05 August 2023 14:02

The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is 4

$14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$.

$\frac{14}{\sin^2 x} - 2\sin^2 x = 21 - 4(1-\sin^2 x)$

$14 - 2\sin^4 x = 21\sin^2 x - 4\sin^2 x(1-\sin^2 x)$

$\sin^2 x = y$.

$6\sin^4 x + 17\sin^2 x - 14 = 0$.

$6y^2 + 17y - 14 = 0$

$6y^2 + 21y - 4y - 14 = 0$

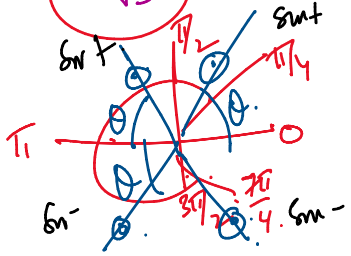
$3y(2y+7) - 2(2y+7) = 0$

$(2y+7)(3y-2) = 0$

$y = -\frac{7}{2}, \frac{2}{3}$ ✓

$0 \le \sin^2 x \le 1$

$\sin x = \pm\frac{\sqrt{2}}{\sqrt{3}}$



$\frac{7\pi}{4} \to$ Quadrant

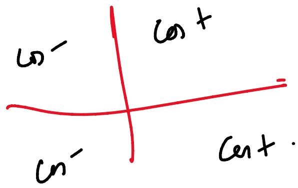
$\frac{7\pi}{4} = \pi + \frac{3\pi}{4}$

$= 2\pi - \frac{\pi}{4}$

$\theta, \pi-\theta$ $\quad \frac{2}{3} = 0.67$

$\sqrt{0.67} = 0.81$

$\sin\frac{\pi}{4} = 0.7$



The number of elements in the set $S = \{\theta \epsilon [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is 16.

$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$

$2\cos^2\theta = 1 + \cos 2\theta$

$3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$.

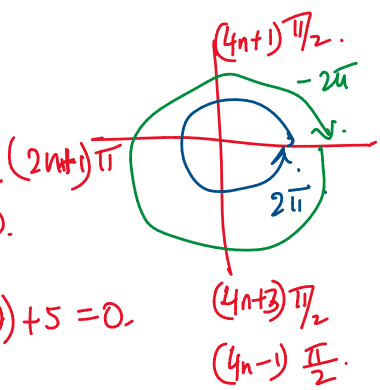
$3\cos^2 2\theta + 6\cos 2\theta - 5(1+\cos 2\theta) + 5 = 0$.

$3\cos^2 2\theta + 6\cos 2\theta - 5\cos 2\theta = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta(3\cos 2\theta + 1) = 0$.

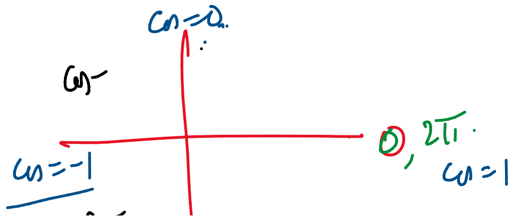
$\theta = \cos(2n+1)\pi/2$



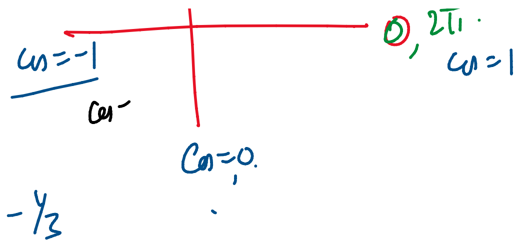
$-4\pi, 4\pi$

$0 - 2\pi$

$\times 4$



$\cos 2\theta\,(3\cos 2\theta + 1) = 0.$



$\cos 2\theta = 0 = \cos(2n+1)\pi/2$

$\cos 2\theta = 0, -1/3.$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}.$

$\cos 2\theta = -1/3$

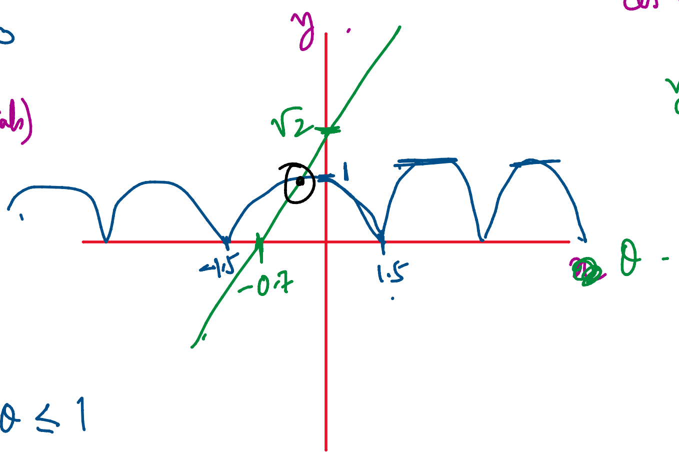
The number of solutions of the equation $2\theta - cos^2\theta + \sqrt{2} = 0$ is R is equal to 1.

$2\theta - \cos^2\theta + \sqrt{2} = 0.$

$\cos^2\theta = 2\theta + \sqrt{2} = y$

$2x - \cos^2 x + \sqrt{2} = 0$

1. algebraic (polynomials)
2. Trigonometric
3. Exponential
4. Logarithmic.



$y = \cos^2\theta$

$y = 2\theta + \sqrt{2}$

$y = \cos^2\theta.$

$-1 \le \cos\theta \le 1$

$0 \le \cos^2\theta \le 1$

The number of solutions of the equation sin x = $\cos^2$ x in the interval (0,10) is 4.

$\sin x = \cos^2 x.$

$\sqrt{5} = 2.2$

$\pi = 3.1$

$\sin x = 1 - \sin^2 x.$

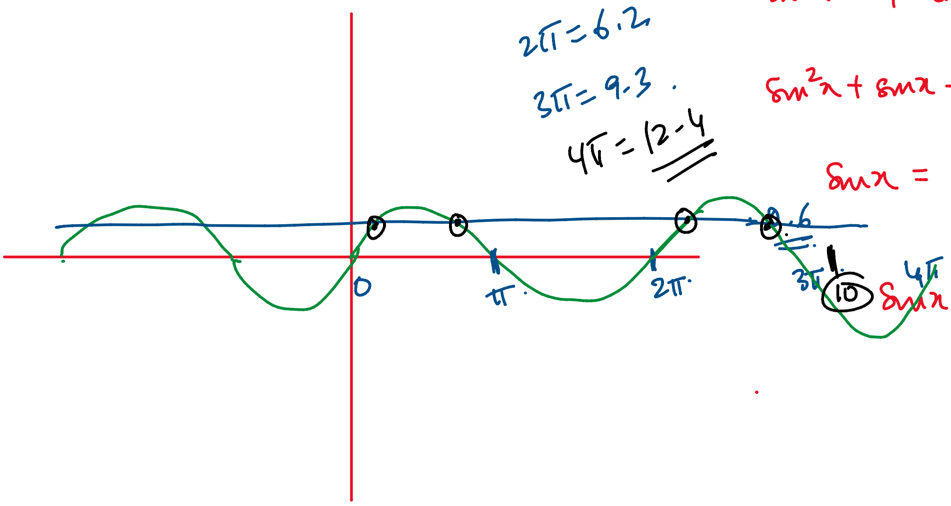
$2\pi = 6.2$

$3\pi = 9.3$

$\sin^2 x + \sin x - 1 = 0.$

$4\pi = 12.4$

$\sin x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}.$



$\sin x = \frac{\sqrt{5}-1}{2}, \frac{-\sqrt{5}-1}{2}.$

+ve, -ve.

0.6. ~~1.6~~

Prove that the equation $k\cos x - 3\sin x = k + 1$ possess a solution iff $k \in (-\infty, 4]$.

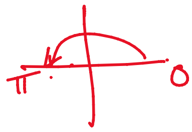
Solve : $\cos\theta + \cos3\theta + \cos5\theta + \cos7\theta = 0$

Solve : $\cos\theta \cos2\theta \cos3\theta = \frac{1}{4}$; where $0 \le \theta \le \pi$.

$\sin^4 x + \cos^4 x = \sin x \cos x.$

$$\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right).\cos x = 0$$

: Solve the equation $(\sin x + \cos x)^{1+\sin 2x} = 2$, when $0 \le x \le \pi$.



$$(\sin x + \cos x)^{(1+\sin 2x)} = 2.$$

$$1+\sin 2x = \sin^2 x + \cos^2 x + 2\sin x\cos x$$
$$= (\sin x + \cos x)^2$$

$$(\sin x + \cos x)^{(\sin x + \cos x)^2} = 2.$$

$$\sin x + \cos x = y$$

$$y^{y^2} = 2.$$

$$y^2 = t \qquad y = \sqrt{t}.$$

$$-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$$

$$y = \pm\sqrt{2}$$

$$-\sqrt{2} \le \sin x + \cos x \le \sqrt{2}$$

$$-\sqrt{2} \le y \le \sqrt{2}$$

$$(\sqrt{t})^t = 2.$$

$$(t^{1/2})^t = 2 \qquad t^{t/2} = 2.$$

$$\sin x + \cos x = \pm\sqrt{2}$$

$$x = \frac{\pi}{4}, \ \pi + \frac{\pi}{4}$$

$$x = \frac{\pi}{4}, \ \frac{5\pi}{4}.$$

$$(t^{t/2})^2 = 2^2$$

$$t^t = 2^2$$

$$t = 2,$$

Solve for x and y : $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + 1/2} \le 1$

$(4n+1)\pi/2$

$\frac{1}{2}$ = min.

$$\cos^2 x = 1 \Rightarrow \cos x = \pm 1$$

Solve for x and y : $2^{\cos^2 x}\sqrt{y^2 - y + 1/2} \le 1$

$\cos^2 x = 1 \Rightarrow \cos x = \pm 1$

$y = \frac{1}{2}$ $\quad x = n\pi$

$(4n+1)\pi/2$

$\cos = -1$ $\quad 2n\pi$ $\quad \cos = 1$

$(2n+1)\pi$

$(4n+3)\pi/2$

min = 2

$0 \le \cos^2 x \le 1$

$\boxed{1 \le \frac{1}{\cos^2 x}} \le \infty$

$\min\left(2^{\frac{1}{\cos^2 x}}\right) = 2$

$\min\left(\frac{1}{\cos^2 x}\right) = 1$

$y^2 - y + \frac{1}{2} = (y - \frac{1}{2})^2 + \frac{1}{4}$

$\ge 0.$

$\min(y^2 - y + \frac{1}{2}) = \frac{1}{4}.$

$\min(LHS) = 2 \times \frac{1}{2} = 1$

$LHS \ge 1$

Method of boundary values / extreme values