Q. $q = AL^{\alpha}K^{\beta}$, $0<\alpha<1$, $0<\beta<1$, $A>0$. $[\alpha+\beta$

$$|H| = \begin{vmatrix} f_{LL} & f_{LK} \\ f_{LK} & f_{KK} \end{vmatrix} = f_{LL}f_{KK} - f_{LK}^2.$$

$$f_L = \frac{\partial f}{\partial L} = A\alpha L^{\alpha-1}K^{\beta}. \Rightarrow f_{LL} = \frac{\partial^2 f}{\partial L^2} = A\alpha(\alpha-1)L^{\alpha-2}K^{\beta}.$$

$$f_K = \frac{\partial f}{\partial K} = A\beta L^{\alpha}K^{\beta-1} \Rightarrow f_{KK} = \frac{\partial^2 f}{\partial K^2} = A\beta(\beta-1)L^{\alpha}K^{\beta-2}.$$

$$f_{LK} = \frac{\partial}{\partial L}\left(\frac{\partial f}{\partial K}\right) = \frac{\partial}{\partial L}\left(A\beta L^{\alpha}K^{\beta-1}\right) = A\alpha\beta L^{\alpha-1}K^{\beta-1}$$

$$|H| = \left[A\alpha(\alpha-1)L^{\alpha-2}K^{\beta}\right]\left[A\beta(\beta-1)L^{\alpha}K^{\beta-2}\right] - \left[A\alpha\beta L^{\alpha-1}K^{\beta-1}\right]^2$$

$$= \left[A\alpha(\alpha-1)L^{\alpha-2}K^{\beta}\cdot\frac{L^2}{L^2}\right]\left[A\beta(\beta-1)L^{\alpha}K^{\beta-2}\frac{K^2}{K^2}\right] - \left[A\alpha\beta L^{\alpha-1}K^{\beta-1}\right]^2$$

$$= \frac{1}{K^2L^2}\alpha(\alpha-1)\beta(\beta-1)\left[\underbrace{AL^{\alpha}K^{\beta}}_{q}\right]\left[\underbrace{AL^{\alpha}K^{\beta}}_{q}\right] - \left[A\alpha\beta L^{\alpha-1}K^{\beta-1}\right]^2$$

$$= \frac{q^2}{K^2L^2}\alpha(\alpha-1)\beta(\beta-1) - \left[A\alpha\beta L^{\alpha-1}K^{\beta-1}\cdot\frac{KL}{KL}\right]^2$$

$$= \frac{q^2}{K^2L^2}\alpha(\alpha-1)\beta(\beta-1) - \frac{\alpha^2\beta^2}{K^2L^2}\left[\underbrace{AL^{\alpha}K^{\beta}}_{=q}\right]^2.$$

$$= \frac{q^2}{K^2L^2}\alpha(\alpha-1)\beta(\beta-1) - \frac{\alpha^2\beta^2}{K^2L^2}q^2.$$

$$= \underbrace{\frac{q^2}{K^2L^2}\alpha\beta}_{>0}\left[(\alpha-1)(\beta-1) - \alpha\beta\right].$$

$$= \frac{q^2\alpha\beta}{K^2L^2}\left[\cancel{\alpha\beta} - \alpha - \beta + 1 - \cancel{\alpha\beta}\right]$$

$$= \frac{q^2\alpha\beta}{K^2L^2}\left[\underbrace{1-\alpha-\beta}\right].$$

$(1-\alpha-\beta \gtrless 0?)$

$1 \gtrless (\alpha+\beta)$

$\boxed{|H| \gtrless 0 \text{ as } [1-\alpha-\beta] \gtrless 0}$.

Given: $0<\alpha<1$, $0<\beta<1$.

Case I: $\alpha=\beta=1/4$, $\alpha+\beta = 1/2 < 1$.

Case I: $\alpha = \beta = 1/4$, $\alpha + \beta = 1/2 < 1$

Case II: $\alpha = \beta = 3/4$, $\alpha + \beta = 3/2 > 1$.

Constrained Opt:

Max: $(x+2)(y+1)$ s.t $b = P_x \cdot x + P_y \cdot y = \phi(x, y)$.

$u(x,y) =$

Obj fn: $u(x, y)$.

Constraint fn: $\phi(x, y)$

Max $u(x, y)$ s.t $\phi(x, y)$.

$L = u(x, y) + \lambda \phi(x, y)$. [$\lambda$ = Lagrange Multiplier]

FOC: $\dfrac{\partial L}{\partial x} = 0 \Rightarrow$

$\dfrac{\partial L}{\partial y} = 0 \Rightarrow$

$\phi_x = \dfrac{\partial \phi}{\partial x}$, $\phi_y = \dfrac{\partial \phi}{\partial y}$.

$\dfrac{\partial L}{\partial \lambda} = 0 \Rightarrow$

SOC: Check: $|\bar{H}| = \begin{vmatrix} 0 & \phi_x & \phi_y \\ \phi_x & u_{xx} & u_{xy} \\ \phi_y & u_{xy} & u_{yy} \end{vmatrix}$

Principal Minor: $|\bar{H}_1| = \begin{vmatrix} 0 & \phi_x \\ \phi_x & u_{xx} \end{vmatrix} = -\phi_x^2 < 0$.

Max: $|\bar{H}| > 0$

Min: $|\bar{H}| < 0$.

Q. $\boxed{q = g(K, L)}$ Homogeneous of degree 2.

Find degree of homogeneity of MP_K.

$$g(\lambda K, \lambda L) = \lambda^2 g(K, L)$$

$MP_K = \dfrac{\partial q}{\partial K}$.

Diff w.r.t K:-

$MP_K = \frac{\partial \cdot}{\partial K}$

Diff w.r.t K :-

$$\frac{\partial g}{\partial(\lambda K)} \cdot \left\{\frac{\partial(\lambda K)}{\partial K}\right\} = \lambda^2 \cdot \frac{\partial g}{\partial K}$$

$$\frac{\partial g}{\partial(\lambda K)} \cdot \lambda = \lambda^2 \cdot \frac{\partial g}{\partial K}$$

$$\frac{\partial g}{\partial(\lambda K)} = \lambda \cdot \left(\frac{\partial g}{\partial K}\right).$$

$$\boxed{MP_{\lambda K} = \lambda \cdot (MP_K)}$$

$\Rightarrow$ MP_K fn is homogeneous of degree 1

$$y = f[x(t)].$$

$$\frac{dy}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt}$$

Q.

$C = a + b(Y-T)$, $a > 0$, $0 < b < 1$.

$I = \bar{I} - k r$, $k > 0$

$\hookrightarrow$ Interest sensitivity

$G = \bar{G}$

$M^d = c_0 + c_1 Y - c_2 r$, $c_0, c_1, c_2 > 0$

$M^s = \bar{M}$.

IS: $Y = C + I + G$

$$Y = a + bY + \bar{I} - kr + \bar{G}$$

$$(1-b)Y = a + \bar{I} - kr + \bar{G}$$

Slope of IS: $(1-b)\,dY = -k \cdot dr$.

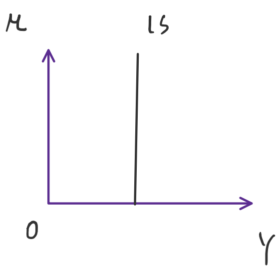
$$\left.\frac{dr}{dY}\right|_{IS} = -\frac{(1-b)}{k} < 0.$$

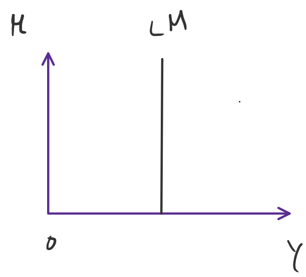
LM: $\bar{M} = c_0 + c_1 Y - c_2 r$.

Diff $0 = c_1 \cdot dY - c_2\, dr$

$$c_1\, dY = c_2\, dr.$$

$$\left.\frac{dr}{dY}\right|_{LM} = \frac{c_1}{c_2} > 0.$$





If: $k = c_2 = 0$

IS: $\left.\frac{dr}{dY}\right|_{IS} = -\frac{(1-b)}{k} \longrightarrow \infty.$

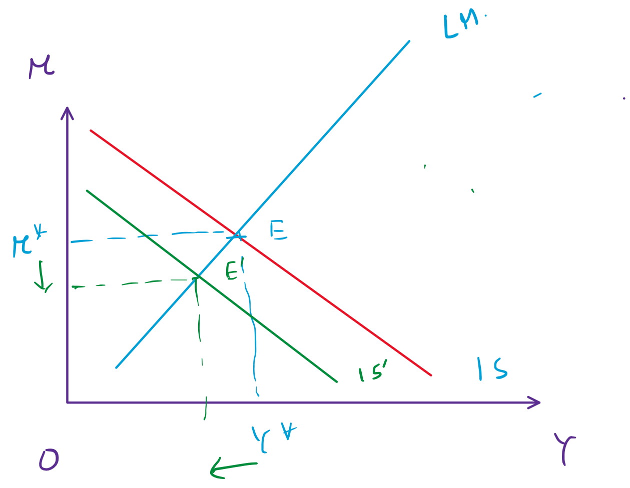
$dY|_{IS}$ k

$$LM: \left.\frac{dr}{dY}\right|_{LM} = \frac{c_1}{c_2} \to \infty .$$

Q. Standard IS-LM:

$$S = Y - C = Y - (a + bY)$$
$$= \underbrace{-a}_{\downarrow} + \underbrace{(1-b)}_{\text{MPS}} Y .$$

Autonomous saving.



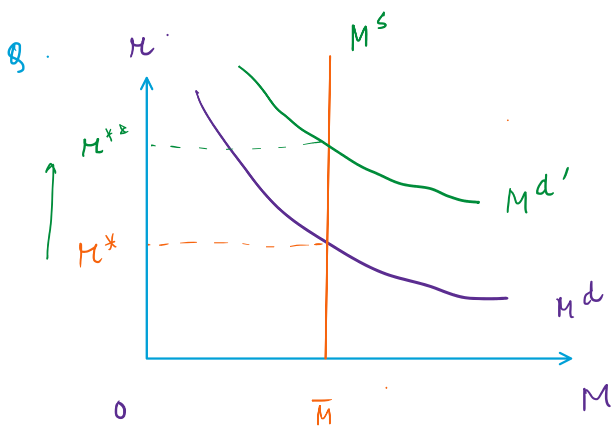
Auto saving ↑ ⇒ S ↑ ⇒ C ↓ ⇒ AD ↓ ⇒ Y ↓ .

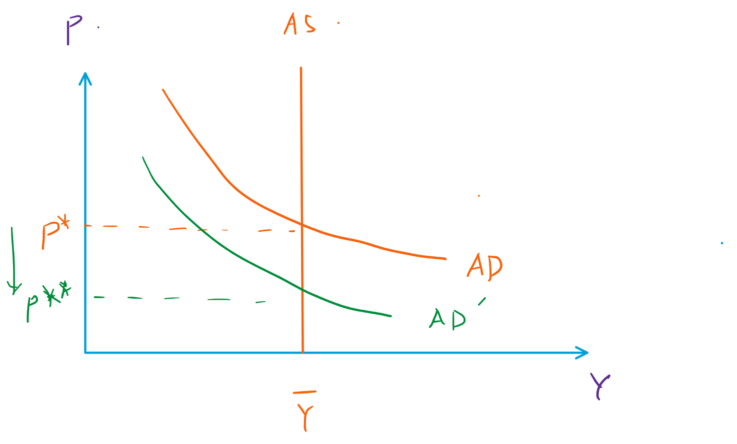
Q. QTM : $M \times v = P \times Y$.

$$\Rightarrow v = \frac{1}{\bar{M}} P \times \bar{Y} . \Rightarrow v \uparrow \Rightarrow P \uparrow .$$

$$\hookrightarrow \frac{M}{P} = \left(\frac{1}{v}\right) Y \Rightarrow \frac{M}{P} = \boxed{k.Y} . \qquad v \uparrow, k \downarrow \Rightarrow M_d \downarrow$$

Q.

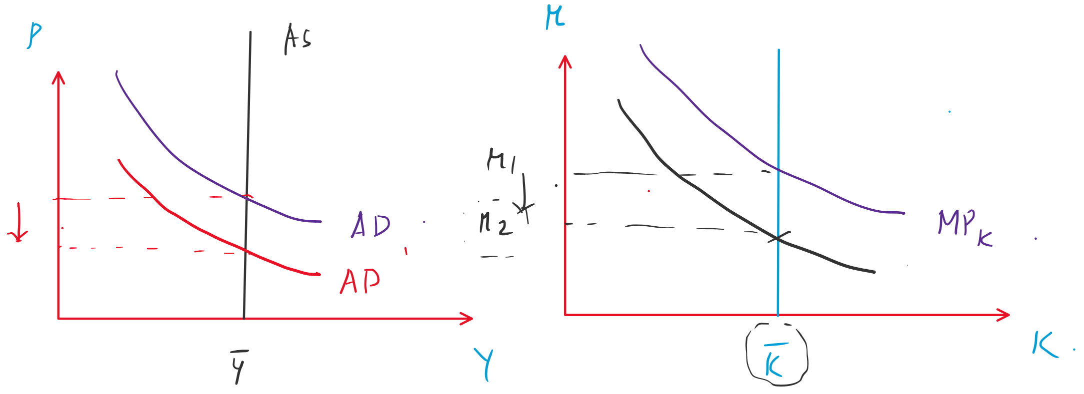




M^d ↑ ⇒ r ↑ ⇒ I ↓ ⇒ AD ↓

Q. T ↑ ⇒ Y_d ↓ ⇒ C ↓ ⇒ AD ↓ ⇒ P ↓ ,

Production fn: $\bar{Y} = F(\bar{K}, \bar{L})$.


P
AS
AD
AD
Y
M
M1
M2
MPK
K