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① Degree of Homogneity of a function.

$$Z = f(x_1, x_2)$$

say let us multiply x_1 and x_2 by 'λ' proportion, the $f(\lambda x_1, \lambda x_2) = \lambda^n Z$

n = degree of homogenuity.

Classification of returns to scale.

① $n = 1$: $f(\lambda x_1, \lambda x_2) = \lambda Z$ [CRS ⇒ Const Returns to scale]

② $n > 1$: $f(\lambda x_1, \lambda x_2) < \lambda Z$ [IRS ⇒ Increasing Return to scale]

③ $n < 1$: $f(\lambda x_1, \lambda x_2) > \lambda Z$ [DRS: Decreasing Returns to scale]

Ex 1. Let $Q = 2L + 3K$ is a production function. Comment on the degree of homoguinity and returns to scale.

Soln. Let us increase L and K by 'λ' propostion, then we get, $2(\lambda L) + 3(\lambda K)$

$$= 2\lambda L + 3\lambda K$$

$$= \lambda [\underbrace{2L + 3K}]$$

$$= \lambda^1 Q \quad [\because Q = 2L + 3K]$$

$\therefore$ The function is homogenous of degree 1 and follows constant returns to scale.

Ex 2: $Q = A \cdot L^{\alpha} K^{\beta}$ is our production function. So comment on degree of homogenuity & Returns to scale.

$$Q = A L^{\alpha} K^{\beta} \longrightarrow ①$$

Soln: $Q = AL^{\alpha}K^{\beta} \longrightarrow (1)$

Suppose 'L' and 'K' is increased by 'λ' proportion, then,

$$A(\lambda L)^{\alpha}(\lambda K)^{\beta}$$

$$= A\lambda^{\alpha}L^{\alpha}\lambda^{\beta}K^{\beta}$$

$$= \lambda^{\alpha+\beta}(AL^{\alpha}K^{\beta})$$

From (1), $\lambda^{\alpha+\beta}\cdot Q$

$\therefore$ Degree of homogenity is $\alpha+\beta$.

(i) if $\alpha+\beta > 1 \Rightarrow$ IRS

(ii) $\alpha+\beta < 1 \Rightarrow$ DRS

(iii) $\alpha+\beta = 1 \Rightarrow$ CRS

Euler's theorem:

$Z = f(x_1, x_2)$ with degree of homogenity 'n'.

Then Euler's theorem states that,

$$f_1 \cdot x_1 + f_2 \cdot x_2 = nZ$$

Ex3: $Q = AL^{\alpha}K^{\beta}$. Verify Euler's theorem.

From ex2 $\Rightarrow$ degree of homogenity is $(\alpha+\beta)$.

from ex2 ⇒ degree of homogen...

To prove Euler's theorem we need

$$\frac{\partial Q}{\partial L}\cdot L + \frac{\partial Q}{\partial K}\cdot K = (\alpha+\beta)Q$$

LHS: $\frac{\partial Q}{\partial L}\cdot L + \frac{\partial Q}{\partial K}\cdot K$

$= A\alpha L^{\alpha-1}K^{\beta}\cdot L + AL^{\alpha}\beta K^{\beta-1}\cdot K'$

$= A\alpha L^{\alpha}K^{\beta} + A\beta L^{\alpha}K^{\beta}$

$= AL^{\alpha}K^{\beta}(\alpha+\beta)$

$= Q(\alpha+\beta)$

$= RHS$

Verified.

Ex4: $Q = 75\left[0.3K^{-0.4} + 0.7L^{-0.4}\right]^{-2.5}$. Verify Euler's theorem.

Solution: increase 'K' and 'L' by 'λ' proportion, we get,

$75\left[0.3(\lambda K)^{-0.4} + 0.7(\lambda L)^{-0.4}\right]^{-2.5}$

$= 75\left[0.3\lambda^{-0.4}K^{-0.4} + 0.7\lambda^{-0.4}L^{-0.4}\right]^{-2.5}$

$= 75\,\lambda^{-0.4(-2.5)}\left[0.3K^{-0.4} + 0.7L^{-0.4}\right]$

$$= 75\,\lambda^{-0.4(-2.5)}\left[0.3K^{-0.4}+0.7L^{-0.4}\right]$$

$$= \lambda^{1}\cdot Q$$

↳ degree of homogeneity is 1.

Euler's theorem states that.

$$\frac{\partial Q}{\partial L}\cdot L+\frac{\partial Q}{\partial K}\cdot K = 1\cdot Q.$$

LHS: $\partial Q/\partial L\cdot L+\partial Q/\partial K\cdot K$

$$=\left\{75\times(-2.5)\left[0.3K^{-0.4}+0.7L^{-0.4}\right]^{-3.5}(0.7)(-0.4)L^{-1.4}\right\}\cdot L$$

$$+\left\{75\times(-2.5)\left[0.3K^{-0.4}+0.7L^{-0.4}\right]^{-3.5}(0.3)(-0.4)K^{-1.4}\right\}\cdot K$$

$$=75(-2.5)\left[0.3K^{-0.4}+0.7L^{-0.4}\right]^{-3.5}\left[-0.28L^{-0.4}-0.12K^{-0.4}\right]$$

$$=75(-2.5)(-0.4)\left[0.3K^{-0.4}+0.7L^{-0.4}\right]^{-2.5}$$

$$75\left[0.3K^{-0.4}+0.7L^{-0.4}\right]^{-2.5}$$

$= 75\left[0.3K^{-0.4} + 0.7L^{-\cdot\cdot}\right]$

$= 1 \cdot Q$

$= \underline{\underline{RHS}}$ Verified

Maximisation/Minimisation for 2-variable functions.

ie, $\boxed{z = f(x_1, x_2)}$

Maximisation.

$\underline{\underline{F.O.C}}$: $f_1 = 0$, $f_2 = 0$

$\underline{\underline{S.O.C}}$: $\Rightarrow f_{11} < 0$, $f_{22} < 0$

$\Rightarrow f_{11} \cdot f_{22} > f_{12}^2$

Minimisation.

$f_1 = 0$, $f_2 = 0$

$\Rightarrow f_{11} > 0$, $f_{22} > 0$

$\Rightarrow f_{11} \cdot f_{22} > f_{12}^2$

$\underline{\underline{Ex}}$: Find the Stationary value & test the nature of x and y for the following $f\underline{n}$:

$$z = 3x^2 + 6xy + 7y^2$$

Soln : F.O.C $f_x = 6x + 6y = 0$ — ①

$f_y = 6x + 14y = 0$ — ②

Solving ① and ②, $6x + 6y = 0$

$6x + 14y = 0$

$\overline{\quad y = 0 \quad}$

$\therefore x = 0$

S.O.C $f_{xx} = 6 > 0$, $f_{yy} = 14 > 0$

$f_{xy} = 6$

$f_{xx} \cdot f_{yy} = 14 \times 6 = 84 > 6^2$

$\therefore$ S.O.C for minimisation is satisfied.

$\therefore$ at $x=0, y=0 \rightarrow z$ is minimised. (ans).

Ex: A bi-product firm produces two commodities x and y, prices of which are 12 and 18 resp. The joint cost fn is $C = 2x^2 + xy + 2y^2$. Obtain the optimal level of its products which maximise profit.

Solution: Acc to the ques, Total Rev, $TR = 12x + 18y$ — ①

and, $TC, C = 2x^2 + xy + 2y^2$ — ②

$\therefore$ Profit function, $\Pi = TR - TC$

$\Pi = (12x + 18y) - (2x^2 + xy + 2y^2)$

∴ Profit of

$$\pi = (12x+18y) - (2x^2+xy+2y^2)$$

$$\pi = 12x+18y-2x^2-xy-2y^2$$

Now, $\pi_x = 12-4x-y=0$ — (1) $\times 4 \Rightarrow 16x+\cancel{4y}=48$

$\pi_y = 18-x-4y=0$ — (2) $\Rightarrow \underset{-}{x}+\underset{-}{\cancel{4y}}=\underset{-}{18}$

$$15x = 30$$

∴ $x = 2$ units

$\Rightarrow$ $y = 4$ units

S.O.C

$\pi_{xx} = -4 < 0$ $\quad \pi_{yy} = -4 < 0$

$\pi_{xy} = -1$

Now, $\pi_{xx}\cdot\pi_{yy} = 16 > (-1)^2$

ie, $\pi_{xx}\pi_{yy} > \pi_{xy}^2$

∴ S.O.C is satisfies for maximisation.

∴ At $x=2$ units and $y=4$ units firm can maximise it's profit.