Binomial Theorem

Problems & Sol^n

Binomial Problems

- This section contains **30 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct

1\. If $\sum_{r=0}^{n}(-1)^r\,{}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\ldots \text{upto } m \text{ terms}\right]=f(n)\left(1-\frac{1}{2^{mn}}\right)$, $\int_{-3}^{3}f(x^3\ln x)\,d(x^3\ln x)$ is equal to

(a) -6 (b) -3

(c) 3 (d) Cannot be determined

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→ alternating series.

$\Rightarrow \sum(-1)^r\,{}^nC_r\left[\left(\frac{1}{2}\right)^r+\left(\frac{3}{4}\right)^r+\left(\frac{7}{8}\right)^r+\ldots m\right]$

$\Rightarrow \left(1-\frac{1}{2}\right)^n+\left(1-\frac{3}{4}\right)^n+\left(1-\frac{7}{8}\right)^n+\ldots m \text{ terms}$

[As $\ln x$ -3 to $+3$ → the value) $x<0$

$\Rightarrow \frac{1}{2^n}+\frac{1}{2^{2n}}+\frac{1}{2^{3n}}+\ldots m$ term.

$\Rightarrow \frac{1}{2^n}\frac{\left[1-\left(\frac{1}{2^n}\right)^m\right]}{\left(1-\frac{1}{2^n}\right)}=\left(\frac{1}{2^n-1}\right)\left(1-\frac{1}{2^{mn}}\right)$

$\therefore f(n)=\frac{1}{2^n-1}$

So, $\int_{-3}^{3}f(x^3\ln x)\,d(x^3\ln x)$

$=\int_{-3}^{3}\frac{1}{(2^{x^3\ln x}-1)}(3x^2\ln x+x^2)\,dx$

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for $\log$ $x<0$ ✗

BSc MSc PhD

2. The coefficient of $(a^3 \cdot b^6 \cdot c^8 \cdot d^9 \cdot e \cdot f)$ in the expansion of $(a+b+c-d-e-f)^{31}$ is

(a) 123210 (b) 23110

(c) 3110 (d) None of these

3. The sum of rational terms in $(\sqrt{2}+\sqrt[3]{3}+\sqrt[6]{5})^{10}$, is

(a) 12632 (b) 1260

(c) 126 (d) None of these

4. If $(1+x-3x^2)^{2145} = a_0 + a_1x + a_2x^2 + \ldots$, then $a_0 - a_1 + a_2 - a_3 + \ldots$ ends with

(a) 1 (b) 3

(c) 7 (d) 9

5. In the expansion of $\left(\sqrt{\frac{q}{p}} + \sqrt[10]{\frac{p^7}{q^3}}\right)^n$, there is a term similar to pq, then that term is equal to

(a) $45pq$ (b) $120pq$

(c) $210pq$ (d) $252pq$

6. Let $(5+2\sqrt{6})^n = I + f$, where $n, I \in N$ and $0 < f < 1$, then the value of $f^2 - f + I \cdot f - I$, is

(a) a natural number (b) a negative integer

(c) a prime number (d) an irrational number

7. If $x + \frac{1}{x} = 1$ and $p = x^{4000} + \frac{1}{x^{4000}}$ and q is the digit at unit place in the number $2^{2^n} + 1$, $n \in N$ and $n > 1$, then $p + q$, is

(a) 8
(b) 6
(c) 7
(d) None of these

8. If the number of terms in $\left(x + 1 + \frac{1}{x}\right)^n$ $(n \in I^+)$ is 401, then n is greater than

(a) 201
(b) 200
(c) 199
(d) None of these

$(1+x+x^2 \ldots)$

$\rightarrow$ term $\rightarrow (2n+1)$

So, $2n+1 = 401$
$n = 200$

Summational P/C

$^5C_2 + {}^5C_3 =$ $^2C_1 + {}^2C_2$
$\Rightarrow 2+1$
$\Rightarrow 3 \Rightarrow {}^3C_2$

9. $\sum_{r=0}^{n-1} \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$ is equal to

(a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$

(c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2(n+1)}$

$\Rightarrow \sum \frac{{}^nC_r}{{}^{n+1}C_{r+1}} = \sum \frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r} \Rightarrow \sum_{r=0}^{n-1} \frac{r+1}{n+1}$

$\Rightarrow \frac{1}{n+1}[1+2+\ldots+n] = \frac{n(n+1)}{2(n+1)} = \frac{n}{2}$

$n = 100 \rightarrow 101$

Largest = middle = 51^{st} term ..

$\Rightarrow {}^{100}C_{50}\left(\frac{b}{2}\right)^{50}\left(\frac{b}{2}\right)^{50}$

$\Rightarrow {}^{100}C_{50}\left(b/2\right)^{100}$

$(a+b)^2$

$= a^2 + 2ab + b^2$

10. The largest term in the expansion of $\left(\frac{b}{2}+\frac{b}{2}\right)^{100}$ is

(a) b^{100}

(b) $\left(\frac{b}{2}\right)^{100}$

(c) ${}^{100}C_{50}\left(\frac{b}{2}\right)^{100}$

(d) ${}^{100}C_{50}b^{100}$

11. If the fourth term of $\left(\sqrt{x^{\left(\frac{1}{1+\log x}\right)}}+\sqrt[12]{x}\right)^6$ is equal to 200 and $x > 1$, x is equal to

(a) $10\sqrt{2}$ (b) 10 (c) 10^4 (d) $\frac{10}{\sqrt{2}}$

12. The coefficient of x^m in

$(1+x)^m + (1+x)^{m+1} + \ldots + (1+x)^n$, $m \le n$, is

(a) ${}^{n+1}C_{m+1}$ (b) ${}^{n-1}C_{m-1}$

(c) ${}^{n}C_{m}$ (d) ${}^{n}C_{m+1}$

13. The number of values of 'r' satisfying the equation $^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is

(a) 1 (b) 2

(c) 3 (d) 4

$^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2} + {}^{39}C_{r^2-1}$

$^{40}C_{3r} = {}^{40}C_{r^2}$

$3r = r^2$

$40 = 3r + r^2$

$r^2 + 3r - 40 = 0$

$r = -8, 5$

But $r = 0, -8$

$r = 3, 5$

$r = 0, 3, 5, -8$

14. The sum $S = {}^{20}C_2 + 2 \cdot {}^{20}C_3 + 3 \cdot {}^{20}C_4 + \ldots + 19 \cdot {}^{20}C_{20}$ is equal to

(a) $1 + 5 \cdot 2^{20}$ (b) $1 + 2^{21}$

(c) $1 + 9 \cdot 2^{20}$ (d) 2^{20}

15. The remainder, if $1 + 2 + 2^2 + 2^3 + \ldots + 2^{1999}$ is divided by 5 is

(a) 0 (b) 1

(c) 2 (d) 3

$S = \frac{1(2^{2000} - 1)}{2 - 1} \Rightarrow (2^{2000} - 1) = (2^2)^{1000} - 1$

$= (5 - 1)^{1000} - 1$

$= (5 \ldots) + 1 - 1$

Remainder → 0

Ans → 2

16. Coefficient of $\frac{1}{x}$ in the expansion of $(1+x)^n(1+1/x)^n$ is

(a) $\frac{n!}{(n-1)!(n+1)!}$ (b) $\frac{2n!}{(n-1)!(n+1)!}$

(c) $\frac{n!}{(2n-1)!(2n+1)!}$ (d) $\frac{2n!}{(2n-1)!(2n+1)!}$

17. The last two digits of the number 19^{9^4} is

(a) 19 (b) 29

(c) 39 (d) 81

18. If the second term in the expansion of $\left(\sqrt[13]{a}+\frac{a}{\sqrt{a^{-1}}}\right)^{n}$ is $14a^{5/2}$, the value of $\frac{{}^{n}C_{3}}{{}^{n}C_{2}}$ is

(a) 4
(b) 3
(c) 12
(d) 6

19. If $6^{83} + 8^{83}$ is divided by 49, the remainder is

(a) 0 (b) 14

(c) 35 (d) 42

20. The sum of all the rational terms in the expansion of $(3^{1/4} + 4^{1/3})^{12}$ is

(a) 91 (b) 251
(c) 273 (d) 283

21. Last four digits of the number $N = 7^{100} - 3^{100}$ is

(a) 2000 (b) 4000

(c) 6000 (d) 8000

22. If 5^{99} is divided by 13, the remainder is

(a) 2 (b) 4 (c) 6 (d) 8

23. The value of $\left\{\frac{3^{2003}}{28}\right\}$, where $\{.\}$ denotes the fractional part function is

(a) 17/28 (b) 19/28

(c) 23/28 (d) 5/28

24. The value of $\sum_{r=0}^{20} r(20-r)({}^{20}C_r)^2$ is equal to

(a) $400\ {}^{37}C_{20}$

(b) $400\ {}^{40}C_{19}$

(c) $400\ {}^{38}C_{19}$

(d) $400\ {}^{38}C_{20}$

25. If $(3+x^{2008}+x^{2009})^{2010}=a_0+a_1x+a_2x^2+\ldots+a_nx^n$, the value of

$a_0-\frac{a_1}{2}-\frac{a_2}{2}+a_3-\frac{a_4}{2}-\frac{a_5}{2}+a_6-\ldots$ is

(a) 1
(b) 2^{2010}
(c) 5^{2010}
(d) 3^{2010}

26. The total number of terms which depend on the value of x in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$ is

(a) $2n + 1$
(b) $2n$
(c) $n + 1$
(d) n

27. The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$, is

(a) 420 (b) 476

(c) 532 (d) 588

28. The number of real negative terms in the binomial expansion of $(1 + ix)^{4n-2}$, $n \in N, n > 0, i = \sqrt{-1}$, is

(a) n

(b) $n + 1$

(c) $n - 1$

(d) $2n$

29. $\sum_{p=1}^{n} \sum_{m=p}^{n} \binom{n}{m}\binom{m}{p}$ is equal to

(a) 3^n

(b) 2^n

(c) $3^n + 2^n$

(d) $3^n - 2^n$

30. The largest real value of x, such that $\sum_{r=0}^{4}\left(\frac{5^{4-r}}{(4-r)!}\right)\left(\frac{x^r}{r!}\right)=\frac{8}{3}$ is

(a) $2\sqrt{2}-5$

(b) $2\sqrt{2}+5$

(c) $-2\sqrt{2}-5$

(d) $-2\sqrt{2}+5$

- This section contains **15 multiple choice questions**. Each question has four choices (a), (b), (c) and (d) out of which **MORE THAN ONE** may be correct.

31. If in the expansion of $(1+x)^m (1-x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, the values of m and n are

(a) 3 (b) 6 (c) 9 (d) 12

32. If the coefficients of rth, $(r+1)$th and $(r+2)$th terms in the expansion of $(1+x)^{14}$ are in AP, then r is /are

(a) 5 (b) 9

(c) 10 (d) 12

33. If n is a positive integer and $(3\sqrt{3}+5)^{2n+1} = \alpha + \beta$, where α is an integer and $0 < \beta < 1$, then

(a) α is an even integer

(b) $(\alpha + \beta)^2$ is divisible by 2^{2n+1}

(c) the integer just below $(3\sqrt{3}+5)^{2n+1}$ divisible by 3

(d) α is divisible by 10

34. If $(8+3\sqrt{7})^n = P + F$, where P is an integer and F is a proper fraction, then

(a) P is an odd integer
(b) P is an even integer
(c) $F(P+F)=1$
(d) $(1-F)(P+F)=1$

35. The value of x for which the 6th term in the expansion of

$$\left\{2^{\log_2\sqrt{(9^{x-1}+7)}} + \frac{1}{2^{\left(\frac{1}{5}\right)\log_2(3^{x-1}+1)}}\right\}^7$$ is 84, is

(a) 4 (b) 3
(c) 2 (d) 1

36. Consider the binomial expansion of $\left(\sqrt{x}+\frac{1}{2\cdot\sqrt[4]{x}}\right)^n$, $n \in N$, where the terms of the expansion are written in decreasing powers of x. If the coefficients of the first three terms form an arithmetic progression, then the statement(s) which hold good is /are

(a) Total number of terms in the expansion of the binomial is 8

(b) Number of terms in the expansion with integral power of x is 3

(c) There is no term in the expansion which is independent of x

(d) Fourth and fifth are the middle terms of the expansion

37. Let $(1+x^2)^2 (1+x)^n = a_0 + a_1 x + a_2 x^2 + \ldots$, if a_1, a_2 and a_3 are in AP, the value of n is

(a) 2 (b) 3

(c) 4 (d) 7

38. 10th term of $\left(3-\sqrt{\frac{17}{4}+3\sqrt{2}}\right)^{20}$ is

(a) an irrational number (b) a rational number

(c) a positive integer (d) a negative integer

39. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots + C_n x^n$, then

$$C_0 - (C_0 + C_1) + (C_0 + C_1 + C_2) - (C_0 + C_1 + C_2 + C_3) + \ldots + (-1)^{n-1}$$

$(C_0 + C_1 + C_2 + \ldots + C_{n-1})$, when n is even integer is

(a) a positive value

(b) a negative value

(c) divisible by 2^{n-1}

(d) divisible by 2^n

40. If $f(n) = \sum_{i=0}^{n} \binom{30}{30-i}\binom{20}{30-i}$, then

(a) maximum value of $f(n)$ is ${}^{50}C_{25}$

(b) $f(0) + f(1) + f(2) + \ldots + f(50) = 2^{50}$

(c) $f(n)$ is always divisible by 50

(d) $f^2(0) + f^2(1) + f^2(2) + \ldots + f^2(50) = {}^{100}C_{50}$

41. Number of values of r satisfying the equation
$^{69}C_{3r-1} - {}^{69}C_{r^2} = {}^{69}C_{r^2-1} - {}^{69}C_{3r}$ is

(a) 1
(b) 2
(c) 3
(d) 7

42. If the middle term of $\left(x + \frac{1}{x}\sin^{-1}x\right)^8$ is equal to $\frac{630}{16}$, the values of x is/are

(a) $-\frac{\pi}{3}$ (b) $-\frac{\pi}{6}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

43. If $b^2 < ac$, the sum of the coefficients in the expansion of $(a\alpha^2 x^2 + 2b\alpha x + c)^n, (a, b, c, \alpha \in R, n \in N)$, is

(a) + ve, if $a > 0$
(b) + ve, if $c > 0$
(c) − ve, if $a < 0$, n is odd
(d) + ve, if $c < 0$, n is even

44. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n \in N$, then

(a) number of terms $= 2n+1$

(b) term independent of $x = 2^{n-1}$

(c) coefficient of $x^{2n-2} = n$

(d) coefficient of $x^2 = n$

45. The coefficient of the $(r+1)$th term of $\left(x+\frac{1}{x}\right)^{20}$, when expanded in the descending powers of x, is equal to the coefficient of the 6th term of $\left(x^2+2+\frac{1}{x^2}\right)^{10}$ when expanded in ascending powers of x. The value of r is

(a) 5 (b) 6

(c) 14 (d) 15

- This section contains **10 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

67. For integer $n > 1$, the digit at unit's place in the number $\sum_{r=0}^{100} r! + 2^{2^n}$ is

68. If $(1 + x + x^2 + x^3)^n = \sum_{r=0}^{3n} a_r\, x^r$ and $\sum_{r=0}^{3n} a_r = k$ and if $\sum_{r=0}^{3n} r\, a_r = \frac{\lambda\, n\, k}{2}$, the value of λ is

69. The number of rational terms in the expansion of $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$ is

70. If $2^{2006} + 2006$ is divided by 7, the remainder is

71. The last two digits of the natural number 19^{9^4} is ab, the value of $b - 3a$ is

72. If $\dfrac{\left[\begin{array}{l} {}^nC_r + 4\cdot{}^nC_{r+1} + 6\cdot{}^nC_{r+2} \\ \qquad + 4\cdot{}^nC_{r+3} + {}^nC_{r+4} \end{array}\right]}{\left[\begin{array}{l} {}^nC_r + 3\cdot{}^nC_{r+1} + 3\cdot{}^nC_{r+2} \\ \qquad + {}^nC_{r+3} \end{array}\right]} = \dfrac{n+\lambda}{r+\lambda}$,

the value of λ is

73. The value of $99^{50} - 99 \cdot 98^{50} + \frac{99 \cdot 98}{1 \cdot 2} (97)^{50} - \ldots + 99$ is

74. If the greatest term in the expansion of $(1+x)^{2n}$ has the greatest coefficient if and only if $x \in \left(\frac{10}{11}, \frac{11}{10}\right)$ and the fourth term in the expansion of $\left(\lambda x + \frac{1}{x}\right)^m$ is $\frac{n}{4}$, the value of $m\lambda$ is

75. If the value of

$$(n+2)\cdot {}^{n}C_{0}\cdot 2^{n+1}-(n+1)\cdot {}^{n}C_{1}\cdot 2^{n}+n\cdot {}^{n}C_{2}\cdot 2^{n-1}-\ldots$$

is equal to $k\,(n+1)$, the value of k is

76. If $(1 + x + x^2 + \ldots + x^9)^4 (x + x^2 + x^3 + \ldots + x^9)$

$= \sum_{r=1}^{45} a_r x^r$ and the value of $a_2 + a_6 + a_{10} + \ldots + a_{42}$ is λ,

the sum of all digits of λ is