Consumers: Max utility subject to budget constraint.

Max $u = u(x_1, x_2)$ s.t $M = P_1 x_1 + P_2 x_2$.

on solving, optimization condition: $\dfrac{\partial u/\partial x_1}{\partial u/\partial x_2} = \dfrac{P_1}{P_2}$.

Q. $u = \underbrace{(x_1+2)}^2 \overbrace{(x_2+3)}^3$. Find $\left.\dfrac{\partial u}{\partial x_1}\right|_{(x_1=3,\, x_2=3)}$

$$\frac{\partial u}{\partial x_1} = (x_2+3)^3 \, 2(x_1+2)$$

$$\left.\frac{\partial u}{\partial x_1}\right|_{(3,3)} = (3+3)^3 \, 2(3+2) = \ldots$$

Q. $f(x,y) = \dfrac{2x-3y}{x+y}$

$f(x) = \dfrac{u(x)}{v(x)}$

$$\frac{\partial f}{\partial x} = \frac{(x+y)\left\{\frac{\partial}{\partial x}(2x-3y)\right\} - (2x-3y)\left\{\frac{\partial}{\partial x}(x+y)\right\}}{(x+y)^2}$$

$$= \frac{(x+y)(2) - (2x-3y)(1)}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{(x+y)(-3) - (2x-3y)(1)}{(x+y)^2}$$

Q. $f(x,y) = \dfrac{x^2-1}{xy}$.

$$\frac{\partial f}{\partial x} = \frac{x \cdot y \cdot \frac{\partial}{\partial x}(x^2-1) - (x^2-1)\frac{\partial}{\partial x}(xy)}{(xy)^2}$$

$$= \frac{x \cdot y(2x) - (x^2-1)y}{x^2y^2}$$

$$\frac{\partial f}{\partial y} = \frac{x^2-1}{x} \frac{\partial}{\partial y}\left(\frac{1}{y}\right) = \frac{x^2-1}{x}\left(-\frac{1}{y^2}\right)$$

$$\frac{\partial f}{\partial y} = \frac{x^2-1}{x} \cdot \frac{\partial}{\partial y}\left(\frac{1}{y}\right) = \frac{x^2-1}{x}\left(-\frac{1}{y^2}\right)$$

Producers: $\pi = P \cdot q - \underbrace{c(q)}_{\text{cost fn.}}$ .... (Demand side) [Product Mkt].

$\frac{\partial \pi}{\partial q} = 0 \Rightarrow$ Max π w.r.t q.

$q = f(L, K)$ ... Production function. ... (Supply side) [Factor Mkt]

$C = wL + rK$ [w, r = Payment to labour & cap]

$\pi = P \cdot f(L, K) - wL - rK$.

$\therefore \frac{\partial \pi}{\partial L} = 0, \frac{\partial \pi}{\partial K} = 0$ .... choose that amt of labour & cap. to maximize profit.

Elasticity of Substitution: [supply side consideration of producers].

Production fn: $q = f(L, K)$.

$\frac{\partial q}{\partial L} = MP_L, \quad \frac{\partial q}{\partial K} = MP_K \Rightarrow \frac{MP_L}{MP_K} = MRTS_{L,K}$.

Relation b/w $MRTS_{L,K}$ & amt of L & K employed in production process ---- We define elasticity of substitution (σ) to capture this.

$$\sigma = \frac{\% \Delta (K/L)}{\% \Delta (MRTS_{L,K})}$$

$$\sigma = \frac{d[\ln(K/L)]}{d[\ln(MRTS_{L,K})]} \quad \text{.... } (*)$$

$$e_p = \frac{\% \Delta Q_d}{\% \Delta P} = \frac{dQ_d/Q_d \times \cancel{100}}{dP/P \times \cancel{100}} = \frac{(dQ_d/Q_d)}{dP/P} = \frac{d[\ln Q_d]}{d[\ln P]}$$

CES Production Fn:-

$q = A[\delta L^{-\rho} + (1-\delta)K^{-\rho}]^{-1/\rho}$.

d [ln P] .

Q: $q = A\left[\delta L^{-\rho} + (1-\delta)K^{-\rho}\right]^{-1/\rho}$, $\delta > 0$ ...... Find σ.

$$MP_L = \frac{\partial q}{\partial L} = A\left(-\frac{1}{\rho}\right)\left[\delta L^{-\rho} + (1-\delta)K^{-\rho}\right]^{-\frac{1}{\rho}-1} \cdot \left\{\delta \cdot (-\rho)\, L^{-\rho-1}\right\}$$

$$MP_K = \frac{\partial q}{\partial K} = A\left(-\frac{1}{\rho}\right)\left[\delta L^{-\rho} + (1-\delta)K^{-\rho}\right]^{-\frac{1}{\rho}-1} \left\{(1-\delta)(-\rho)\, K^{-\rho-1}\right\}.$$

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{\delta}{1-\delta} \cdot \left(\frac{L}{K}\right)^{-\rho-1} = \left(\frac{\delta}{1-\delta}\right)\left(\frac{K}{L}\right)^{(\rho+1)}$$

$$\ln\left(MRTS_{L,K}\right) = \overbrace{\ln\left(\frac{\delta}{1-\delta}\right)} + (\rho+1)\ln\left(\frac{K}{L}\right).$$

Diff: $$d\left[\ln\left(MRTS_{L,K}\right)\right] = 0 + (\rho+1)\cdot d\left[\ln\left(\frac{K}{L}\right)\right]$$

$$\Rightarrow d\left[\ln\left(MRTS_{L,K}\right)\right] = (\rho+1)\, d\left[\ln\left(\frac{K}{L}\right)\right]$$

$$\frac{1}{\rho+1} = \frac{d\left[\ln\left(\frac{K}{L}\right)\right]}{d\left[\ln\left(MRTS_{L,K}\right)\right]} = \sigma.$$

Q. $u = x^a y^b$ ..... Find Marshallian demand curves for x & y & Indirect utility fn.

$\Rightarrow \left[\text{Max } u = x^a y^b \text{ subject to } M = P_x \cdot x + P_y \cdot y\right]$

utility maximization problem. → Variables: x, y.

Parameters: M, P_x, P_y.

$$L = x^a y^b + \lambda\left[M - P_x \cdot x - P_y \cdot y\right] \text{ ..... Lagrangian.}$$

$\frac{\partial L}{\partial x} = 0 \Rightarrow a \cdot x^{a-1} y^b + \lambda(-P_x) = 0 \Rightarrow a \cdot x^{a-1} y^b = \lambda P_x$ --- (i)

$\frac{\partial L}{\partial y} = 0 \Rightarrow b \cdot x^a y^{b-1} + \lambda(-P_y) = 0 \Rightarrow b \cdot x^a y^{b-1} = \lambda P_y$ --- (ii)

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow M - P_x \cdot x - P_y \cdot y = 0 \Rightarrow M = P_x \cdot x + P_y \cdot y$ --- (iii).

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \quad M - P_x \cdot x - P_y \cdot y = 0 \quad \Rightarrow M = P_x \cdot x + P_y \cdot y \quad \text{--- (iii)}$

Opt $x \Rightarrow x^*$, Opt $y \Rightarrow y^*$

(i) $\div$ (ii) :- $\quad \frac{a \cdot x^{a-1} y^b}{b \cdot x^a y^{b-1}} = \frac{P_x}{P_y} \Rightarrow y = \left(\frac{b}{a}\right)\left(\frac{P_x}{P_y}\right)(x)$

$$\frac{a}{b}\left(\frac{y}{x}\right) = \frac{P_x}{P_y}$$

Put in (iii) : $M = P_x \cdot x + P_y \left(\frac{b}{a}\right)\left(\frac{P_x}{P_y}\right)(x)$

Solving : $\begin{cases} x^* = \left(\frac{a}{a+b}\right)\left(\frac{M}{P_x}\right) \\ y^* = \left(\frac{b}{a+b}\right)\left(\frac{M}{P_y}\right) \end{cases} \rightarrow$ Marshallian demands.

Opt x, Opt y.

$u = x^a y^b \quad \Rightarrow \quad u^* = x^{*a} y^{*b}$

$= u(x, y) \qquad u^* = \left\{\left(\frac{a}{a+b}\right)\left(\frac{M}{P_x}\right)\right\}^a \left\{\left(\frac{b}{a+b}\right)\left(\frac{M}{P_y}\right)\right\}^b$

$= u^*(M, P_x, P_y)$

↳ Indirect utility function.