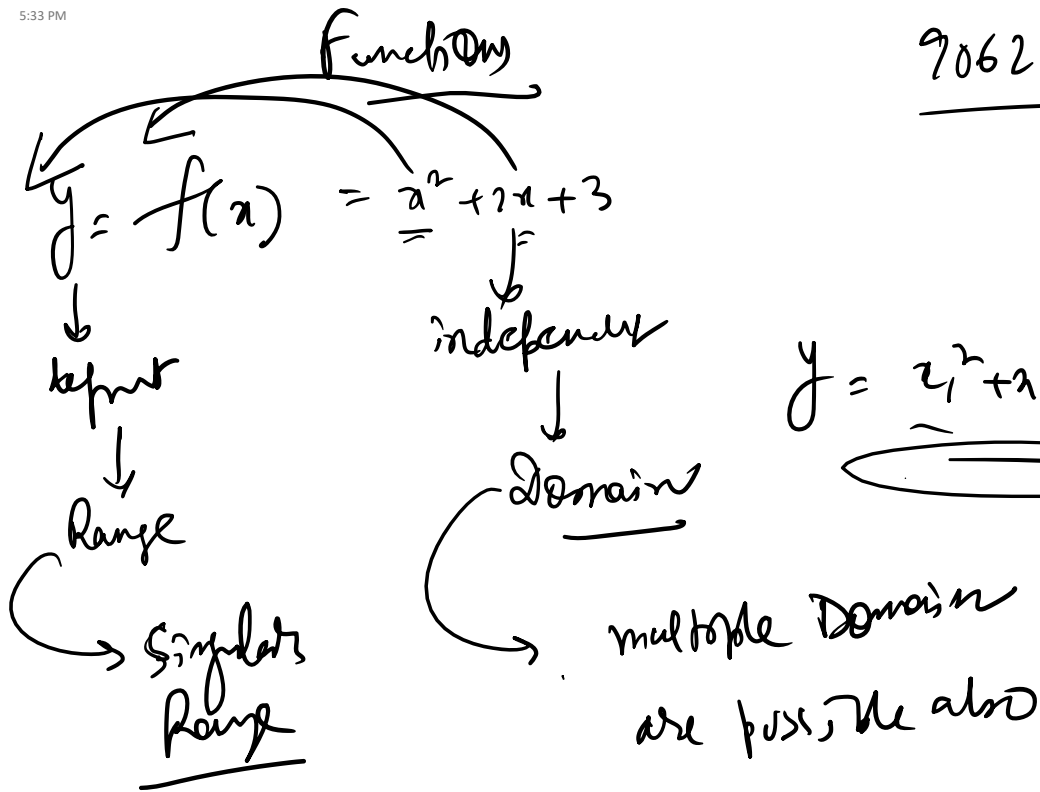


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max min  
min max

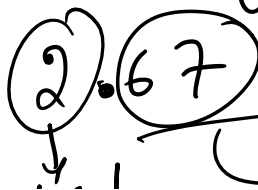
$3 \leq x_1 \leq 10$   
 $2 \leq x_2 \leq 15$   
 $2.5 \leq x_3 \leq 17$

max of min min of max

Gif

$y = [x]$

$y = [2.67] \rightarrow 2$



fractional part

fractional part

$\alpha$   
 $\downarrow$   
 Integer  $\rightarrow$  fractional part

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$$0 \leq x < 1 \quad [x] \rightarrow 0$$

$$+99 \leq x < 100 \quad [x] \rightarrow 99$$

$$-7 \leq x < -6 \quad [x] \rightarrow -7$$

So hit lower value is always applicable.

$[x] + [y] \leq [x+y]$

If  $\frac{1}{2}$  from part to form an example.

$(T/F)$

9062395723

$$[x] - [y] = [x - y]$$

$$[2.67] + [3.99]$$

$$\rightarrow 2 + 3$$

$$= 5$$

$\neq$

$$(2.67 + 3.99)$$

$$[6.66] \rightarrow 6$$

ISI 2014

$$\left[ \frac{3}{4} \right] + \left[ \frac{3}{4} + \frac{1}{100} \right] + \left[ \frac{3}{4} + \frac{2}{100} \right] + \dots + \left[ \frac{3}{4} + \frac{25}{100} \right]$$

$$= ?$$

0/42/3

$$\begin{aligned}
 & \left[ \underline{0.75} \right] + \left( \underline{0.75} + \underline{0.01} \right) + \left( \underline{0.75} + \underline{0.02} \right) + \dots + \left[ \underline{2} + \underline{0.25} \right] \\
 & = 0 + 0 + 0 + \dots + (1) \\
 & = 0 + 1 = \textcircled{1}
 \end{aligned}$$

FPF fractional part function

$$I = [x]$$

$$x = I + f$$

$\{x\} \rightarrow$  sign of fractional part.

$$f = x - I$$

fractional part =  $\left( \begin{array}{l} \text{Integer} \\ \text{number} \end{array} - \text{Integral part.} \right)$

$$0 \leq x < 1$$

$$\textcircled{1} \leq x < 2$$

$$\textcircled{-1} \leq x < 0$$

$$-2 \leq x < -1$$

$$x$$

$$x - 1$$

$$\rightarrow x + 1 \Rightarrow x - (-1)$$

$$\rightarrow x + 2 \Rightarrow x - (-2)$$

$$0.67 \rightarrow 0$$

$$2.63 \rightarrow \textcircled{2}$$

$$1.05$$

$$1.07$$

$$1.29 - 1 = 0.29$$

#

$$4n = x + x$$

$$2x = 2x$$

$$2x = 0$$

$$x = 0$$

$$x = \{x\} + [x]$$

$$\# \quad 4\{x\} = x + [x]$$

$$4\{x\} = [x] + \{x\} + [x]$$

$$3\{x\} = 2[x]$$

$$\{x\} = \frac{2[x]}{3}$$

We know,

$$0 \leq \{x\} \leq 1$$

$$0 \leq 2[x] \leq 3$$

$$0 \leq [x] < \frac{3}{2}$$

$$0 \leq [x] < 1.5$$

$$\text{either } 0 \leq [x] < 1 \quad [x] = 0$$

$$\text{or } 1 \leq [x] < 1.5 \quad [x] = 1$$

$$\begin{matrix} 0-1 \\ 1-2 \end{matrix}$$

$$\text{If } [x] = 1 \quad \text{then } \{x\} = \frac{2}{3}$$

$$\text{Then } x = [x] + \{x\}$$

$$= 1 + \frac{2}{3} = \frac{5}{3}$$

$$\text{And } [x] = 0, \{x\} = 0$$

$$x = 0$$

So, the value  $0, \frac{5}{3}$

# # Range Based Problems

$$y = 2x + 3 \quad 0 \leq x \leq 10$$

$y = 3$  at  $x = 0$   
 $y = 23$  at  $x = 10$

$3 \leq y \leq 23$

ISI 2012

$$f(x) = \frac{e^x}{1+(x)}$$

$x > 0$

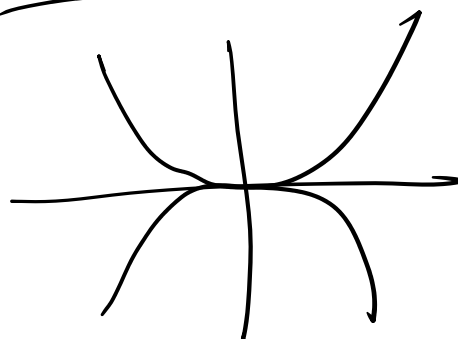
NOTE Positive = Non-negative  
 $> 0$   $\geq 0$

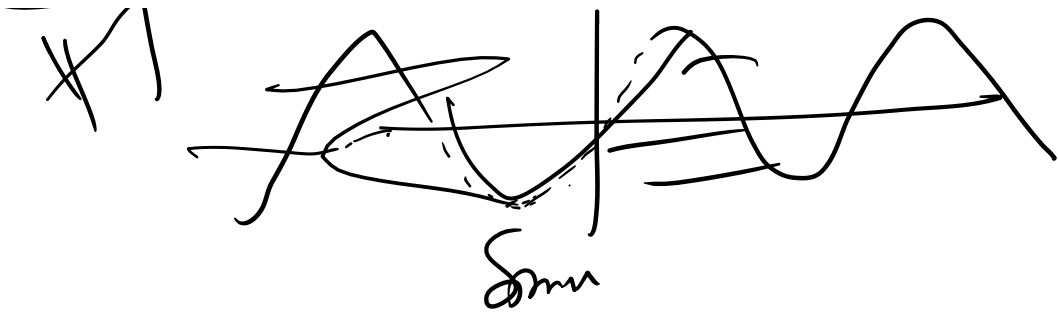
$90423$   
 $95723$

Even vs Odd functions

(In a  $\odot$  like Advanced  $\odot$  way)

Symmetry of Graphs





$$f(n) = f(-n) \text{ Even}$$

$$f(n) = -f(-n) \text{ odd}$$

①  $X$  of 2 odd  $\rightarrow$  Even

$X$  of 2 Even  $\rightarrow$  Even

②

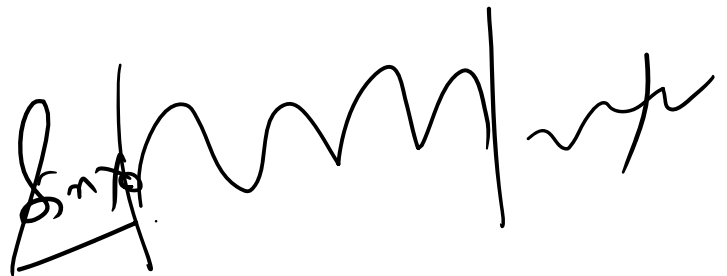
$X$  of odd & Even  $\rightarrow$  odd

③

Energy odd/Even becomes  
square after squaring  
~~test~~

Time base  
( $t$ )

Periodic function



$$y = f(\sin x)$$

$$\sin x = \sin(2\pi t)$$

$$f(\sin x) = f(\sin(2\pi t))$$

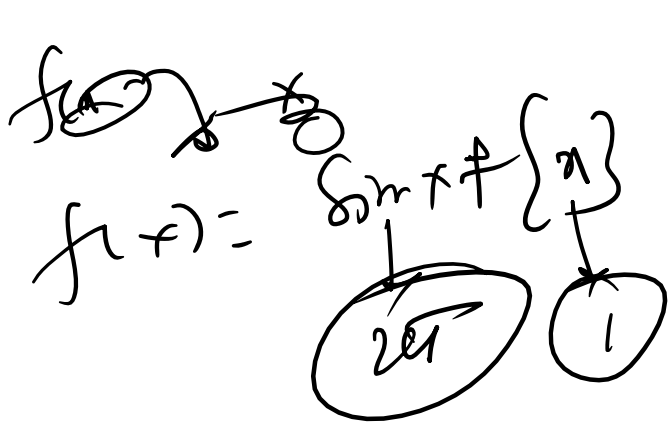
$0 < 100$  of  $f(x)$

Q If  $f(x) = 1e \times$   
 $f(x) = 2^3 \times 5$

Show no strict  
situation systems

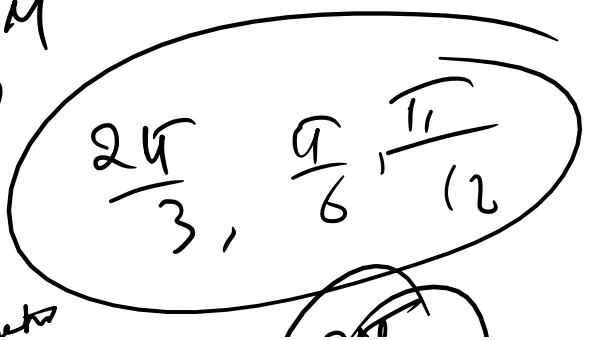


Note How to Take Common between 2 periods



LCM of  $\frac{24}{1}$   
 $\rightarrow 24$

1 # fractional LCM  
LCM??  
~~2~~  
3, 6, ...



Calc of Numerator  
 HCF of denominator

$$\frac{2x}{3}$$

Q. 15 For

$$f(x) = \sin x + \cos(\sqrt{4-a^2})$$

$\sqrt{4-a^2}$  is a rational number only

Let's have  $(4-a^2)$  is a perfect square

$$a = 0, 2, -2$$

$$\begin{aligned} 4-a^2 &= 0 \\ a^2 &= 4 \\ a &= \pm 2 \end{aligned}$$

~~$$f(x) = \cos(\sin x) = (\cos x)$$~~

~~$$f(x) = \cos^{-1}$$~~

$$f(x+1) = f(x)$$



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$$f(x) = \cos^{-1}(\cos x) \\ \cos^{-1}(\cos(x+\pi))$$

$$(2\pi, \pi) \\ \downarrow \\ \text{LCM} \rightarrow 2\pi$$

$$f(x) = \cos^{-1}(\cos x) \\ f(x+\pi) = \cos^{-1}(\cos(x+\pi))$$

$$\text{LCM } 2\pi \\ \text{HCF } \pi$$

#  $f(x+\pi) = f(x)$   
Here period is  $\pi$

$$f(x+3) = f(x)$$

$$f(x) = x^2 + x + 3 \\ f(x_1, x_2, x_3) = 20(x_1, x_2, x_3)$$

~~Given~~  
 $f(f(x)) = ?$

$$f(x) = 2x^2 + 3 \\ \dots (x+3) + 3$$

$f(f(x)) = :$

$$f(x) = 2x^2 + 3$$

$$f(f(x)) = 2(2x^2 + 3)^2 + 3$$
$$= 4x^4 + 6x^2 + 6$$

$$f(f(x)) = 4(2x^2 + 3) + 15$$