Thursday, June 8, 2023 7:02 PM

STATISTICS

Probability

Random variable

Randomness

50

Urban Area

High Income Group

India

B1 B2

B50

C1

lower middle class

School

USA middle income

57

C3

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Success Rate is higher

Changing p

Instances

Normal

Poisson | Binomial

Level of Selection Procedures

(I) Integration $\int$ $\iint$ $\iiint$

(II) Partial derivative

(III) Differentiations

(IV) Matrix + Determinant

(V) Beta & Gamma functions

$$f(x) = \alpha x^2 + \beta x + \gamma$$

120

CR Statistical function

SR Stratified fraction

UR — with Constant → SR

without n → UR

Replacement → with

→ without

IIIIIIIIIIIIIIIIIIIIIIIII88888888888888

$^{N}C_n$ → SRSWOR (order ignored)

TTGGGGGGGT

$^{N}P_n$ → SRSWR (order considered)

N^n → SRSWR

TGT

8888888888888888888888888888888888

Random sampling

$X \to$ CRV $\quad f_X(x) = \frac{1}{2} \quad 0 \le x \le 2$

$\qquad\qquad\qquad\qquad 0 \quad$ otherwise

$E\left(|X - E(X)|\right)$

$$E(X) = \int_0^2 x \cdot \frac{x}{2}\, dx = \left(\frac{x^3}{6}\right)\Big|_0^2 = 4/3$$

$$E(X) = \int_0^2 x \cdot x/2 \, dx = \left. \frac{x^3}{6} \right|_0^2$$

$$E(|X - E(X)|) = E(X - 4/3) = \int_0^{4/3} (-x + 4/3)\, x/2 \, dx + \int_{4/3}^2 (x - 4/3)\, x/2 \, dx$$

$$= \left. \left(-\frac{x^3}{6} + \frac{x^2}{3} \right) \right|_0^{x=4/3} + \left. \left(\frac{x^3}{6} - \frac{x^2}{3} \right) \right|_{4/3}^2$$

$$=$$

$f_X(x) = x e^{-x}$ $x > 0$

find the ratio of 1st to 2nd moment

$$\boxed{\int_0^\infty x^{r-1} e^{-x} dx}$$

$r = 0, -1, -2 \ldots$

$n = 1, 2 \ldots$ $\Gamma(n) = (n-1)!$

$$\Gamma(2) = 1 = \int_0^{+\infty} x e^{-x} dx$$

$$E(X) = \int_0^{+\infty} x \cdot x e^{-x} dx = \int_0^{+\infty} x^2 e^{-x} dx = \Gamma(3) = 2! = 2$$

$$E(X^2) = \int_0^{+\infty} x^2 x e^{-x} dx = \int_0^{+\infty} x^3 e^{-x} = \Gamma(4) = 3! = 6$$

$x < 30$ P
$x \geq 30$ t ^{2} + x^2

A bag 10 white 15 black

marbles in a selection of 10 marbles

X - white

X - white marbles

without replacement. $\dfrac{Var(X)}{E(X)}$??

Hypergeometric

Finite Population — Catalyst (1) → Distribution; Cat (2)

without replacement

(i) Finite population
(ii) 2 form
(iii) Draws
(iv) WOR

UNIFORM

All outcomes are equally likely

P density is same

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$M = 25 \quad k = 10 \quad n = 10$

$E(X) = \dfrac{nk}{M} = 4$

$V(X) = \dfrac{nk(M-k)(M-n)}{M^2(M-1)} = 3/2$

$\dfrac{V(X)}{E(X)} = \dfrac{3/2}{4} = 3/8$

X CRV $\quad f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$

$E[X \mid X > 0]$

$$E[X \mid X \geq 0]$$

$X \to N(0,1)$

So, the density function of the conditional distribution

$$f(x \mid X \geq 0) = \frac{f(x)}{P(X \geq 0)} = \frac{f(x)}{1/2} = 2f(x)$$

the conditional expectation is

$$\int_0^\infty x f(x \mid X \geq 0)\,dx = \int_0^\infty 2x \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

$$= -\frac{2}{\sqrt{2\pi}} e^{-x^2/2}\Big|_{x=0}^{x=\infty} = \frac{2}{\sqrt{2\pi}} = \frac{\sqrt{2}}{\sqrt{\pi}}$$

X, Y $\quad f_T(t) = \frac{1}{2\theta} \quad -\theta < t < \theta$

$= 0, \quad$ otherwise

$\mathrm{Var}(XY) = \frac{64}{9}$ then $\theta = ?$

X, Y both have Uniform distribution $(-\theta, \theta)$

$$\frac{64}{9} = \mathrm{Var}(XY) = E[(XY)^2] - (E(XY))^2$$

$$= E(X^2)E(Y^2) - (E(XY))^2$$

$$= \left(\frac{\theta - (-\theta)^2}{12} + \theta^2\right)^2 - \theta^2 \cdot 0^2 = \theta^4/9$$

$\theta^4 = 64 \quad \theta = 2\sqrt{2}$

$\theta^4 = 64 \quad \theta = 2\sqrt{2}$

Joint moment x, y

$$F(x,y) = \int_a^b \int_c^d f(x, y)\, dx\, dy$$

$$\int \int x^2 y \, dx \, dy$$

Inside	Inside
Outside	outside