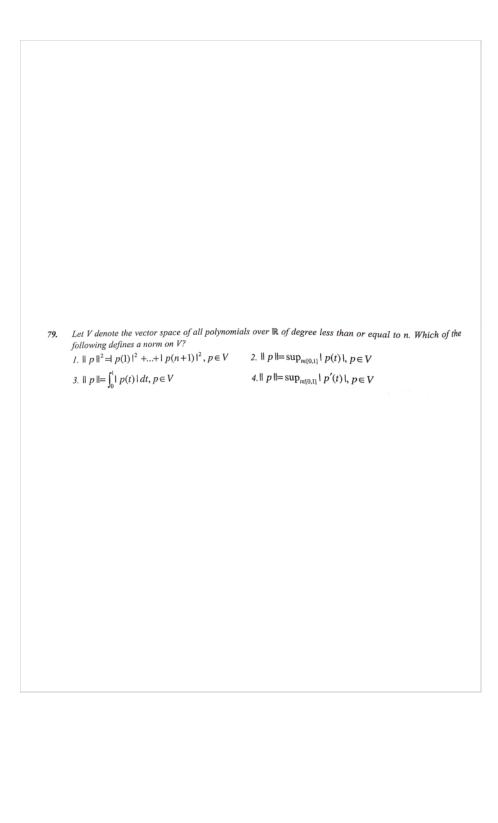
3.  $A = \begin{pmatrix} \cos \frac{\pi}{6} & 0 & \sin \frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin \frac{\pi}{6} & 0 & \cos \frac{\pi}{6} \end{pmatrix}$ 

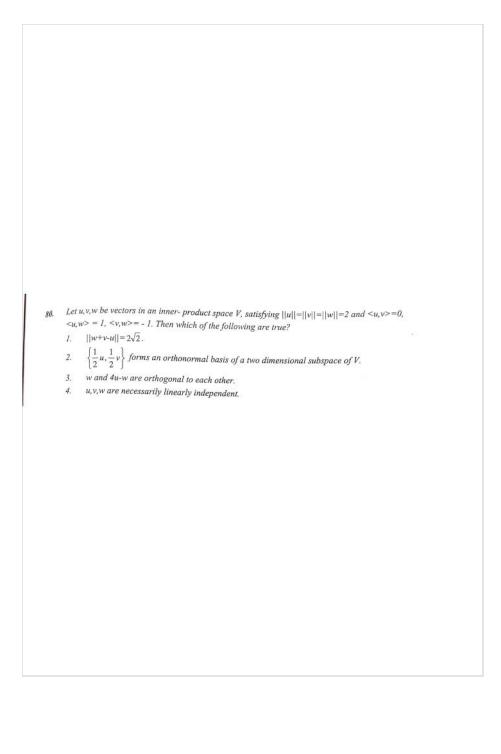
4.  $A = \begin{pmatrix} \cos \frac{\pi}{2} & \sin \frac{\pi}{2} & 0 \\ -\sin \frac{\pi}{2} & \cos \frac{\pi}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 

3. 
$$A = \begin{pmatrix} \cos\frac{\pi}{6} & 0 & \sin\frac{\pi}{6} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{6} & 0 & \cos\frac{\pi}{6} \end{pmatrix}$$

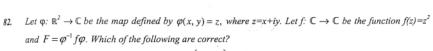
4. 
$$A = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & 0\\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0\\ 0 & 0 & 1 \end{bmatrix}$$



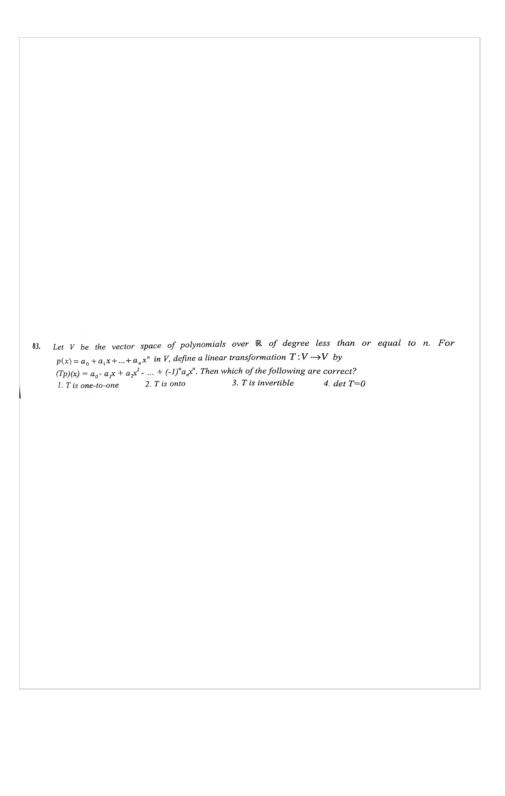




81.	Let A diago	A be a 4×4 matrix over $\mathbb{C}$ such that $rank(A)=2$ and $A^3=A^2\neq 0$ . Suppose that $A$ is not onalizable. Then  One of the Jordan blocks of the Jordan canonical form of $A$ is $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ . $A^2=A\neq 0$ .
	3. 4.	There exists a vector $v$ such that $Av \neq 0$ but $A^2v = 0$ . The characteristic polynomial of $A$ is $x^4 - x^3$ .

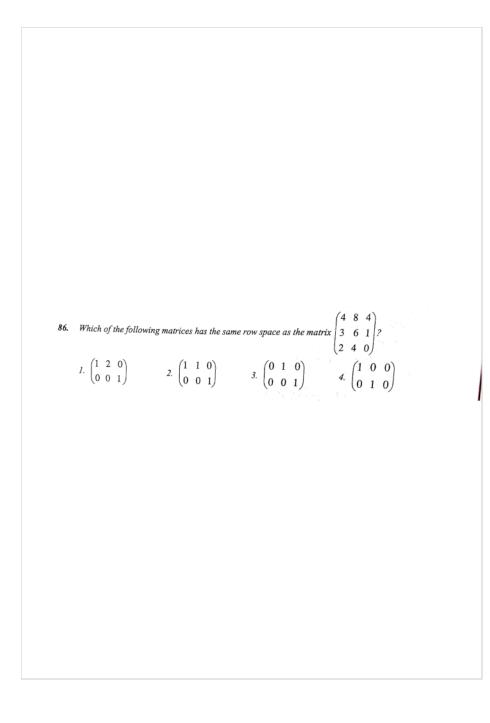


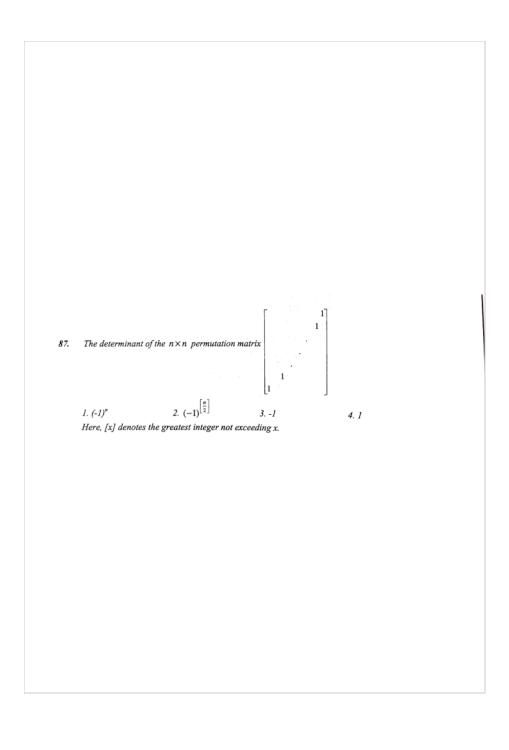
- 1. The linear transformation  $T(x, y) = 2\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$  represents the derivative of F at (x, y).
- 2. The linear transformation  $T(x, y) = 2 \begin{pmatrix} x & y \\ y & x \end{pmatrix}$  represents the derivative of F at (x,y).
- 3. The linear transformation T(z)=2z represents the derivative of f at  $z \in \mathbb{C}$ .
- 4. The linear transformation T(z)=2z represents the derivative of f only at 0.

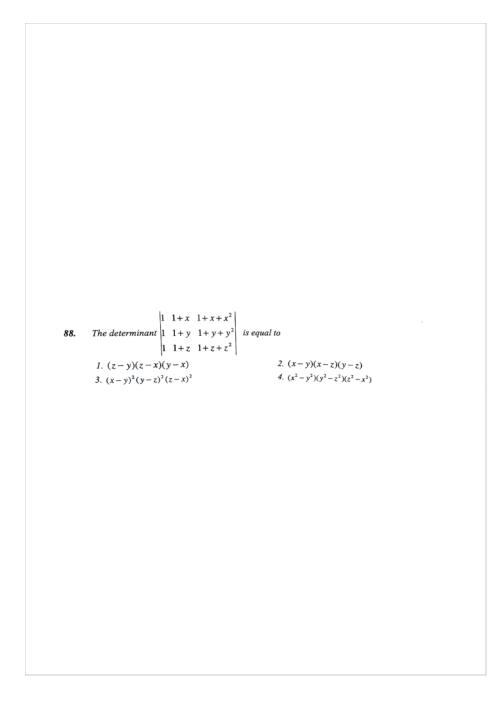


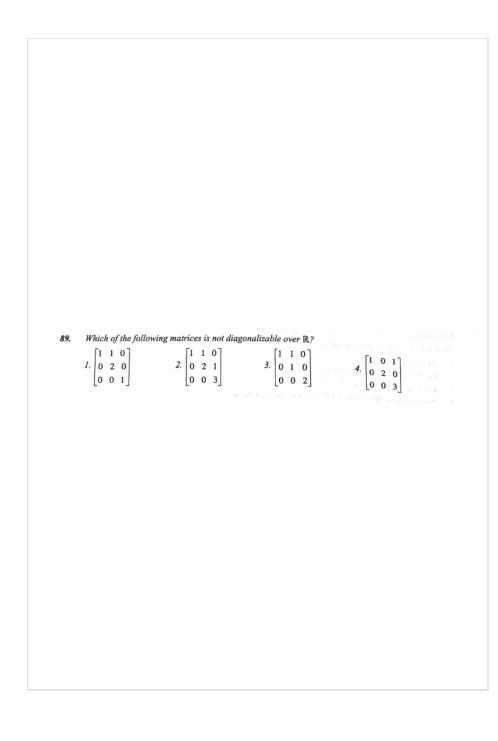
84.	Consider a homogeneous system of linear equations Ax= Then which of the following statements are always true?  1. Ax=0 has a solution.  2. Ax=0 has no non-zero solution.  3. Ax=0 has a non-zero solution.  4. Dimension of the space of all solutions is at least n-m.	0, where A is an m	n×n real matrix and n > m.

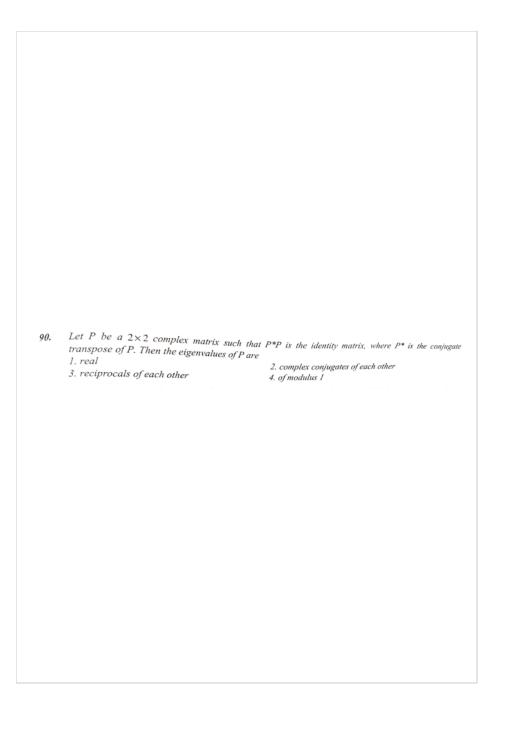
85.	Let A, B be $n \times n$ matrices such that $BA + B^2 = I - BA^2$ , where I is the $n \times n$ identity matrix. Which of the following is always true?  1. A is nonsingular 2. B is nonsingular 3. $A+B$ is nonsingular 4. $AB$ is nonsingular



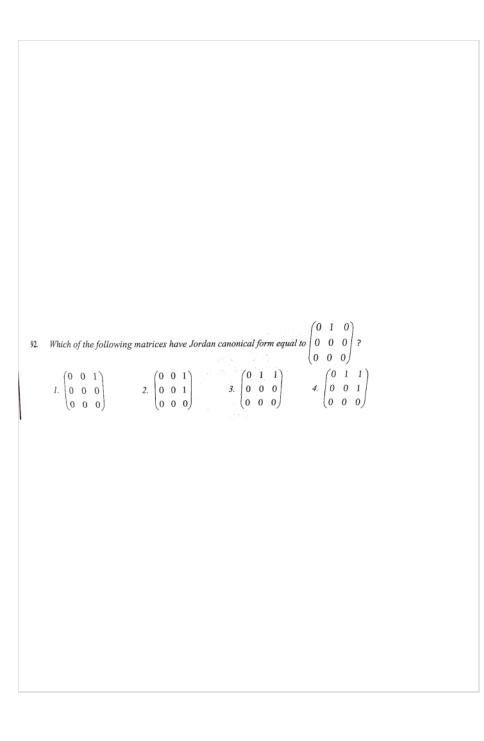




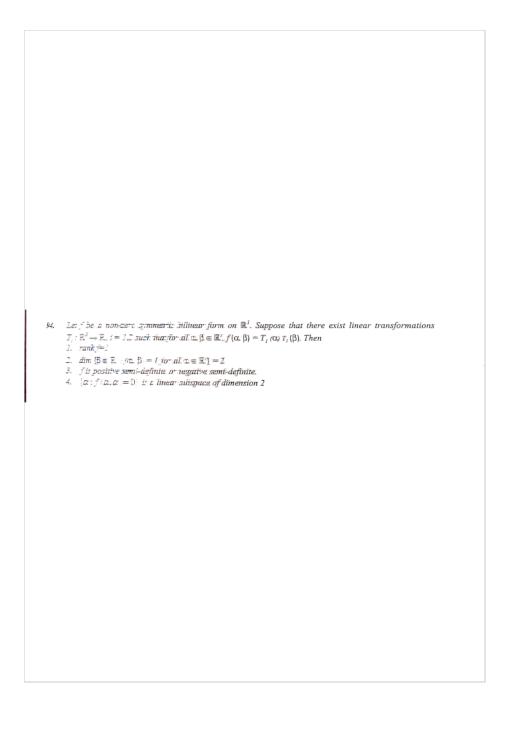


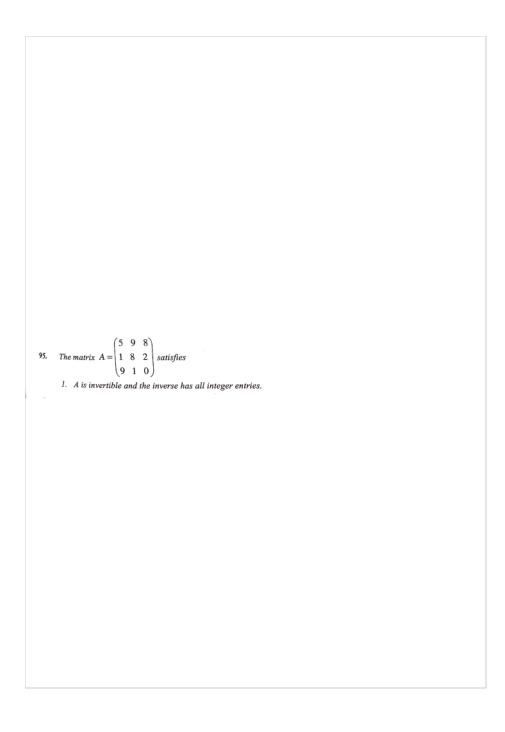


91.	Let $A$ be a real $n \times n$ orthogonal matrix, that is, $A^tA = AA^t = I_n$ , the $n \times n$ identity matrix. Which of the following statements are necessarily true?  1. $\langle Ax, Ay \rangle = \langle x, y \rangle  \forall x, y \in \mathbb{R}^n$ 2. All eigenvalues of $A$ are either $+1$ or $-1$ .  3. The rows of $A$ form an orthonormal basis of $\mathbb{R}^n$ .  4. $A$ is diagonalizable over $\mathbb{R}$ .	



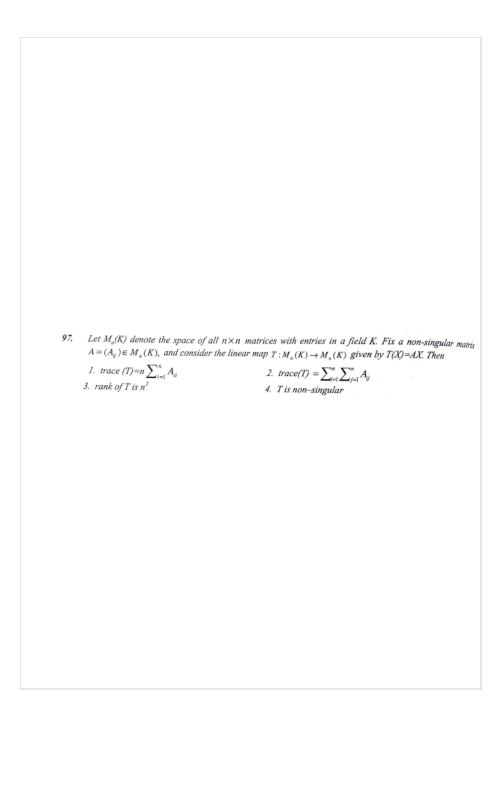
93.	Let A be a 3×4 and b be a 3×1 matrix with integer entries. Suppose that the system Ax=b has a complex solution. Then  1. Ax=b has an integer solution  2. Ax=b has a rational solution  3. The set of real solutions to Ax=0 has a basis consisting of rational solutions.  4. If b≠0, then A has positive rank.	

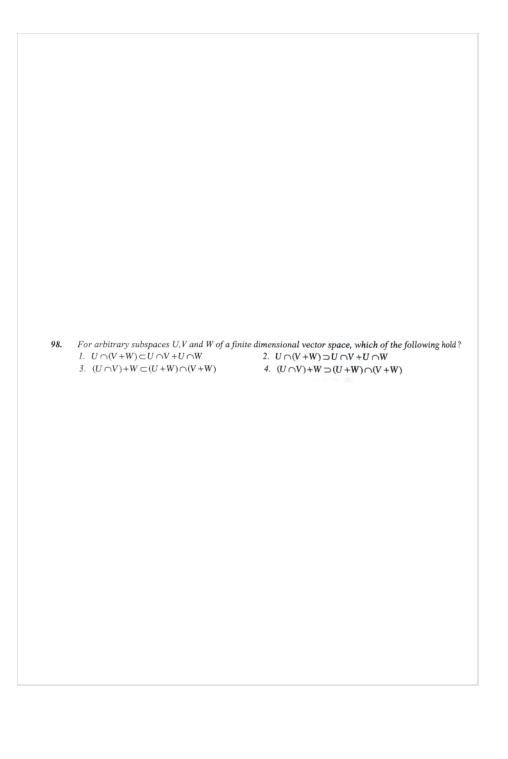






96.	Let A be $5\times 5$ matrix and let B be obtained by changing one element of A. Let r and s be the ranks of and B respectively. Which of the following statements is/are correct?  1. $s \le r+1$ 2. $r-1 \le s$ 3. $s = r-1$ 4. $s \ne r$





99.	Let A be a $4\times7$ real matrix and B be a $7\times4$ real matrix such that $AB=I_4$ , where $I_4$ is the $4\times4$ identity matrix. Which of the following is/are always true?  1. $\operatorname{rank}(A)=4$ 2. $\operatorname{rank}(B)=7$ 3. $\operatorname{nullity}(B)=0$ 4. $\operatorname{BA}=I_7$ , where $I_7$ is the $7\times7$ identity matrix

