Questions Of Random Variables

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Random variables   Statistics

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Questions Of Rando...

$$E(x) = \int_a^b \frac{x}{b-a}\,dx$$

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1. Let X be a uniform (0,1) random variable. Compute $E(X^n)$

(A) $n+1$ (B) $n-1$

(C) $\frac{1}{n+1}$ (D) $\frac{1}{n-1}$

$$E(x) = \int_0^1 x \cdot \frac{1}{1-0}\,dx = \int_0^1 x\,dx$$

$$E(x^n) = \int_0^1 x^n\,dx = \left(\frac{x^{n+1}}{n+1}\right)_0^1 = \frac{1}{n+1}$$

Similar as above.

2. Compute E(X) if X has a density function given by

$$f(x) = \begin{cases} \frac{1}{4} x e^{-\frac{x}{2}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(A) 1
(B) 2
(C) 3
(D) 4

3. Compute E(X) if X has a density function given by

$$f(x) = \begin{cases} \frac{5}{x^2} & x > 5 \\ 0 & x \le 5 \end{cases}$$

(A) 2
(B) 0
(C) 5
(D) None of the above

4. Let X be a discrete random variable with values x = 0,1,2, and probabilities P(X = 0) = 0.25, P(X = 1) = 0.50, and P(X = 2) = 0.25, respectively. Find E(X).

(A) 1
(B) 0
(C) 4
(D) 8

5. Y is an exponential random variable with variance Var [Y] = 25. What is $E[Y^2]$?

(A) 10 (B) 50

(C) 30 (D) 20

$$f_Y(y) = \lambda e^{-\lambda y} \quad y \geq 0$$

$$= 0 \text{, otherwise.}$$

$$E(Y) = \frac{1}{\lambda} \quad Var(Y) = \frac{1}{\lambda^2}$$

$$\lambda = 1/5 \qquad E(Y) = 1/\lambda = 5$$

$$E(Y^2) = V(Y) + (E(Y))^2 = 50$$

6. X is an Erlang (η, λ) random variable with parameter $\lambda = 1/3$ and expected value $E[X] = 15$. What is the value of the parameter η?

(A) 0
(B) 1
(C) 5
(D) None of the above

$$f_X(x) = \frac{\lambda^n x^{n-1} \cdot e^{-\lambda x}}{(n-1)!}$$

$= 0$, otherwise.

$x \geq 0$

$\lambda > 0$

$E(x) = \frac{n}{\lambda}$ $\quad V(x) = \frac{n}{\lambda^2}$

$\lambda = 1/3$ $\quad E(x) = \frac{n}{\lambda} = 15$

$n = 5$

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7. Let X be a random variable with CDF

$$F_x(x) = \begin{cases} 0 & x < -1, \\ x/4 + 1/2 & -1 \le x < 1 \\ 1 & 1 \le x. \end{cases}$$

find E[X]

(A) 0 (B) 55

(C) 53 (D) 22

for $E(X) = \int_{-\infty}^{\infty} x f_x(x) dx$

$= \frac{x}{4}\Big|_{x=-1} + \frac{x^2}{8}\Big|_{-1}^{1} + \frac{x}{4}\Big|_{x=1}$

$= -1/4 + 0 + 1/4 = 0$

8. Y is an exponential random variable with parameter $\lambda = 0.2$. Given the event $A = \{Y < 2\}$. Find the conditional expected value $E[Y \mid A]$.

(A) $\frac{5+7e^{-2/5}}{1-e^{-2/5}}$ (B) $\frac{5-7e^{-2/5}}{1-e^{-2/5}}$

(C) $\frac{5-7e^{-2/5}}{1+e^{-2/5}}$ (D) None of these

$E(Y) = \int_{-\infty}^{\infty} y\, f_{Y|A}(y)\, dy$

$= \frac{1/5}{1-e^{-2/5}} \int_0^2 y\, e^{-y/5}\, dy$

Using By-Parts

$E[Y|A] = \frac{1/5}{1-e^{-2/5}} \left(\left(-5ye^{-y/5}\right)_0^2 + \int_0^2 5e^{-y/5}\, dx \right)$

$= \frac{\frac{1}{5}}{1-e^{-2/5}} \left(-10e^{-2/5} - 25e^{-y/5} \Big|_0^2 \right)$

$= \frac{5-7e^{-2/5}}{1-e^{-2/5}}$

9. Suppose that X is an absolutely continuous random variable with by

$f(x) = \begin{cases} 2x, & x \in (0,1) \\ 0, & \text{otherwise.} \end{cases}$ Find E[X]

(A) 1/3 (B) 2/3

(C) 4/3 (D) None of the above

$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^1 2x^2\, dx = 2/3$

(C) 4/3

(D) None of the above

$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^1 2x\, dx = 1/3$

10. Suppose that X is an absolutely continuous random variable with density given by

$$f(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & \text{otherwise.} \end{cases}$$

Find the expectation of e^X.

(A) e + 1

(B) e

(C) e − 1

(D) None of the above

11. Find the expected value of the number on a die when thrown.

(A) 1.5 (B) 20

(C) 3.5 (D) 6

12. A random variable X has $E(X) = 2$ and E variance.

(A) 4

(B) 3

(C) 5

(D) 8

13. If the covariance between two random variables X and Y is zero then

(A) X and Y are independent

(B) Knowing the value of X provides no information about the value of Y

(C) E (X) = E (Y) = 0

(D) None of the above

14. The probability density function of the random variable X follows the following probability law :

$$p(x) = \frac{1}{2\theta}\exp\left(-\frac{|x-\theta|}{\theta}\right), -\infty < x < \infty$$

(A) $e^{\theta t}\left(1+\theta^2 t^2\right)^{-1}$ (B) $e^{\theta t}\left(1-\theta^2 t^2\right)^{-1}$

(C) $e^{-\theta t}\left(1-\theta^2 t^2\right)^{-1}$ (D) None of these

mgf

$M_X(t) = E(e^{tx}) = \int_{-\infty}^{\infty} \frac{1}{2\theta} \exp\left(-\frac{|x-\theta|}{\theta}\right) e^{tx}\,dx$

So, for $x \in (-\infty, \theta)$: $x - \theta < 0$, $\theta - x > 0$

$$= \frac{e^{\theta t}}{2(\theta t+1)} + \frac{e^{\theta t}}{2(1-\theta t)} = \frac{e^{\theta t}}{1-\theta^2 t^2}$$

15. Find the moment generating whose moments are

$$\mu_r' = (r+1)!\,2^r$$

(A) $(1+2t)^{-2}$ (B) $(1-2t)^2$

(C) $(1-2t)^{-2}$ (D) None of these

(15) MGF $\sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r' = \sum_{r=0}^{\infty} \frac{t^r}{r!}(r+1)!\,2^r$

(15) MGF

$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \cdot \mu_r' = \sum_{r=0} \frac{}{r!}$

$= \sum_{r=0}^{\infty} (r+1)(2t)^r$

$M_x(t) = 1 + 2(2t) + 3(2t)^2 + 4(2t)^3 + \dots$

$= (1-2t)^{-2}$

$(1-a)^{-2}$

$(1-a)^{-1}$

16. If a dice is thrown at random, what is the expected value of face value ?

(A) 1/2 (B) 3/2

(C) 7/2 (D) 5/2

17. If two dice are thrown, what is the expected value of sum of the face values ?

(A) 2 (B) 5

(C) 10 (D) 7

18. If X denotes number of failure preceding first success, with a probability of success P, find E(X)

(A) P + 1 (B) P − 1

(C) $\frac{1-P}{9}$ (D) $\frac{P}{P-1}$

Let X first trials are failure

(X+1) th is the success

p → prob of success (1 − p)

$f(x) = (1-p)^x p$

$E(X) = \sum_{0}^{\infty} x(1-p)^x \cdot p = p(1-p) \sum_{0}^{\infty} x(1-p)^{x-1} = \frac{1-p}{p}$

19. If $f(x) = 30x^4(1 - x)$, $0 \le x < 1$ is the p.d.f of a r.v X. Find E(X).

(A) 1/7 (B) 5/7

(C) 2/7 (D) 9/7

Ans [illegible] $\int x f(x) dx$

Ans   Because – $\int x f(x)\,dx$

20. If $f(x) = \frac{1}{x^2}$, $1 \le x < \infty$ is the p.d.f of a r.v. X. Find E(X).

(A) 2/7   (B) 5/2

(C) 1/9   (D) Does not exist

21. If X is a r.v. with pdf f (x) = $\frac{x}{6}$ when x = 1, 2, 3 and f(x) = 0, otherwise. Find $E(X + 2)^2$.

(A) $\frac{28}{6}$ (B) $\frac{58}{6}$

(C) $\frac{22}{6}$ (D) $\frac{11}{6}$

$E(g(x)) = \sum g(x) f(x)$

$E(x+2)^2 = \sum_{1}^{3} (x+2)^2 \frac{x}{6}$

$= 9 \cdot \frac{1}{6} + 16 \cdot \frac{2}{6} + 25 \cdot \frac{3}{6}$

$= \frac{9 + 32 + 75}{6} = \frac{116}{6} = \boxed{\frac{58}{6}}$

22. If X is a r.v. having the pdf $f(x) = q^{x-1} p$, $x = 1, 2, 3 \ldots\ldots$; $p + q = 1$. Find the mgf of X?

(A) $\frac{pe^t}{1-qe^t}$ (B) $\frac{pe^{-t}}{1-qe^t}$

(C) $\frac{qe^t}{1+pe^t}$ (D) None of these

$$M_X(t) = E(e^{tx}) = \sum e^{tx} f(x) = \sum_{1}^{\infty} e^{tx} q^{x-1} p$$

$$= \frac{p}{q} \sum_{1}^{\infty} (qe^t)^x$$

$$= \frac{p}{q}\left[qe^t + (qe^t)^2 + (qe^t)^3 + \ldots\right]$$

$$= \frac{p}{q}\left[qe^t\right]\left[1 + qe^t + (qe^t)^2 + \ldots\right]$$

$$= p\, e^t (1 - qe^t)^{-1}$$

$$= \left(\frac{pe^t}{1-qe^t}\right)$$

23. If f(x) = 8. If $f(x) = 1/2\, e^{-|x|}$, $-\infty < x < \infty$ is the p.d.f or a r.v X. Find the m.g. f of X.

(A) $\frac{1}{1+t^2}$ (B) $\frac{1}{1-t^2}$

(C) $\frac{1}{2t}$ (D) $\frac{t^2}{2}$

Similarly as above

$$\int e^{tx} f(x)\, dx$$

2

Similarly as above

$$M_X(t) = E(e^{tx}) = \int e^{tx} f(x)\,dx$$

$$= \int e^{tx} \frac{1}{2} e^{-|x|}\,dx$$

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24. Find the variance of the r.v whose m.g.f is $\frac{e^{-t}}{12}(2 + e^{t} + 6e^{3t} + 3e^{6t})$

(A) $\left(\frac{101}{12}\right)+\left(\frac{25}{12}\right)^2$

(B) $\left(\frac{101}{12}\right)-\left(\frac{25}{12}\right)^3$

(C) $\left(\frac{101}{12}\right)-\left(\frac{25}{12}\right)^2$

(D) $\left(\frac{25}{12}\right)-\left(\frac{101}{12}\right)^2$

25. A drunk man performed a random walk over the position 0, ± 1 ± 2.... The drunk man starts from the point 0. He takes successive unit steps with probability p at right and probability (1 −p) at left. His steps are independent. X be a location of the drunk man after taking n − steps.

Find the distribution of $\frac{(n+x)}{2}$ and find out E(X).

(A) n(2p −1) (B) n(2p + 1)

(C) (n + 1)p (C) (n − 1)2p

R → To Right after taking n steps.

R ~ Bin (n, p)

L ~ Bin (n, −p)

X: Position after n-steps..

R+l = n

r−l = X

2R = n + X

R = $\frac{n+X}{2}$ ~ Bin(n, p)

$E\left(\frac{n+x}{2}\right) = np$

$E(x) = 2\left[np - \frac{n}{2}\right] = 2n\left(p - \frac{1}{2}\right) = n(2p-1)$

Ref of Myth session

26. Moment generating function of gamma distribution about origin $M_x(t)$ is

(A) $(1+t)^{\lambda}$, $|t| < 1$
(B) $(1-t)^{\lambda}$, $|t| < 1$
(C) $(1+t)^{-\lambda}$, $|t| < 1$
(D) $(1-t)^{-\lambda}$, $|t| < 1$

$M_x(t) = E(e^{tx})$

$= \int_0^\infty e^{tx} f(x)\,dx$

$= \int_0^\infty e^{tx} \frac{1}{\Gamma(\lambda)} e^{-x} x^{\lambda-1}\,dx$

$= \frac{1}{\Gamma\lambda} \int_0^\infty e^{-(1-t)x} x^{\lambda-1}\,dx$

$= \frac{1}{\Gamma\lambda} \frac{\Gamma\lambda}{(1-t)^{\lambda}}, \quad |t|<1$

So, $M_x(t) = (1-t)^{-\lambda} \quad |t|<1$

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27. Let X and Y are i.i.d. with $P[X = x] = \frac{1}{x} - \frac{1}{x+1}$, x = 1, 2.... Find E [min(X, Y)].

(A) $\frac{\pi^2}{12}$
(B) $\frac{\pi^2}{2}$
(C) $\frac{\pi^2}{6}$
(D) $\frac{\pi^2}{3}$

$T = \min(X, Y)$ ... $P(X \geq t, Y \geq t)$

(C) $\frac{\ }{6}$ (D) $\frac{\ }{3}$

$T = \min(X, Y)$

$P(T=t) = P(X=t, Y>t) + P(X=t, Y=t) + P(Y=t, X>t)$

$= P(X=t)P(Y>t) + P(X=t)P(Y=t) + P(Y=t)P(X>t)$

Now, $P(Y \le t) = P(Y=1) + P(Y=2) + \cdots + P(Y=t)$

$\Rightarrow \left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \cdots + \left(\frac{1}{t}-\frac{1}{t+1}\right)$

$= 1 - \frac{1}{t+1} = \frac{1}{t+1}$

Similarly $P(X>t) = \frac{1}{t+1}$

$P(Y>t) = \frac{1}{t+1}$

$P(T=t) = \left(\frac{1}{t}-\frac{1}{t+1}\right)\frac{1}{t+1} + \frac{1}{t} - \frac{1}{t+1} + \left(\frac{1}{t} - \frac{1}{t+1}\right)^2$

$\Rightarrow \frac{1}{t(1+t)^2} + \frac{1}{t^2(1+t)}$

$E(T) = \frac{1}{(n+1)}\left(\frac{1}{t} - \frac{1}{t+1}\right)$

$= \frac{\pi^2}{6} - 1 + 1$

$= \boxed{\pi^2/6}$

28. A bag contains a coin of value M and a number of other coins whose aggregate value is m. A person draws coins one at a time till the draws the coin of value M. Find the value of his Expectation.

(A) $M - \frac{m}{2}$ (B) $\frac{m}{2} + m$

(C) $M + \frac{m}{2}$ (D) None of these

29. Let X and Y be two r.v.'s with means zero, variance unity and correlation coefficient P, then

(A) $E[\text{Max}(X^2, Y^2)] \leq 1 + \sqrt{1 - P^2}$ (B) $E[\text{Max}(X^2, Y^2)] \geq 1 + \sqrt{1 - P^2}$

(C) $E[\text{Min}(X^2, Y^2)] \leq 1 - \sqrt{1 + P^2}$ (D) None of these

30. The unit interval (0,1) is divided into two subintervals picking a point at random from inside the Interval. Denoting by Y and Z, the lengths of the larger and the shorter subintervals respectively.

(A) $E[Y / Z] = 2$ (B) $E[Y / Z] = 4$

(C) $E[Y / Z] = \infty$ (D) None of these

1. If the moments of a variate X are defined by $E(X^r) = 0.6$, $r = 1,2,3\ldots$ then which of the following holds ?

(A) $P(X = 0) = 0.4$

(B) $P(x = 1) = 0.6$

(C) $P(X \geq 2) = 0.$

(D) $P(X = 3) = 0.8$

2. Let $F_m(x)$ be the distribution function defined by $F_m(x) = 0$ for $x \le -n$

$= \frac{x+n}{2n}$ for $-n < x < n$

$= 1$ for $x \ge n$

then which of the following are true ?

(A) F(x) is not a distribution function

(B) F(x) is distribution function

(C) $\lim_{n\to\infty} F_n(x) = 0.6$

(D) $\lim_{n\to\infty} F_n(x) = \frac{1}{2}$

3. Let X have the pmf $f(x) = \frac{x}{6}$, x = 1,2,.3 then which of the following are true ?

(A) mean = 7/3

(B) variance = 5/9

(C) E(X) = 0.6

(D) $E(X^2) = 5$

4. If X is a random variable and 'a' is constant, then which of the following holds ?

(A) $E[\psi(aX)] = aE[\psi(X)]$

(B) $E[\psi(X)+a] = E[\psi(X)]$

(C) $E[a\psi(X)] = aE[\psi(X)]$

(D) $E[\psi(X)+a] = E[\psi(X)] + a$

5. Which of the following holds ?

(A) If $X \geq 0$ then $E(X) \geq 0$

(B) Let X and Y be two random variables such that $Y \leq X$ then $E(Y) \leq E(X)$

(C) If $X \geq 0$ then $E(X) \leq 0$

(D) All of the above

6. If is a random variable, then Among the following which of them are correct ?

(A) $Var(aX + b) = a\,V(X) + b$

(B) $|E(X)| \leq E|X|$

(C) $V(a\,X + b\,|) = a^2 V(X)$ where a and b are constants.

(D) All of these

7. If X and Y are two random variables, and if they are independent, then which of the following holds?

(A) $Cov(X, Y) = 0$

(B) $Cov(X, Y) \neq 0$

(C) $E[h(X).k(Y)] = E[h(X)]\,E[k(Y)]$

where h (.) is a function of X alone and k (.) is a function of Y alone, provided expectations on both sides exist.

(D) All of the above

8. The random variable x has pdf f(x) = 3/x(2 − x) when $0 \le x \le 2$ and 0 otherwise. Then which of the following are true?

(A) Coefficient of skewness $\beta_1 = 0$

(B) Coefficient of the kurotsis $\beta_2 = \frac{15}{7}$

(C) Coefficient of skewness $\beta_1 = 1/9$

(D) Coefficient of the kurtosis $\beta_2 = 9/7$

9. Given $f(x,y)=\begin{cases} 1/\pi & \text{if } x^2+y^2\le 1 \\ 0 & \text{If } x^2+y^2>1 \end{cases}$,then which of the following holds ?

(A) X and Y are uncorrelated
(B) X and Y are independent
(C) X and Y are not independent
(D) X and Y are not uncorrelated

10. A class of second graders has a mean height of five feet with a standard deviation of one inch. At least what percent of the class must be between 4'10" and 5'2"?

(A) 70% (B) 72%

(C) 73% (D) 75%

1. Let X have the pmf $f(x) = \frac{x}{10}$, x = 1, 2, 3, 4. Then E(X) equals to _______.

2. Let X be a random variable with the following probability distribution :

x	:	–3	6	9
Pr (X = x)	:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

$E(2X + 1)^2$ = ________.

3. *The expectation of the number on a die when thrown is equal to* _______.

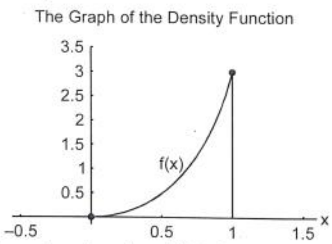
4. Two unbiased dice are thrown. Then the expected value of the sum of number of points on them is ________.

5. In four tosses of a coin, let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. the expected value of X equals to _______.

6. A coin is tossed until a head appear, Then the expectation of the number of tossed required is ________.

7. Suppose X has the density function

$$f(x) = \begin{cases} 3x^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$



The expected area of a random isosceles right train with hypotenuse X equals to ______.

8. Let variate X have the distribution

$$P(X = 0) = P(X = 2) = p;\ P(X = 1) = 1 - 2p,\ \text{for } 0 \le p \le \frac{1}{2}$$

The value of p such that the Var (X) a maximum is ________.

9. Let r.v. X have a density function f(.), cumulative distribution function F(.), mean μ and variance σ^2. Define $Y = \alpha + \beta X$, where α and β are constants satisfying $-\infty < \alpha < \infty$ and $\beta > 0$. The correlation coefficient ρ_{XY} between X and Y equals to ______.

10. An urn contain balls numbered 1, 2, 3. First a ball is drawn from the urn and then a fair coin is tossed the number of times as the number shown on the drawn ball. Then the expected number of heads is _______.

11. A couple decides to have 3 children, If none of the 3 is a girl, they will try again; and if they still don't get a girl, they will try once more. If the random variable X denote the number of children the couple will have following this scheme, then the expected value of X is ________.

12. A lot of 8 TV sets includes 3 that are defective. If 4 of the sets are chosen at random for shipment to a hotel, number of expected defective sets is _______.

13. Let X have the density function $f(x)=\begin{cases}\frac{2x}{k^2} & 0\le x\le k\\ 0 & \text{otherwise}\end{cases}$. The value of k such that the variance of X equal to 2 is ________.

14. If the probability density function of the random variable is

$$f(x) = \begin{cases} 1 - |x| & \text{for } |x| < 1 \\ 0 & \text{otherwise,} \end{cases}$$

then the variance of X equals to ________.

15. Let the probability density function of a random variable X be

$$f(x) = \begin{cases} 630x^4(1-x)^4 & \text{if } < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

then the approximate value of $P(|X-\mu| \le 2\sigma)$ when one uses the chebyshev inequality is __________.

16. What is the moment generation function of the random variable X whose probability density function is given by

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Then the variance of X is _______.

17. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} be^{-bx} & \text{for } x>0 \\ 0 & \text{otherwise,} \end{cases}$$

Where b > 0, If M(t) is the moment generating function of X, then is M (– 6b) = ______.

18. Let the random variable X have moment generation function $M(t) = (1 - t)^{-2}$ for $t < 1$. Then the third moment of X about the origin is _________.

19. If the moment generating of a random variable X is

$$M(t) = \sum_{j=0}^{\infty} \frac{e^{(tj-1)}}{j!}$$

the probability of the event $X = 2$ equals to _________.

20. Given that X has moment generating function

$$M(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{4}e^{t} + \frac{1}{4}e^{2t}$$

$P(|X| < 1) =$ _________.