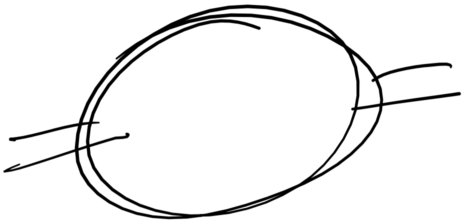
STAT 7.3.2024

07 March 2024 20:38

Statistics

Advanced Level Confidence Interval



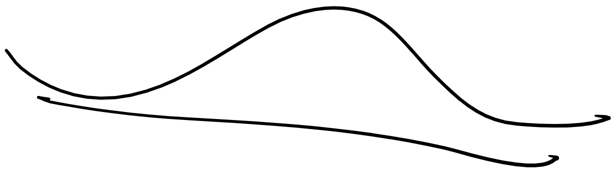
9062395123

Random is not always Random

Level of Randomness

e^{-1}, ln 27814, 78 $e^{\ln 32}$

1, 2, 94



~~ISI~~

~~CSIR~~

~~ISS~~

~~UPSC Stat~~

95%
99%
Account Some

ISI 2017

X_1, X_2 from $N(0,\theta)$ $\quad \theta > 0$

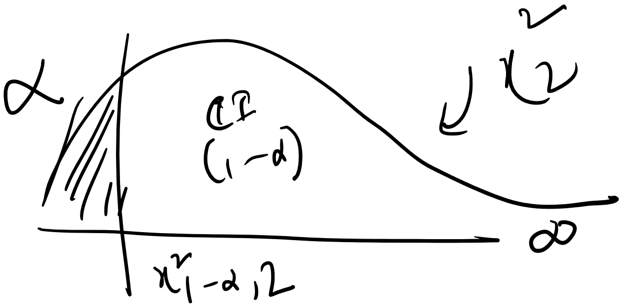
then the value of K for which the interval $\left(0, \frac{X_1^2+X_2^2}{K}\right)$ is a 95% CI for θ?

Ans: $\frac{X_1}{\sqrt{\theta}} \sim N(0,1) \Rightarrow \frac{X_1^2}{\theta} \sim \chi^2_1$

$X_2 \sim N(0,\theta) \Rightarrow \frac{X_2}{\sqrt{\theta}} \sim N(0,1)$

$\Rightarrow \frac{X_2^2}{\theta} \sim \chi^2_1$

Let, $Y = \frac{X_1^2+X_2^2}{\theta} \sim \chi^2_2$



Now, $(1-\alpha) = 95\%$, $\alpha = 5\%$ $\quad \chi^2_{0.95,2} = 0.1026$

$0.95 = P\left(0.1026 < \chi^2_2 < \infty\right)$

$= \cancel{P\left(\frac{1}{\infty} < \frac{\theta}{X_1^2}\right)}$

$= P\left(0.1026 < \frac{X_1^2+X_2^2}{\theta} < \infty\right)$

$\Rightarrow P\left(\frac{1}{\infty} < \frac{\theta}{X_1^2+X_2^2} < \frac{1}{0.1026}\right)$

$= P\left(0 < \theta < \frac{X_1^2+X_2^2}{}\right)$

$$\Rightarrow P\left(0 < \theta < \frac{x_1^2+x_2^2}{0.1026}\right)$$

$$\theta \in \left(0, \frac{x_1^2+x_2^2}{0.1026}\right)$$

$$k = -2\ln(0.95)$$
$$= 0.1026$$

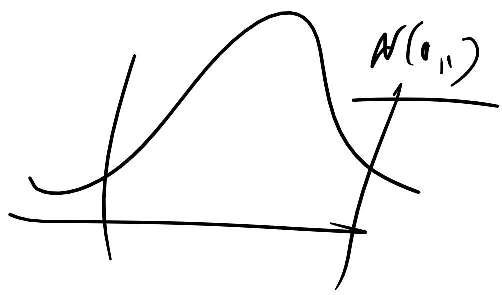
Normal Approx to Binomial

$$\theta \in \left(\hat\theta \pm z_{\alpha/2}\sqrt{\frac{\hat\theta(1-\hat\theta)}{n}}\right)$$

$$\Rightarrow \hat\theta \pm 1.96 \sqrt{\frac{\hat\theta(1-\hat\theta)}{n}}$$ 95%

Normal Approximation to Poisson dis

$$\left(\bar X \pm 1.96\sqrt{\frac{\bar X}{n}}\right)$$



Q3. $x_1, x_2 \ldots x_n \longrightarrow N(\mu_1, \sigma^2)$

& $y_1, y_2, y_3 \longrightarrow N(\mu_2, \sigma^2)$

(x_1, x_2, x_3) & (y_1, y_2, y_3) are independent.

Let the observed value of $\left[\sum^3 x_i - \frac{1}{3}(x_1+x_2+x_3)\right]^2$

Let the observed value of $\sum_{i=1}^{3}\left[x_i - \frac{1}{3}(x_1+x_2+x_3)\right]^2$ & $\sum_{i=1}^{3}\left[y_i - \frac{1}{3}(y_1+y_2+y_3)\right]^2$ be 1 & 5.

The length of shortest 90% CI of $\mu_1-\mu_2$ = ??

Ans: t-dist $\boxed{\sigma_1^2=\sigma_2^2=\sigma^2}$

$n_1=3,\ n_2=3,\ s_1^2=\frac{1}{2},\ s_2^2=\frac{5}{2}$

$$S_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{3/2}$$

$$\text{Length} = 2\left[t_{n_1+n_2-2,\frac{\alpha}{2}}\cdot S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right]$$

$$= 2\left[2.13\times S_p\sqrt{\frac{2}{3}}\right]$$

$$= 4.26$$

90% CI for $\mu_1-\mu_2$ is 4.26

X_1, X_2 iid

PDF

$$f(x) = e^{-(x-\theta)} \quad x>\theta$$
$$=0, \text{ otherwise}$$

Let, $Y=\min\{X_1,X_2\}$

find CI for θ of the type $[Y-b, Y]$

$0\le b<\infty$

Ans: cdf of $x_1, x_2 \ldots$

$f(x) = e^{-(x-\theta)}$ $\quad x > \theta$

$F(x) = 1 - e^{-(x-\theta)}$, $x \geq \theta$

Pdf of $Y = \min(x_1, x_2)$

$f_{x_{(1)}}(x) = f_Y(y) = n\left[1 - F(y)\right]^{n-1} f_Y(y)$

$= 2\left[1 - F(y)\right] f_Y(y)$

$= 2\left[e^{-(y-\theta)}\right] e^{-(y-\theta)}, \; y > \theta$

$= \boxed{2 e^{-2(y-\theta)}}$

Now,

$0.95 = P(Y - b \leq \theta \leq Y)$

$= P(\theta \leq Y \leq \theta + b)$

$= \int_{\theta}^{\theta+b} f_{(y)}(y)\, dy$

$= \int_{\theta}^{\theta+b} 2 e^{-2(y-\theta)} dy$

Let, $2(y-\theta) = t$

$2dy = dt$

$y = \theta \quad t = 0$

$y = \theta + b \; , \; t = 2b$

$= \int_{0}^{2b} e^{-t} dt$

$= \left[-e^{-t}\right]_0^{2b}$

$2b \Rightarrow e^{-2b} = \ln(0.05)$

$= L$

$\ln,\ 0.95 = 1 - e^{-2b} \Rightarrow e^{-2b} = \ln(0.05)$

$b = 1.49$

CI $\left[Y - 1.49,\ Y \right] \longrightarrow$ at 0.95

?

$X_1, X_2, \dots X_n$ $\boxed{n \geq 5}$

$$f(x;\theta) = e^{-x+\theta} \quad x \geq \theta$$
$$= 0 \quad \text{o/w}$$

a) 95% CI of θ how to be of finite length

b) $\min\{x_1, x_2 \dots x_n\} + \frac{1}{n}\ln(0.05)$, $\min(x_1, x_2 \dots x_n)$

is a 95% of θ

c) 95% of CI $\rightarrow$ length (1)

d) 95% of CI of θ $\rightarrow$ len (2)

$$f(x, \mu) = e^{\theta - x} \quad x > \theta$$

PDF of $X_{(1)} = \min(x_1, x_2 \dots x_n)$

$$f_{X_{(1)}}(x) = n\left[1 - F(x)\right]^{n-1} f(x)$$
$$= n\left[e^{\theta - x}\right]^{n-1} e^{\theta - x} \quad x > \theta$$
$$= n e^{n(\theta - x)}$$