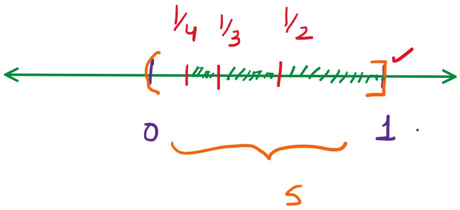
Q. $S = (0, 1]$ and $T = \left\{ \frac{1}{n},\ n = 1, 2, 3, \ldots \right\}$

Check if $(S - T)$ is open/closed.

$T = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots \right\}$



$$S - T = \underbrace{\left(\tfrac{1}{2}, 1\right) \cup \left(\tfrac{1}{3}, \tfrac{1}{2}\right) \cup \left(\tfrac{1}{4}, \tfrac{1}{3}\right) \cup \ldots\ldots}_{\text{union of infinite no. of open intervals (open sets)}}$$

$\Rightarrow$ Open.

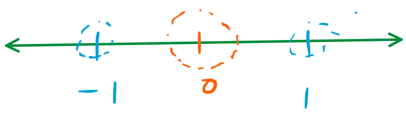
Q. If $Y = \left\{ \frac{x}{1+|x|},\ x \in \mathbb{R} \right\}$, then set of all limit pts of Y is:

(a) $(-1, 1)$ (b) $(-1, 1]$ (c) $[0, 1]$ ~~(d)~~ $[-1, 1]$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$x \to \infty$, $\frac{x}{1+|x|} \to 1$, $x \to -\infty$, $\frac{x}{1+|x|} \to -1$

$Y = (-1, 1)$.



Set of limit pts $= [-1, 1]$.

Q. Let $S = \left\{ \frac{2}{x+1},\ x \in (-1, 1) \right\}$. Set of all limit points of S is:

(a) $(0, \infty)$ (b) $(1, \infty)$ (c) $[0, \infty)$ ~~(d)~~ $[1, \infty)$

$S = (1, \infty)$

$-1 < x < 1$

$0 < x+1 < 2$

$\frac{1}{2} < \frac{1}{x+1} < \infty$

$1 < \frac{2}{x+1} < \infty$

Set of limit pts $= [1, \infty)$

Q. Let $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \,\middle|\, m, n \in \mathbb{N} \right\}$. Which of the following statements are true?

(a) S is closed.

(b) S is not open

(c) $S = \phi$

(d) 0 is the limit point of S.

$m \to \infty$ $\frac{1}{7^m} \to 0$, $n \to \infty$. $\frac{1}{3^n} \to 0$.

But $0 \notin S \Rightarrow S$ is not closed.

$m = n = 1$, $\frac{1}{3} + \frac{1}{7} = \frac{10}{21} \Rightarrow$ not an interior point

not an open set.

Compact set:

If S is a compact set $\Rightarrow S$ is closed and bounded.

Q. $S = \left\{ \frac{x^2}{1+x^2} : x \in \mathbb{R} \right\}$. Check if S is compact.

$x^2 \geq 0 \Rightarrow 1 + x^2 \geq 0 \Rightarrow \frac{x^2}{1+x^2} \geq 0$.

and $\frac{x^2}{1+x^2} < 1 \Rightarrow S = [0, 1) \Rightarrow$ Not closed.

Bounded. $\Rightarrow$ Bounded but not closed.

$\Rightarrow S$ is not compact.

Q. If K is a non-empty closed subset of $\mathbb{R}$. Then check if the set $\{x + y : x \in K, y \in [1, 2]\}$ is closed in $\mathbb{R}$

Eg: $K = [a, b]$. $Y = [1, 2] \Rightarrow$ compact.

N = New set $= K + Y = \{x + y : x \in K, y \in [1, 2]\}$.

$N = \{a + 1, \ldots\ldots \; b+2\}$

$$N = \{a+1, \ldots\ldots, b+2\}$$

all elements of set N are limit pts $\Rightarrow$ N is closed.

Is N compact? Yes.

HW

Q. Let $S = \{x \in \mathbb{R} \mid x^6 - x^5 \leq 100\}$ and $T = \{x^2 - 2x, x \in (0,\infty)\}$

Then the set $S \cap T$ is:

(a) closed, bounded on $\mathbb{R}$

(b) closed, not bounded on $\mathbb{R}$

(c) bounded, not closed on $\mathbb{R}$

(d) neither closed nor bounded.

$$x^6 - x^5 = 0$$