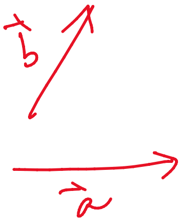
08 December 2023 18:04

Group C: Vector Analysis

[Marks: 20] [16 classes]

• Triple product, vector equations, applications to geometry and mechanics — concurrent forces in a plane, theory of couples, system of parallel forces. Introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions of one variable.



Resultant.

✓ Dot product

✓ Cross product.

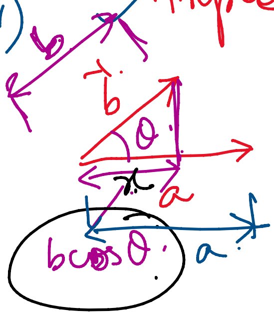
Triple product / Box product

Dot product. (Scalar)

$\vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

Scalar.

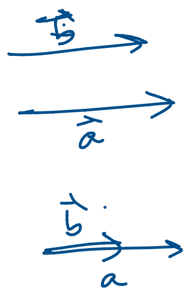


$\cos\theta = \frac{x}{b}$.

$x = b\cos\theta$.

projection of $\vec{b}$ on $\vec{a}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$



Case 1.

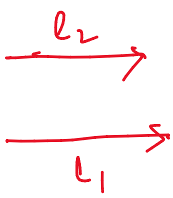
If $\theta = 90^\circ$ $(\vec{a}\perp\vec{b})$ check for Orthogonality.

$\cos\theta = 0 \Rightarrow \boxed{\vec{a}\cdot\vec{b} = 0}$

Case 2.

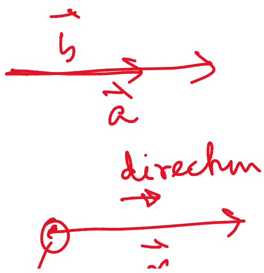
If $\theta = 0$ ($\vec{a}$ & $\vec{b}$ are parallel or collinear)

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$



$\vec{a} = 2\hat{i} + 3\hat{j}$

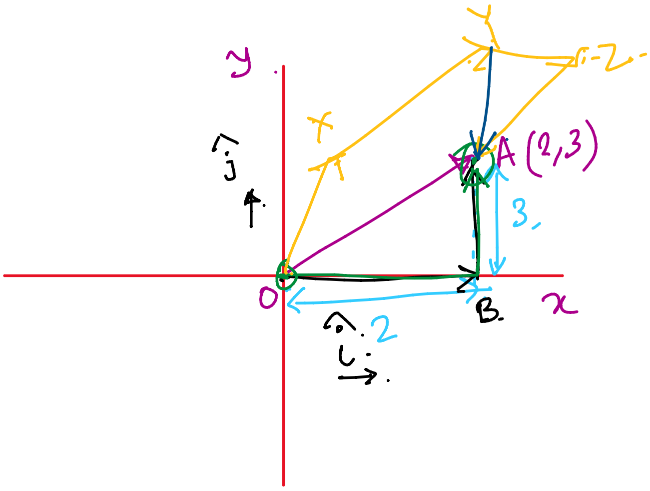
$\vec{b} = 4\hat{i} + 6\hat{j}$



direction($\vec{b}$)

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = a + \lambda b$



$\vec{OX} + \vec{XY} + \vec{YZ} + \vec{ZA} = \vec{OA}$

Polygon law

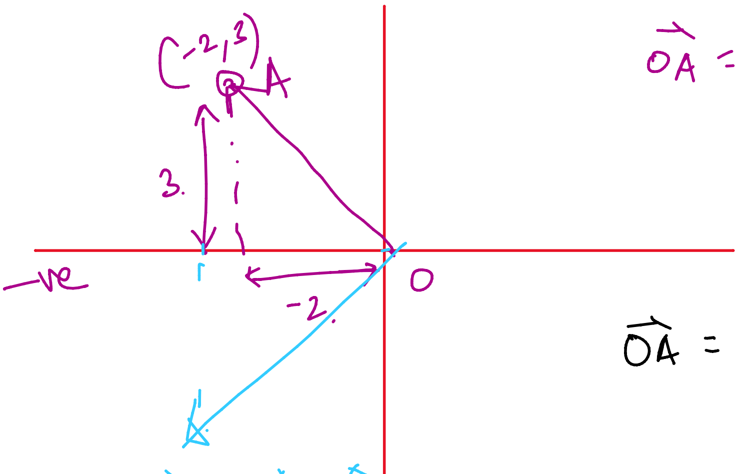
$\vec{OB} = 2\hat{i}$
$\vec{BA} = 3\hat{j}$ } paths

$\vec{OB} + \vec{BA} \rightarrow$ I reach A.

$\vec{OA} \rightarrow$ I " "

$\boxed{\vec{OA} = \vec{OB} + \vec{BA}}$ vector eqn

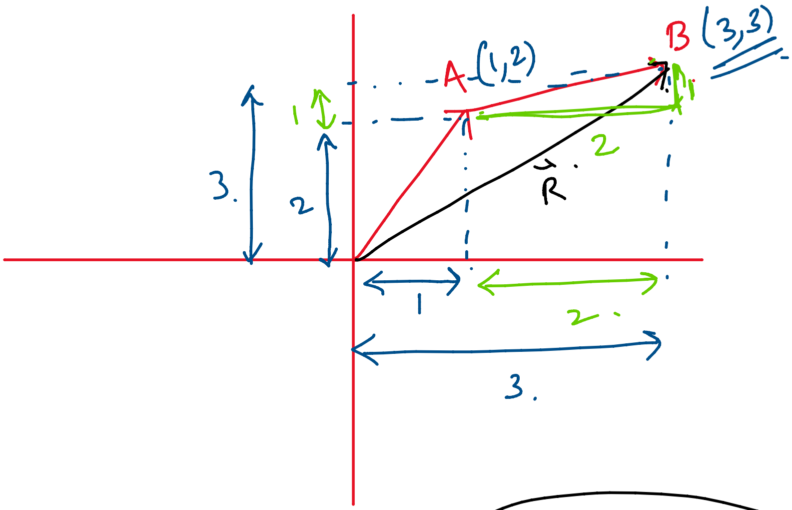
Triangle law.



$\vec{OA} = -2\hat{i} + 3\hat{j}$

$\vec{OA} = 2\hat{i} - 3\hat{j}$

$\vec{OA} = -2\hat{i} - 3\hat{j}$



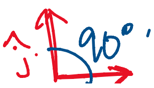
$\vec{R} = \vec{OA} + \vec{AB}$

$\vec{OB}$

$\vec{OA} = \hat{i} + 2\hat{j}$

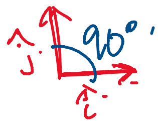
$\vec{AB} = 2\hat{i} + \hat{j}$

$\vec{R} = 3\hat{i} + 3\hat{j}$



$\vec{a} = 2\hat{i} + 3\hat{j}$ $\quad$ $\vec{b} = 1\hat{i} - 4\hat{j}$

2×1
$+ 3 \times (-4)$



$\vec{a} = (2)\hat{i} + (3)\hat{j}$

$+3 \times (-4)$

$\vec{a} \cdot \vec{b} = (2\hat{i} + 3\hat{j}) \cdot (\hat{i} - 4\hat{j})$

$= 2\,\hat{i}\cdot\hat{i} - 8\,\cancel{\hat{i}\cdot\hat{j}} + 3\,\cancel{\hat{j}\cdot\hat{i}} - 12\,\hat{j}\cdot\hat{j} = 2 - 12$

$= -10$

$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0$

$= 1 \times 1 = 1$

$\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90° = 0$

$\hat{j}\cdot\hat{i} = 0$

$\hat{j}\cdot\hat{j} = 1$

$\boxed{\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}}$ commutative

$2 \times 3 = 3 \times 2$

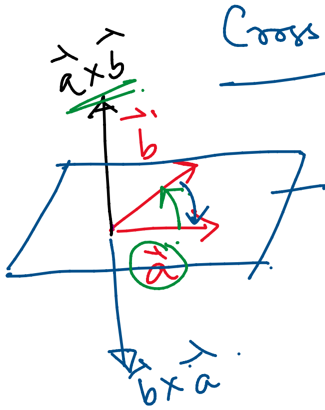
order doesn't matter

$2+3 = 3+2$

$2 \times 3 = 3 \times 2$

$\vec{a} = 3\hat{i} - 2\hat{j}$ $\quad \vec{b} = -2\hat{i} + 4\hat{j}$

Cross Pdt $\rightarrow$ vector which is $\perp$ to both $\vec{a}$ & $\vec{b}$.



$\boxed{\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}}$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

If $\theta = 0 \Rightarrow |\vec{a} \times \vec{b}| = 0$

$\Downarrow$

$\vec{a}$ & $\vec{b}$ are collinear

$\vec{a} = 2\hat{i} + \hat{j}$

$\vec{b} = -\hat{i} + 3\hat{j}$

$\vec{a} \times \vec{b} = (2\hat{i} + \hat{j}) \times (-\hat{i} + 3\hat{j})$

$= -2\,\cancel{\hat{i}\times\hat{i}} + 6\,\hat{i}\times\hat{j} - \hat{j}\times\hat{i} + 3\,\cancel{\hat{j}\times\hat{j}}$

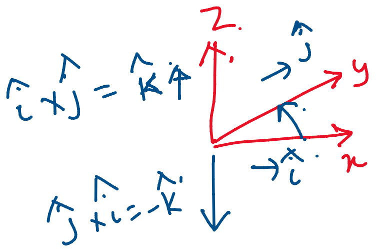
$|\hat{i} \times \hat{i}| = 1 \times 1 \times \sin 0$

$= 0.$

$|\hat{j} \times \hat{j}| = 0.$

$\hat{i} \times \hat{j} = \hat{k}$

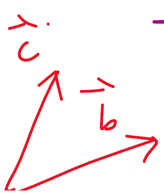
$\hat{j} \times \hat{i} = -\hat{k}$



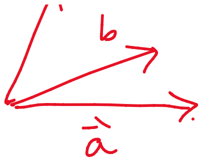
$\vec{a} \times \vec{b} = 6\hat{k} - (-\hat{k}) = 7\hat{k}$

Box (Triple) product $\rightarrow$ product of 3 vectors

$(\vec{a} \times \vec{b}) \cdot \vec{c}$ definition

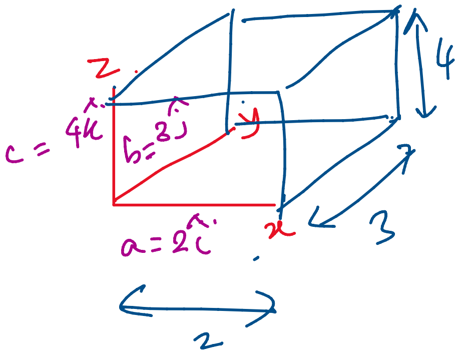


scalar



$(\vec{a} \times \vec{b}) \cdot \vec{c}$

vector $\cdot\ \vec{c} \Rightarrow$ scalar



$\vec{a} \times \vec{b} = 6\hat{k}$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = 6\hat{k} \cdot 4\hat{k} = 24$

vol of the box