Production and Cost.

[Supply side Analysis].

Production fn: $q = f(L, K)$.

$L \to$ Labour

$K \to$ Capital.

$q \to$ Quantity produced & supplied by a firm.

Producers are present in 2 markets:

(i) Product Mkt [sell the output produced] => supply side of the mkt (chapter on Mkts).

(ii) Factor Mkt [they buy L & K for production process]. (chapter on Production & Cost)

Obj of a Producer: π- maximization.

By defn: $\pi = R - C = P \cdot q - C$.

(i) When the producer is in the product mkt: Max π choosing 'q'.

(ii) When the producer is in the factor mkt: Max π choosing L, K.

Costs of Producer $\to$ Payment to labour: wage rate (w)

$\to$ Payment for capital: rental rate (r)

$\therefore$ $\boxed{C = wL + rK}$ => Cost Expression for the firm.

$\therefore$ For Factor Market: [Choose opt amt of L & K s.t π is max]

We know, for the firm: Production fn: $q = f(L, K)$

Cost exp: $wL + rK$. [w, r = parameters]

$\pi = R - C = P \cdot q - C = \boxed{P f(L, K) - wL - rK}$. [fn of L, K only]

FOC: $\frac{\partial \pi}{\partial L} = 0 \Rightarrow P \cdot \left(\frac{\partial q}{\partial L}\right) - w = 0 \Rightarrow \boxed{P \cdot MP_L = w}$

$\frac{\partial \pi}{\partial K} = 0 \Rightarrow P \cdot \left(\frac{\partial q}{\partial K}\right) - r = 0 \Rightarrow \boxed{P \cdot MP_K = r}$

$\to$ solving this gives L^* & K^*.

Here $\left.\begin{array}{l} L^* = L^*(w, r, P) \\ K^* = K^*(w, r, P) \end{array}\right\} \Rightarrow$ Labour demand, & Capital demand.

(i) $\Rightarrow P \cdot MP_L = w \Rightarrow P = \dfrac{w}{MP_L}$.

(ii) $\Rightarrow P \cdot MP_K = r \Rightarrow \dfrac{w}{MP_L} \cdot MP_K = r \Rightarrow \boxed{\dfrac{MP_L}{MP_K} = \dfrac{w}{r}}$

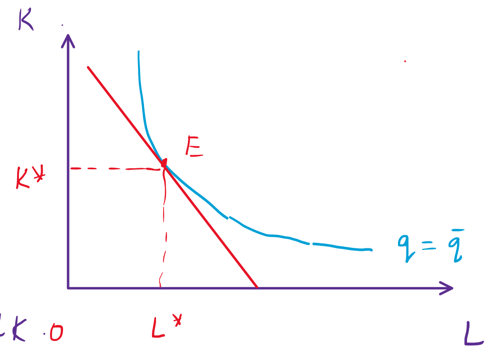
MRTS (slope of isoquant)

slope of cost exp.

Optimization condition.

At opt: $\boxed{MRTS = \dfrac{w}{r}}$

MRTS: Slope of the isoquant.



Check: $q = f(L, K)$

Diff: $dq = \left(\dfrac{\partial q}{\partial L}\right) \cdot dL + \left(\dfrac{\partial q}{\partial K}\right) \cdot dK$.

For isoquant $dq = 0 \Rightarrow 0 = MP_L \cdot dL + MP_K \cdot dK \cdot 0$

$$\frac{dK}{dL} = -\frac{MP_L}{MP_K} \Rightarrow \left|\frac{dK}{dL}\right| = MRTS = \frac{MP_L}{MP_K}.$$

Cost Expression: $C = wL + rK$.

$$dC = w \cdot dL + r \cdot dK.$$

$$dC = 0 \Rightarrow \frac{dK}{dL} = -\frac{w}{r} \Rightarrow \left|\frac{dK}{dL}\right| = \frac{w}{r}.$$

Another approach: Cost Minimization

Fix a level of output $q = \bar{q}$. Determine the minimum level of cost required to produce it.

$\therefore \min\limits_{\{L,K\}} C = wL + rK$ s.t $\bar{q} = f(L, K)$.

$$\mathcal{L} = wL + rK + \lambda\left[\bar{q} - q(L, K)\right].$$

FOC: $\dfrac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow w - \lambda\left(\dfrac{\partial q}{\partial L}\right) = 0 \Rightarrow w - \lambda MP_L = 0$ ---- (i)

FOC: $\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow \quad w - \lambda\left(\frac{\partial q}{\partial L}\right) = 0 \quad \Rightarrow \quad w - \lambda MP_L = 0 \text{ ---- (i)}$

$\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow \quad r - \lambda\left(\frac{\partial q}{\partial K}\right) = 0 \quad \Rightarrow \quad r - \lambda \cdot MP_K = 0 \text{ ---- (ii)}.$

$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow \quad \bar{q} - q(L,K) = 0 \Rightarrow \quad \bar{q} = q(L,K).$

(i): $\frac{w}{MP_L} = \lambda$ (ii) $\frac{r}{MP_K} = \lambda$

Combining: $\frac{w}{MP_L} = \frac{r}{MP_K} \Rightarrow \boxed{\frac{MP_L}{MP_K} = \frac{w}{r}} \Rightarrow$ Optimization condition

We will obtain: $L^* = L^*(w, r, q)$

$K^* = K^*(w, r, q)$

$$C = w\, L^*(w, r, q) + r \cdot K^*(w, r, q).$$

$$= C(w, r, q) \Rightarrow \text{Cost Function}.$$

[Minimum amount of cost reqd to produce a given level of q]