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Topics: Problems ND
Rank Correlation
Expectation of r.v.

1. The wage distribution of workers in a factory is normal with mean Rs 400 and s.d Rs 50.
If the wages of 80 workers be less than RS 350, what is the total number (N) of workers in the factory?
Then find the number of workers whose income is above RS 450.
[Given that $\int_0^1 \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = 0.34$]

Solution:
Let the total number of workers be N.
Then

$$N \cdot P(x < 350) = 80$$

$$N \cdot \phi\left(\frac{350-400}{50}\right) = 80$$

$$N \cdot \phi(-1) = 80$$

$$N[1 - \phi(1)] = 80$$

$$\phi(1) = \int_{-\infty}^{1} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \underbrace{\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt}^{0.5}$$

$$\left[\phi(1) = \int_{-\infty}^{1} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt + \int_{0}^{1} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt\right.$$

$$\phi(1) = 0.5 + 0.34 = \boxed{0.84}$$

$$N \times (1 - 0.84) = 80$$

$$N(0.16) = 80$$

$$N = \frac{80}{0.16} = 500 \text{. (Number of workers).}$$

$$P(x > 450) = 1 - P(x \le 450)$$

$$= 1 - \phi\left(\frac{450 - 400}{50}\right)$$

$$= 1 - \phi(1)$$

$$= 1 - 0.84$$

$$= 0.16$$

$\therefore$ No. of workers with wage above Rs 450 $= 500 \times 0.16 = 80$ (ans)

2. The mean of a normally distributed variable x is 10 and $P(x > 14) = 0.0228$.

Find $P(x \le 11)$

Given that $\phi(0.5) = 0.6915$, $\phi(2) = 0.9772$

where $\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$]

$P(x > 14) = 0.0228$

$1 - P(x \leq 14) = 0.0228$

$1 - \phi\left(\frac{14-10}{\sigma}\right) = 0.0228$

$1 - 0.0228 = \phi\left(\frac{4}{\sigma}\right)$

$0.9772 = \phi(4/\sigma)$

$\phi(2) = \phi(4/\sigma)$

$2 = 4/\sigma$

$\boxed{\sigma = 2}$

mean = 10

$\sigma = 2$

$P(x \leq 11) = ?$

$\downarrow$

③ If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and s.d Rs 70 and Rs 5 respectively, find the expected number of workers whose weekly wages are

(i) between Rs 66 and Rs 72 $\quad \underline{P(66 \leq x \leq 72)}$

(ii) less than Rs 66 $\quad$ (iii) more than Rs 72.

Given that $\int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \underline{0.1554}$ and $\underline{0.2881}$ according as $\underline{z = 0.4}$ and $\underline{0.8}$.

(i) $P(66 \leq x \leq 72)$ $\quad \phi\left(\frac{72-70}{}\right)$ $\quad \phi\left(\frac{66-70}{}\right)$

(i) $P(66 \le x \le 72) = \phi\left(\frac{72-70}{5}\right) - \phi\left(\frac{66-70}{5}\right)$

$= \phi(0.4) - \phi(-0.8)$

$= \phi(0.4) - [1 - \phi(0.8)]$

$= \phi(0.4) - 1 + \phi(0.8)$

from question $\int_0^{0.4} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt = 0.1554$

$\therefore \phi(0.4) = \int_{-\infty}^{0.4} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt$

$= \underbrace{\int_{-\infty}^{0} \left(\frac{1}{\sqrt{2\pi}} e^{-t^2/2}\right) dt}_{} + \underbrace{\int_{0}^{0.4} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt}_{}$

$\phi(0.4) = 0.5 + 0.1554 = 0.6554$

$\phi(0.8) = 0.5 + 0.2881 = 0.7881$

$\therefore P(66 \le x \le 72) = \phi(0.4) - 1 + \phi(0.8)$

$= 0.6554 + 0.7881 - 1$

$= 1.4435 - 1$

$= 0.4435$

$\therefore$ No. of workers with weekly wages

∴ No. of workers with salary ... between 66 and 72 are,

$$= N \cdot P(66 \le x \le 72)$$

$$= 10,000 \times 0.4435 = 4435 \text{ (ans)}.$$

(ii) No. less than Rs 66.

(iii) No. more than Rs 72