Pg Math 15th June 2023 Only Questions

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Pg Math 15th June...

$T(p(x)) = p'(x)$

$T(1) = 0 = 0.1 + 0.x + 0.x^2 + 0.x^3$

$T(x) = 1 = 1.1 + 0.x + 0.x^2 + 0.x^3$

$T(x^2) = 2x = 0.1 + 2.x + 0.x^2 + 0.x^3$

$T(x^3) = 3x^2 = 0.1 + 0.x + 3x^2 + 0.x^3$

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matrix of T on basis B

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1. Let W be the vector space of all real polynomials of degree at most 3. Define $T: W \to W$ by $(Tp)(x) = p'(x)$, where p' is the derivative of p. The matrix of T in the basis $\{1, x, x^2, x^3\}$, considered as column vectors, is given by

1. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ 2. $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}$ 3. $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 4. $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

2. Let $S = \{A : A = [a_{ij}]_{5\times 5}, a_{ij} = 0 \text{ or } 1 \ \forall i, j, \sum_j a_{ij} = 1 \ \forall i \text{ and } \sum_i a_{ij} = 1 \ \forall j\}$. Then the number of elements in S is

1. 5^2 2. 5^5 3. $5!$ 4. 55

We can interchange Rows ..

$A =$ $5!$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & & & 0 \\ 0 & & 1 & & \\ & & & & 1 \end{bmatrix}$$

3. Let ξ be a primitive fifth root of unity. Define $A = \begin{pmatrix} \xi^{-2} & 0 & 0 & 0 & 0 \\ 0 & \xi^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{pmatrix}$

For a vector $v = (v_1, v_2, v_3, v_4, v_5) \in \mathbb{R}^5$, define $|v|_A = \sqrt{|vAv^T|}$, where v^T is transpose of v. If $w = (1, -1, 1, 1, -1)$, then $|w|_A$ equals

1. 0 2. 1 3. -1 4. 2

4. The dimension of the vector space of all symmetric matrices of order $n \times n$ $(n \geq 2)$ with real entries and trace equal to zero is

1. $\left(\frac{n^2 - n}{2}\right) - 1$ 2. $\left(\frac{n^2 + n}{2}\right) - 1$ 3. $\left(\frac{n^2 - 2n}{2}\right) - 1$ 4. $\left(\frac{n^2 + 2n}{2}\right) - 1$

5. Let D be a non-zero $n \times n$ real matrix with $n \geq 2$. Which of the following implications is valid?

1. det(D) = 0 implies rank (D)=0 ✗
2. det(D) = 1 implies rank (D) ≠ 1 ✓
3. rank(D)=1 implies det(D) ≠ 0 ✗
4. rank(D)=n implies det(D) ≠ 1 ✗

$n \geq 2$ Let $n = 2$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ det = 0   R(D) = 1 → Non zero Rows

$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ R(D) = 2   det = 1

$D = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ R = 2   det = −1+1 = 0

6. Suppose A, B are $n\times n$ positive definite matrices and I be the $n\times n$ identity matrix. Then which of the following are positive definite.

1. $A+B$ 2. ABA 3. A^2+I 4. AB

7. Let N be a 3×3 non-zero matrix with the property $N^3=O$. Which of the following is/are true?

1. N is not similar to a diagonal matrix.
2. N is similar to a diagonal matrix.
3. N has one non-zero eigenvector.
4. N has three linearly independent eigenvectors.

$N^3=0$

N is nonzero nilpotent → never diagonalizable

N is NOT similar to diagonal matrix.

0 is Eigenvalue of N

0 is EV of N

$\lambda=0$

N can't have

AM of 0 = 3

8. Let $a_{ij}=a_ia_j$, $1\le i, j\le n$, where $a_1,\ldots,a_n$ are real numbers. Let $A=((a_{ij}))$ be the $n\times n$ matrix $((a_{ij}))$. Then

1. it is possible to choose $a_1,\ldots,a_n$ so as to make the matrix A non-singular.
2. the matrix A is positive definite if $(a_1,\ldots,a_n)$ is a non-zero vector.
3. the matrix A is positive semidefinite for all $(a_1,\ldots,a_n)$.
4. for all $(a_1,\ldots,a_n)$, zero is an eigenvalue of A.

9. Let T be a linear transformation on the real vector space $\mathbb{R}^n$ over $\mathbb{R}$ such that $T^2 = \lambda T$ for some $\lambda \in \mathbb{R}$. Then

1. $\| Tx \| = | \lambda | \| x \|$ for $x \in \mathbb{R}^n$.
2. If $\| Tx \| = \| x \|$ for some non-zero vector $x \in \mathbb{R}^n$, then $\lambda = \pm 1$
3. $T = \lambda I$, where I is the identity transformation on $\mathbb{R}^n$.
4. If $\| Tx \| > \| x \|$ for a non-zero vector $x \in \mathbb{R}^n$, then T is necessarily singular.

10. Let M be the vector space of all 3×3 real matrices and let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$. Which of the following are subspaces of M?

1. $\{X \in M : XA = AX\}$
2. $\{X \in M : X + A = A + X\}$
3. $\{X \in M : trace(AX) = 0\}$
4. $\{X \in M : \det(AX) = 0\}$

11. Let $W = \{ p(B) : p$ is a polynomial with real coefficients$\}$, where $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. The dimension d of the vector space W satisfies

1. $4 \le d \le 6$
2. $6 \le d \le 9$
3. $3 \le d \le 8$
4. $3 \le d \le 4$

|A| = ?
Only 2 diagonal all are zeros then this formula is applicable.

12. The determinant of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ is

6×6

1. [illegible]
2. -9
3. [illegible] -27
4. 1

$a_{ij} = \alpha$, $i = j$
$a_{ij} = \beta$, $i + j = n + 1$, $n = 6 \times 6$
$a_{ij} = 0$ otherwise.

$|A| = (\alpha^2 - \beta^2)^{n/2}$
$= (1^2 - 2^2)^{6/2}$
$= -27$

13. For a positive integer n, let P_n denote the space of all polynomials p(x) with coefficients in $\mathbb{R}$ such that deg $p(x) \leq n$, and let B_n denote the standard basis of P_n given by $B_n = \{1, x, x^2, \ldots, x^n\}$. If $T : P_3 \to P_4$ is the linear transformation defined by $T(p(x)) = x^2 p'(x) + \int_0^x p(t)dt$ and $A = (a_{ij})$ is the 5×4 matrix of T with respect to standard bases B_3 and B_4, then

1. $a_{32} = \frac{3}{2}$ and $a_{33} = \frac{7}{3}$
2. $a_{32} = \frac{3}{2}$ and $a_{33} = 0$
3. $a_{32} = 0$ and $a_{33} = \frac{7}{3}$
4. $a_{32} = 0$ and $a_{33} = 0$

14. Let A be a 5×4 matrix with real entries such that the space of all solutions of the linear system $AX^t = [1,2,3,4,5]^t$ is given by $\{[1+2s, 2+3s, 3+4s, 4+5s]^t : s \in \mathbb{R}\}$. (Here, M^t denotes the transpose of a matrix M). Then the rank of A is equal to

1. 4
2. 3
3. 2
4. 1

det A = product of E.v.

tr(A) = Σ EV

15. Let A be a 3×3 matrix with real entries such that $\det(A)=6$ and the trace of A is 0. If $\det(A+I)=0$, where I denotes the 3×3 identity matrix, then the eigenvalues of A are

1. -1,2,3
2. -1,2,-3
3. 1,2,-3
4. -1,-2,3

EV of A ≠ −1, 2, 3 |A| = −6 ≠ 6

−1, 2, −3   tr(A) = −1+2−3 = −2 ≠ 0

1, 2, −3   det −6 ≠ 6

−1, −2, 3 |A| = 6   tr(A) = 0

16. Suppose the matrix $A = \begin{pmatrix} 40 & -29 & -11 \\ -18 & 30 & -12 \\ 26 & 24 & -50 \end{pmatrix}$ has a certain complex number $\lambda \neq 0$ as an eigenvalue. Which of the following numbers must also be an eigenvalue of A?

1. $\lambda + 20$
2. $\lambda - 20$
3. $20 - \lambda$
4. $-20 - \lambda$

Sum 1st Row element = 0

$A\begin{bmatrix}1\\1\\1\end{bmatrix} = 0\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$ → EV of A

$\lambda \neq 0$ is an E.V. of A

Σ EV = trace

tr A = 0 + λ + λ₁

λ₁ = −λ + 20 = (20 − λ) must be an eigenvalue of A

B7 / B8

Companion matrix $C_A(x) = m_A(x) = x^4 - 5x^2 + 4$
$= (x^2 - 4)(x^2 - 1)$
$= (x+2)(x-2)(x+1)(x-1)$

EV = 2, −2, 1, −1

All EV are distinct → diagonalizable

17. Let $A = \begin{bmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Then a Jordan canonical form of A is

1. $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

2. $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

Jordan Canonical form A =
J(−1)   J(1)

1. $\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ ✓ 2. ✗ $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ 4. $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

Jordan Canonical form $A =$ $\begin{bmatrix} J(-1) & & & \\ & J(1) & & \\ & & J(2) & \\ & & & J(-2) \end{bmatrix}$ $= \begin{bmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & -2 \end{bmatrix}$

18. *Let n be a positive integer and let H_n be the space of all $n \times n$ matrices $A = (a_{ij})$ with entries in $\mathbb{R}$ satisfying $a_{ij} = a_{rs}$, whenever $i + j = r + s (i, j, r, s = 1, \ldots, n)$. Then the dimension of H_n, as a vector space over $\mathbb{R}$, is*

1. n^2 2. n^2-n+1 3. $2n+1$ 4. $2n-1$

19. Consider a matrix $A = (a_{ij})_{n \times n}$ with integer entries such that $a_{ij} = 0$ for $i > j$ and $a_{ii} = 1$ for $i = 1, \ldots, n$. Which of the following properties must be true?

1. A^{-1} exists and it has integer entries
2. A^{-1} exists and it has some entries that are not integers
3. A^{-1} is a polynomial function of A with integer coefficients
4. A^{-1} is not a power of A unless A is the identity matrix

20. Let J be the 3×3 matrix all of whose entries are 1. Then

1. 0 and 3 are the only eigenvalues of A
2. J is positive semidefinite, i.e., $\langle Jx, x \rangle \geq 0$ for all $x \in \mathbb{R}^3$
3. J is diagonalizable
4. J is positive definite, i.e., $\langle Jx, x \rangle > 0$ for all $x \in \mathbb{R}^3$ with $x \neq 0$.

21. Let A, B be complex $n \times n$ matrices. Which of the following statements are true?

1. If A, B and $A+B$ are invertible, then $A^{-1} + B^{-1}$ is invertible.
2. If A, B and $A+B$ are invertible, then $A^{-1} - B^{-1}$ is invertible.
3. If AB is nilpotent, then BA is nilpotent.
4. Characteristic polynomials of AB and BA are equal if A is invertible.

det(A) = 0 ∃ non-zero minor order = (2)

ρ(A) = 2. 0 → EV of A (∵ det A = product of E.V.)

22. Let ω be a complex number such that $\omega^3 = 1$, but $\omega \neq 1$. If $A = \begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{bmatrix}$, then which of the following statements are true?

1. A is invertible ✗
2. rank$(A)=2$ ✓
3. 0 is an eigenvalue of A ✓
4. there exist linearly independent vectors $v, w \in \mathbb{C}^3$ such that $Av=Aw=0$ ✗

GM of 0 = 3 − ρ(A − 0·I) = 3 − 2 = (1)

∃ only 1 linearly independent vector v such that Av = 0

(4) ✗

23. Let A be a 4×4 matrix with real entries such that -1,1,2,-2 are its eigenvalues. If $B = A^4 - 5A^2 + 5I$, where I denotes the 4×4 identity matrix, then which of the following statements are correct?

1. det(A+B)=0
2. det(B)=1
3. trace of A-B is 0
4. trace of A+B is 4

24. Let $M_2(\mathbb{R})$ denote the set of 2×2 real matrices. Let $A \in M_2(\mathbb{R})$ be of trace 2 and determinant -3. Identifying $M_2(\mathbb{R})$ with $\mathbb{R}^4$, consider the linear transformation $T: M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T(B)=AB$. Then which of the following statements are true?

1. T is diagonalizable
2. 2 is an eigenvalue of T
3. T is invertible
4. $T(B)=B$ for some $0 \neq B$ in $M_2(\mathbb{R})$

25. Let A be a 2×2 non-zero matrix with entries in $\mathbb{C}$ such that $A^2=O$. Which of the following statements must be true?

1. PAP^{-1} is diagonal for some invertible 2×2 matrix P with entries in $\mathbb{R}$.
2. A has two distinct eigenvalues in $\mathbb{C}$
3. A has only one eigenvalue in $\mathbb{C}$ with multiplicity 2
4. $Av=v$ for some $v\in \mathbb{C}^2$, $v\neq 0$

26. Consider the linear transformation $T:\mathbb{R}^7\to\mathbb{R}^7$ defined by $T(x_1,x_2,\ldots,x_6,x_7)=(x_7,x_6,\ldots,x_2,x_1)$. Which of the following statements are true?

1. The determinant of T is 1
2. There is a basis of $\mathbb{R}^7$ with respect to which T is a diagonal matrix
3. $T^7=I$
4. The smallest n such that $T^n=I$, is even

27. Let λ,μ be distinct eigenvalues of a 2×2 matrix A. Then, which of the following statements must be true?

1. A^2 has distinct eigenvalues
2. $A^3=\dfrac{\lambda^3-\mu^3}{\lambda-\mu}A-\lambda\mu(\lambda+\mu)I$
3. trace of A^n is $\lambda^n+\mu^n$ for every positive integer n
4. A^n is not a scalar multiple of identity for any positive integer n

28. Let A, B be $n \times n$ real matrices. Which of the following statements is correct?

1. rank (A+B) = rank (A) + rank (B)
2. rank (A+B) ≤ rank (A) + rank (B)
3. rank (A+B) = min {rank (A), rank (B)}
4. rank (A+B) = max {rank (A), rank (B)}

29. Let ξ be a primitive cube root of unity. Define $A=\begin{bmatrix}\xi^{-1} & 0\\ 0 & \xi\end{bmatrix}$. For a vector $\mathbf{v}=(v_1, v_2, v_3)\in \mathbb{R}^3$ define $|\mathbf{v}|_A=\sqrt{|\mathbf{v}A\mathbf{v}^T|}$, where $\mathbf{v}^T$ is transpose of $\mathbf{v}$. If $\mathbf{w}=(1,1,1)$ then $|\mathbf{w}|_A$ equals

1. 0
2. 1
3. -1
4. 2

30. The dimension of the vector space of all symmetric matrices $A=(a_{ij})$ of order $n\times n(n\geq 2)$ with real entries, $a_{11}=0$ and trace zero is

1. $(n^2+n-4)/2$
2. $(n^2-n+4)/2$
3. $(n^2+n-3)/2$
4. $(n^2-n+3)/2$

$A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $B=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$ $A+B=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

R(A) = 2

R(B) = 2 R(A+B) = 0

R(A+B) ≠ min{R(A), R(B)}

R(A+B) ≠ max{R(A), R(B)}

(1-50)

31. *Let N be the vector space of all real polynomials of degree at most 3. Define $S : N \to N$ by $S(p(x)) = p(x+1)$, $p \in N$. Then the matrix of S in the basis $\{1, x, x^2, x^3\}$ considered as column vectors is given by:*

1. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$
2. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
3. $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 3 & 3 & 3 & 3 \end{bmatrix}$
4. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

32. Which of the following matrices are positive definite?

1. $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ 2. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ 3. $\begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$ 4. $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$

33. Let A be a non-zero linear transformation on a real vector space V of dimension n. Let the subspace $V_0 \subset V$ be the image of V under A. Let $k = \dim V_0 < n$ and suppose that for some $\lambda \in \mathbb{R}$, $A^2 = \lambda A$. Then

1. $\lambda = 1$
2. $\det A = |\lambda|^n$
3. λ is the only eigenvalue of A
4. there is a nontrivial subspace $V_1 \subset V$ such that $Ax = 0$ for all $x \in V_1$

34. Let C be an $n \times n$ real matrix. Let W be the vector space spanned by $\{I, C, C^2, ..., C^{2n}\}$. The dimension of the vector space W is

1. $2n$ 2. atmost n 3. n^2 4. atmost $2n$

35. Let V_1, V_2 be subspaces of a vector space V. Which of the following is/are necessarily a subspace of V?

1. $V_1 \cap V_2$
2. $V_1 \cup V_2$
3. $V_1 + V_2 = \{x + y : x \in V_1, y \in V_2\}$
4. $V_1/V_2 = \{x \in V_1 \text{ and } x \notin V_2\}$

36. Let N be a non-zero 3×3 matrix with the property $N^2 = O$. Which of the following is/are true?

1. N is not similar to a diagonal matrix.
2. N is similar to a diagonal matrix.
3. N has one non-zero eigenvector.
4. N has three linearly independent eigenvectors.

37. Let n be a positive integer and let $M_n(\mathbb{R})$ denote the space of all $n \times n$ real matrices. If $T: M_n(\mathbb{R}) \to M_n(\mathbb{R})$ is a linear transformation such that $T(A)=0$ whenever $A \in M_n(\mathbb{R})$ is symmetric or skew-symmetric, then the rank of T is

1. $\frac{n(n+1)}{2}$ 2. $\frac{n(n-1)}{2}$ 3. n 4. 0

38. Let $S: \mathbb{R}^3 \to \mathbb{R}^4$ and $T: \mathbb{R}^4 \to \mathbb{R}^3$ be linear transformations such that $T \circ S$ is the identity map of $\mathbb{R}^3$. Then

1. $S \circ T$ is the identity map of $\mathbb{R}^4$
2. $S \circ T$ is one–one, but not onto.
3. $S \circ T$ is onto, but not one-one.
4. $S \circ T$ is neither one–one nor onto.

39. Let V be a 3-dimensional vector space over the field $F_3 = \mathbb{Z}/3\mathbb{Z}$ of 3 elements. The number of distinct 1- dimensional subspaces of V is

1. 13   2. 26   3. 9   4. 15

40. Let V be the inner product space consisting of linear polynomials, $p: [0,1] \to \mathbb{R}$ (i.e., V consists of polynomials p of the form $p(x) = ax + b;\ a,\ b \in \mathbb{R}$), with the inner product defined by

$\langle p, q \rangle = \int_0^1 p(x)q(x)\,dx$ for $p,q \in V$. An orthonormal basis of V is

1. $\{1, x\}$ 2. $\{1, x\sqrt{3}\}$ 3. $\{1, (2x-1)\sqrt{3}\}$ 4. $\left\{1, x-\frac{1}{2}\right\}$

41. Let $f(x)$ be the minimal polynomial of the 4X4 matrix $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. Then the rank of the 4 X 4 matrix $f(A)$ is

1. 0 2. 1 3. 2 4. 4

42. Let a, b, c be positive real numbers such that $b^2 + c^2 < a < 1$. Consider the 3X3 matrix $A = \begin{bmatrix} 1 & b & c \\ b & a & 0 \\ c & 0 & 1 \end{bmatrix}$

1. All the eigenvalues of A are negative real numbers.
2. All the eigenvalues of A are positive real numbers.
3. A can have a positive as well as a negative eigenvalue.
4. Eigenvalues of A can be non-real complex numbers.

43. The system of equations $x - y - z = 1$, $2x + 3y - z = 5$, $x + 2y - kz = 4$, where $k \in \mathbb{R}$, has an infinite number of solutions for

1. $k = 0$ 2. $k = 1$ 3. $k = 2$ 4. $k = 3$

44. Let n be an integer, $n \ge 3$ and let $u_1, u_2, \ldots, u_n$ be n linearly independent elements in a vector space over $\mathbb{R}$. Set $u_0 = 0$ and $u_{n+1} = u_1$. Define $v_i = u_i + u_{i+1}$ and $w_i = u_{i-1} + u_i$ for $i = 1,2,\ldots,n$. Then

1. $v_1, v_2, \ldots, v_n$ are linearly independent, if $n = 2010$.
2. $v_1, v_2, \ldots, v_n$ are linearly independent, if $n = 2011$.
3. $w_1, w_2, \ldots, w_n$ are linearly independent, if $n = 2010$.
4. $w_1, w_2, \ldots, w_n$ are linearly independent, if $n = 2011$.

45. Let V and W be finite-dimensional vector spaces over $\mathbb{R}$ and let $T_1: V \to V$ and $T_2: W \to W$ be linear transformations whose minimal polynomials are given by $f_1(x) = x^3+x^2+x+1$ and $f_2(x) = x^4 - x^2 - 2$. Let $T: V \oplus W \to V \oplus W$ be the linear transformation defined by $T((v, w)) = (T_1(v), T_2(w))$ for $(v,w) \in V \oplus W$ and let $f(x)$ be the minimal polynomial of T. Then

1. $\deg f(x) = 7$ 2. $\deg f(x) = 5$ 3. nullity $(T) = 1$ 4. nullity $(T) = 0$

46. Let $a, b, c, d \in \mathbb{R}$ and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$

for $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2$. Let $S: \mathbb{C} \to \mathbb{C}$ be the corresponding map defined by $S(x + iy) = (ax + by) + i(cx + dy)$

for $x, y \in \mathbb{R}$. Then

1. S is always $\mathbb{C}$ – linear, that is $S(z_1 + z_2) = S(z_1) + S(z_2)$ for all $z_1, z_2 \in \mathbb{C}$ and $S(\alpha z) = \alpha S(z)$ for all $\alpha \in \mathbb{C}$ and $z \in \mathbb{C}$.
2. S is $\mathbb{C}$ – linear if $b = -c$ and $d = a$.
3. S is $\mathbb{C}$ – linear only if $b = -c$ and $d = a$.
4. S is $\mathbb{C}$ – linear if and only if T is the identity transformation.

47. Let $A = [a_{ij}]$ be an $n \times n$ complex matrix and let A^* denote the conjugate transpose of A. Which of the following statements are necessarily true?

1. If A is invertible, then $tr(A^*A) \neq 0$, i.e., the trace of A^*A is non zero.
2. If $tr(A^*A) \neq 0$, then A is invertible.
3. If $|tr(A^*A)| < n^2$, then $|a_{ij}| < 1$ for some i,j.
4. If $tr(A^*A) = 0$, then A is the zero matrix.

48. Let n be a positive integer and V be an $(n+1)$-dimensional vector space over $\mathbb{R}$. If $\{e_1, e_2, ..., e_{n+1}\}$ is a basis of V and $T: V \to V$ is the linear transformation satisfying $T(e_i) = e_{i+1}$ for $i = 1, 2, ..., n$ and $T(e_{n+1}) = 0$. Then

1. trace of T is non-zero.
2. rank of T is n.
3. nullity of T is 1
4. $T^n = T \circ T \circ ... \circ T$ (n times) is the zero map.

49. Let A and B be $n \times n$ real matrices such that $AB = BA = O$ and $A + B$ is invertible. Which of the following are always true?

1. rank (A) = rank (B)
2. rank (A) + rank $(B) = n$.
3. nullity (A) + nullity $(B) = n$.
4. $A - B$ is invertible.

50. Let n be an integer ≥ 2 and let $M_n(\mathbb{R})$ denote the vector space of $n \times n$ real matrices. Let $B \in M_n(\mathbb{R})$ be an orthogonal matrix and let B^t denote the transpose of B. Consider $W_B = \{B^t AB : A \in M_n(\mathbb{R})\}$. Which of the following are necessarily true?

1. W_B is the subspace of $M_n(\mathbb{R})$ and dim $W_B \leq$ rank (B).
2. W_B is the subspace of $M_n(\mathbb{R})$ and dim W_B = rank (B) rank (B^t).
3. $W_B = M_n(\mathbb{R})$.
4. W_B is not a subspace of $M_n(\mathbb{R})$.