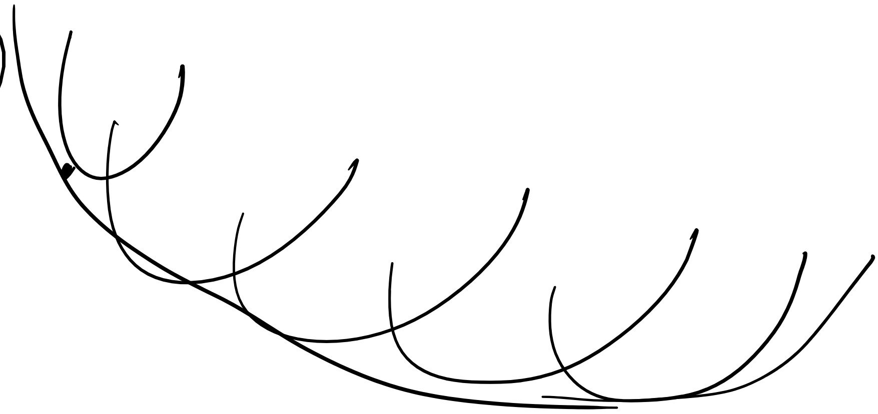
DIFF CALCULUS 8/2/2024

08 February 2024 19:39

Envelope

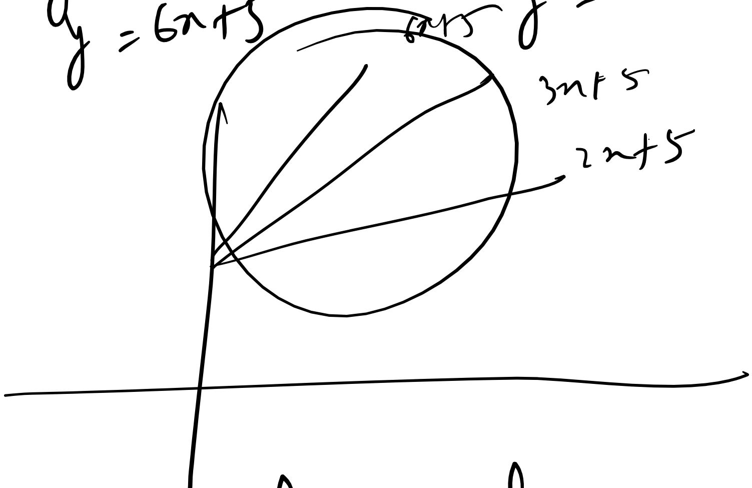


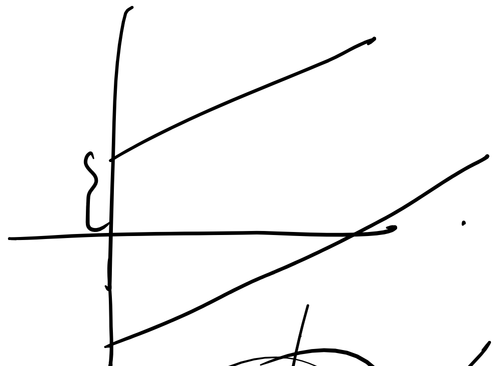
$y = mx + 5$

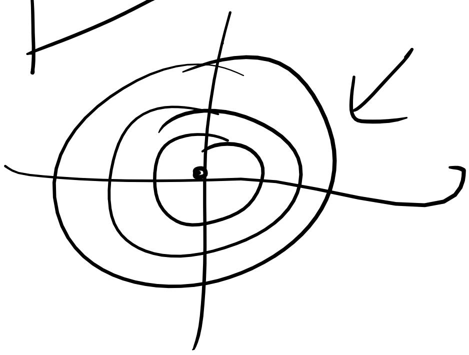
$y = 2x+5$
$y = 3x+5$
$y = 6x+5$

$y = 2x+5$
$y = 2x-5$

family of Curves





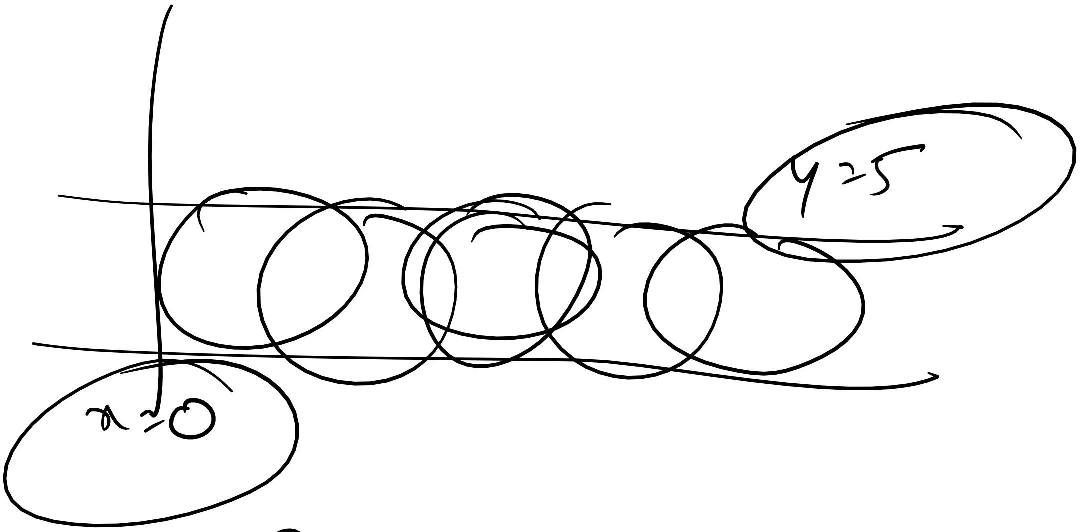


neighbourhood where the family is Same

$f(x, y, \alpha) = 0 \rightarrow$ family of Curves ..

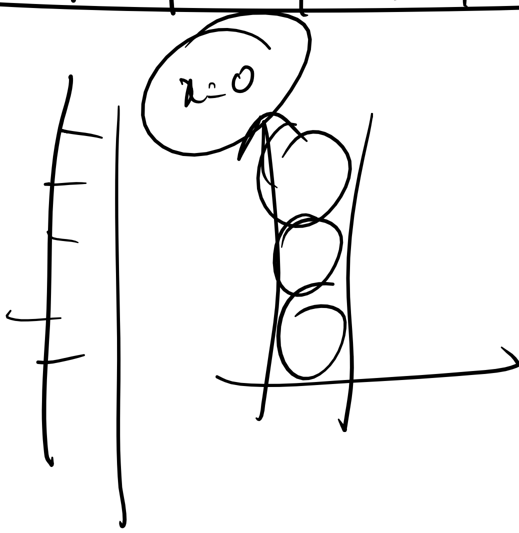
$\boxed{9062395123}$

$y^2 = 4ax$



Panorama

Vertic



ISI 1994

#

$$\boxed{y = mx + \frac{a}{m}}$$

a = parameter

$$\frac{\partial y}{\partial m} = x - \frac{a}{m^2} = 0$$

$\boxed{m^2 = \frac{a}{[illegible]}}$ a [illegible]

$\Rightarrow m$ $\boxed{m^2 = \frac{a}{x}}$ $a \cdot \frac{x}{a}$

$y^2 = m^2x^2 + \frac{a^2}{m^2} + 2mx\cdot\frac{a}{m}$

$y^2 = m^2x^2 + ax + 2ax$

$y^2 = \frac{a}{x}\cdot x^2 + a^2\cdot\frac{x}{a} + 2ax = \boxed{4ax}$

$$\boxed{y^2 = 4ax}$$

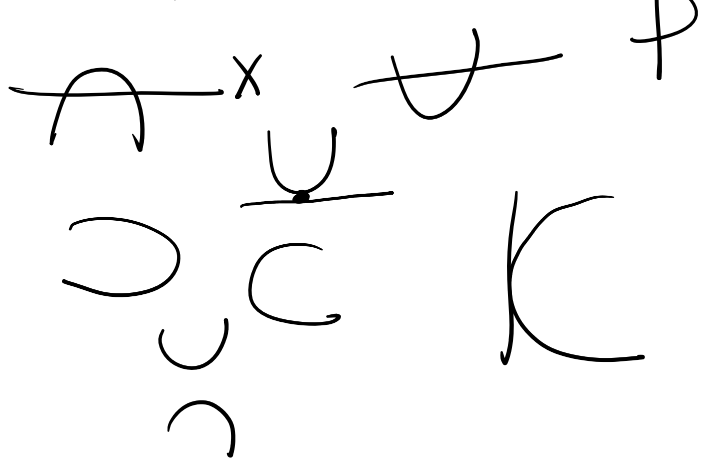
\# $y = mx + a\sqrt{1+m^2}$

$(y-mx)^2 = a^2(1+m^2)$

$(x^2-a^2)m^2 - 2mxy + (y^2-a^2) = 0$

This is a quadratic equation ... of the Parameter m

$\boxed{D=0}$



Laughing Buddha

P

Neet

$D = b^2 - 4ac$

$\Rightarrow (-2xy)^2 - 4(x^2-a^2)(y^2-a^2) = 0$

$\Rightarrow (-2xy)^2 - 4(x^2-a^2)(y^2-a^2) = 0$

Or, $4x^2y^2 - 4(x^2-a^2)(y^2-a^2) = 0$

$x^2y^2 - (x^2-a^2)(y^2-a^2) = 0$

$x^2y^2 - (x^2y^2 - a^2x^2 - a^2y^2 + a^4) = 0$

$a^2x^2 + a^2y^2 - a^4 = 0$

$a^2(x^2+y^2-a^2) = 0$

$\boxed{x^2+y^2=a^2}$ Circle

$\boxed{x\cos^n\theta + y\sin^n\theta = a}$

$x.n\cos^{n-1}\theta(-\sin\theta) + yn\sin^{n-1}\theta\cos\theta = 0$

$\cancel{x\sin\theta\cos\theta}\left(y\sin^{n-2}\theta - x\cos^{n-2}\theta\right) = 0$

$= 0$

$\frac{x}{y} = \left(\frac{\sin\theta}{\cos\theta}\right)^{n-2}$

$\frac{\sin\theta}{\cos\theta} = \frac{x^{\frac{1}{n-2}}}{y^{\frac{1}{n-2}}}$

$\boxed{\sin\theta = kx^{\frac{1}{n-2}}}$

$\boxed{\cos\theta = ky^{\frac{1}{n-2}}}$

Squaring both

$1 = k^2\left(x^{\frac{2}{n-2}} + y^{\frac{2}{n-2}}\right)$

$k = \frac{1}{\sqrt{x^{\frac{2}{n-2}} + y^{\frac{2}{n-2}}}}$

$$\sin\theta = \frac{x^{\frac{1}{n-2}}}{\sqrt{x^{\frac{2}{n-2}}+y^{\frac{2}{n-2}}}} \qquad \cos\theta = \frac{K}{\sqrt{\cancel{x}^{\frac{2}{n-2}}+y^{\frac{2}{n-2}}}}$$

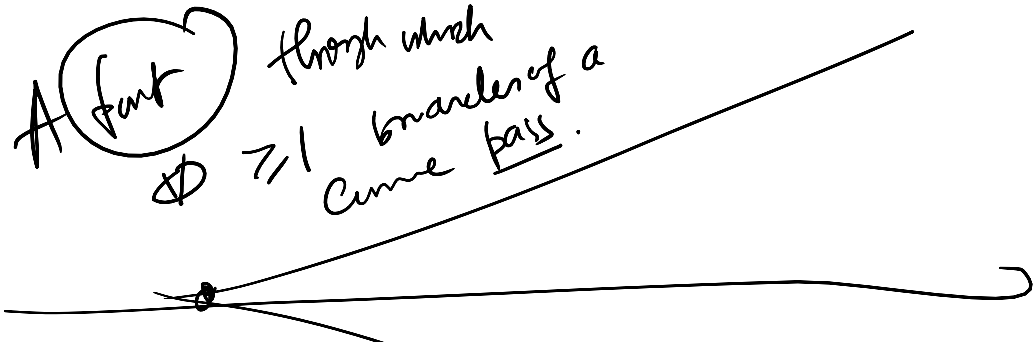
In eq (1)

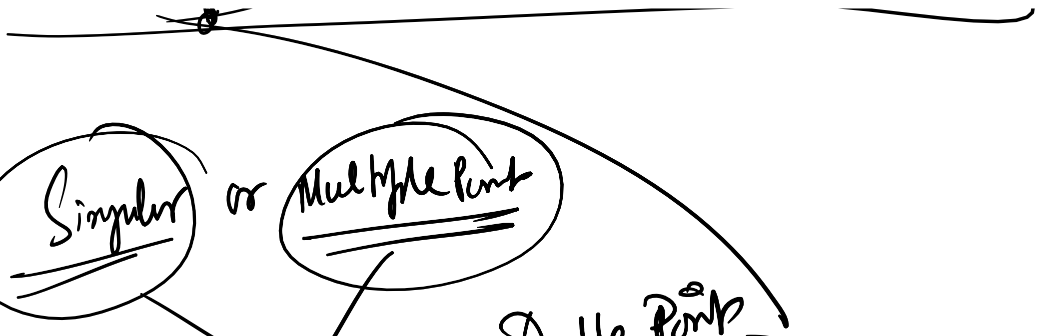
Envelope need to be a Sin Curv..

Envelope → Parametric Removal ..

Singular Point
- Node
- Cusp
- Isolated Point

A point through which ≥1 branches of a curve pass.



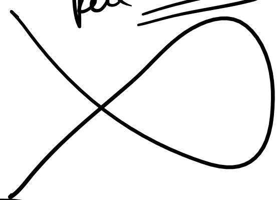


Singular or Multiple Point

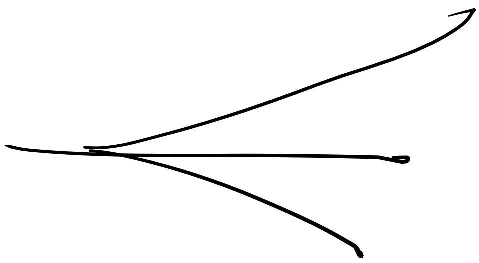
Double Point

Node

Real limbs



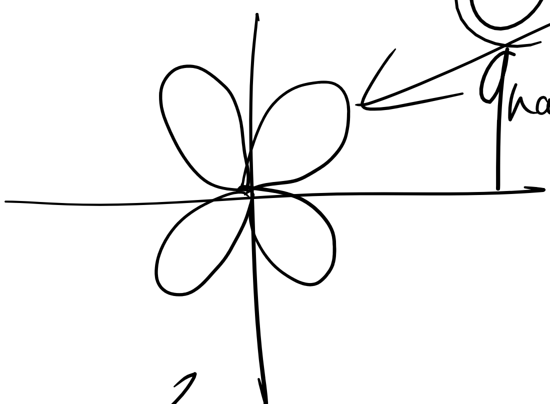
Cusp



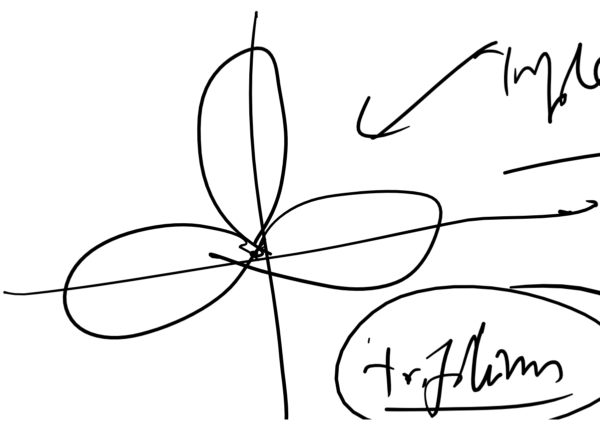
Ordinary vs Singular

$(x^2+y^2)^3 = 4a^2x^2y^2$

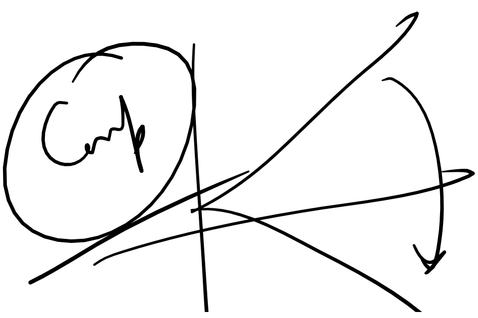
Quadruple Point



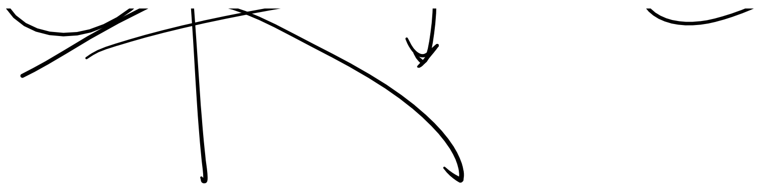
Triple Point



Cusp

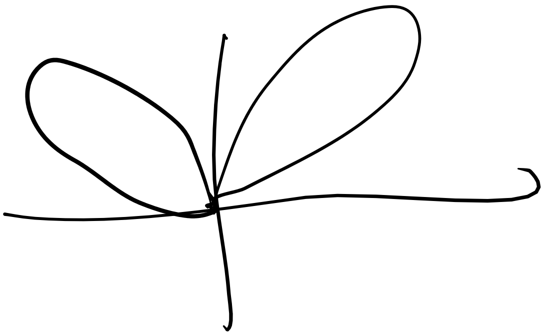


Trifolium



(trifolium)

$(x^2+y^2)^2 = a(x^3-3xy^2)$



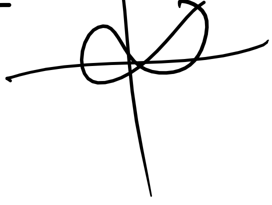
$\left[f_{xy}(\alpha,\beta)\right]^2 > f_{xx}\cdot f_{yy}$ — Node

$=$ — Cusp

$<$ — Isolated Point

Acnode — $y^2+x^2+x^3=0$

Lemniscate



Coste

Q $\boxed{x^3+y^3-3axy=0}$ $(0,0)$

find nature of double point?

The curve has

$3x^2-3ay=0$
$3y^2-3ax=0$
$3y^2=3ay$
$x^2=ay$
$y^2=ax$
$x^2=a$

$f_x = 3x^2-3ay$ $f_{xx}=6x$

$f_y = 3y^2-3ax$ $f_{yy}=6y$

$f_{xy}=-3a$

$f(0,0)=0$ $f_x=0=f_y$

$(0,0) \to$ Double Point

$x^4=a^2y^2$
$x^4=a^2 \cdot ax$
$x^4-a^3x=0$
$x(x^3-a^3)=0$
$\boxed{x=0}$ $\boxed{x^3=a^3}$

$f_{xy}=-3a$ $f_{xx}=6\cdot 0=0$

$f_{yy}=0$

$(f_{xy})^2=9a^2>0.0 \to \boxed{\text{NODE}}$

T/F ~~Intersection~~ Can the Parameter be even derivative of a double Point $\boxed{T/F}$

① Ans: $y^2=ax^2+bx^3$

$f(x,y)=y^2-ax^2-bx^3$

$f_x = -2ax-3bx^2$

$-2a-6bx$

$f_x = -2ax - 3bx^2$

$f_{xx} = -2a - 6bx$

$f_y = 2y \qquad f_{xy} = 0$

$f_{yy} = 2$

$\phi = 0^2 - (-4a) = 4a \qquad a \gtreqless 0$

N / C / SP

The parameter is the decider.

IIT JAM MATH DIF...