Stat 12 10 23

Thursday, October 12, 2023 6:37 PM

STATISTICAL Distributions

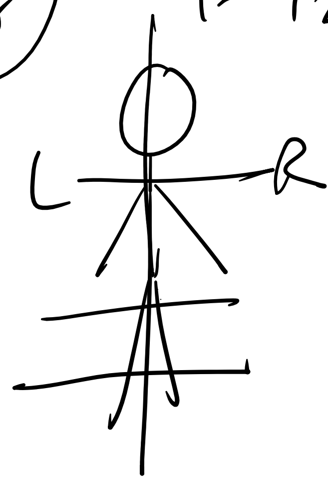
Toss until person

H, TH, TTH, TTTH, TTTTH, TTTTTH;

1 - 3 - - - 4

$\frac{1}{2}, \frac{1}{2}\cdot\frac{1}{2}, \left(\frac{1}{2}\right)^3, \left(\frac{1}{2}\right)^4$ - - - -

∞ GP $\rightarrow \frac{a}{1-r}$ $\quad \frac{1/2}{1-1/2} = \frac{1/2}{1/2} = 1$



Vertical Symmetry

$(n+1)p-1 < r < (n+1)p$

modes

90623-95723

Admired Poisson..

$X \sim P(\lambda)$

$(1+x)^{-1}$

Explanation ...

$e^x = e^1 = 2.718$

$X \longrightarrow P(X = \text{Even}) = ?$

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(x = \text{Even}) = p(0) + p(2) + p(4) + \dots$$

$$= e^{-\lambda}\left[1 + \frac{\lambda^2}{2!} + \frac{\lambda^4}{4!} + \dots\right]$$

now

$$e^{\lambda} = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \dots$$

$$e^1 = 1 + 1 + (0.718)$$

$$e^{-\lambda} = 1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \dots$$

$$e^{\lambda} + e^{-\lambda} = 2\left[1 + \frac{\lambda^2}{2!} + \frac{\lambda^4}{4!} + \dots\right]$$

$$p(x = \text{Even}) = e^{-\lambda}\left[\frac{1}{2}(e^{\lambda} + e^{-\lambda})\right]$$

$$\Rightarrow \frac{1}{2}(1 + e^{-2\lambda})$$

Application of derivative ...

1. P.D
2. D.F
3. D.I
4. ~~I~~ B.G.F

\# $X \sim P(\lambda)$

$$M_X(t) = e^{\lambda(e^t - 1)} \qquad \lambda > 0.$$

$$E(X) = \lambda$$

Central moment $\Rightarrow$ MGF about Mean $\to \lambda$

$$M_{X-\lambda}(t) = E\left[e^{t(X-\lambda)}\right] = e^{-\lambda t} E(e^{tX})$$

$$= \boxed{e^{-\lambda t} M_X(t)}$$

Now, $M_{X-\lambda}(t) = e^{-\lambda t} \cdot e^{\lambda(e^t - 1)} = e^{\lambda(e^t - 1 - t)}$

$$\boxed{M(t) = e^{\lambda(e^t - 1 - t)}}$$

$$\frac{dM(t)}{dt} = e^{\lambda(e^t - 1 - t)} \cdot \lambda(e^t - 1)$$

$$= M(t) \cdot \lambda(e^t - 1)$$

$$\frac{d}{d\lambda} M(t) = e^{\lambda(e^t - 1 - t)} (e^t - 1 - t)$$

$$= M(t)(e^t - 1 - t)$$

~~\#~~

Preferential Trial $\to$ 10 $\to$ 3/4/5

Pre-decided Success $\to$ 5 $\to$ 6/8/15

Question ... $X, Y \rightarrow$ independent R.V.

$X \sim P(\lambda)$, $X+Y \sim P(\lambda+\mu)$

find the distribution of Y.

Ans: $X \sim P(\lambda)$, $X+Y \sim P(\lambda+\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$ --- (1)

$M_{X+Y}(t) = e^{(\lambda+\mu)(e^t-1)}$ --- (2)

So, $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$

$\Rightarrow e^{(\lambda+\mu)(e^t-1)} = e^{\lambda(e^t-1)} M_Y(t)$

So, $M_Y(t) = \dfrac{e^{(\lambda+\mu)(e^t-1)}}{e^{\lambda(e^t-1)}} = \boxed{e^{\mu(e^t-1)}}$

Show: Let, X is a Poisson variate mean λ

$E(X^2) = \lambda E(X+1)$

Ans: $E(X^2) = Var(X) + [E(X)]^2 = \lambda + \lambda^2$

$= \lambda(1+\lambda) = \lambda[1 + E(X)]$

$E(X^2) = \lambda[E(X)+1] = \lambda E(X+1)$

If $\lambda = 1$, $E|X-1| =$ MD of PD about mean

$= \sum_{x=0}^{\infty} |x-1| \, p(x)$

$\infty \qquad \qquad \infty$

$$= \sum_{x=0}^{\infty} \left(\frac{e^{-1} 1^x}{x!}\right) |x-1| = e^{-1} \sum_{x=0}^{\infty} \frac{|x-1|}{x!}$$

$$= e^{-1}\left[1 + 0 + \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots\right]$$

$$= e^{-1}\left[1 + \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots\right]$$

Now

$$\frac{n}{(n+1)!} = \frac{(n+1) - 1}{(n+1)!} = \frac{(n+1)}{(n+1)!} - \frac{1}{(n+1)!}$$

$$= \frac{1}{n!} - \frac{1}{(n+1)!}$$

$$\therefore E|x-1| = e^{-1}\left[1 + \left(1 - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \dots\right]$$

$$= e^{-1}[1+1] = \frac{2}{e} \text{ Mean}$$

Note many times + ~~23~~

Pending + Previous Structure

Negative Binomial Distribution

10 → 4 $\quad {}^{10}C_4\, p^6 q^4$

+ Geometric Dist

Notes on B/G..

Improper Integral.. → 3 types

∞ ... $[0, \infty)$

Improper ...

$\int_0^\infty e^{-x}\,dx$ $\qquad \int_0^1 \frac{1}{\sqrt{x}}\,dx$ $\qquad$ dom $[0, \infty)$

(I) When Domain → finite

$$\int_2^\infty \frac{1}{x^2-1}\,dx \qquad \int_{-\infty}^1 \frac{1}{(3-x)^2}$$

(II) finite value → finite dom.

$$\int_1^3 \frac{dx}{x-1} \qquad \int_2^4 \frac{dx}{(x-4)^2}$$

Conv / Div

(III) Mix of I + II

$$\int_0^\infty \frac{dx}{\sqrt{x(1+x)}}$$

$m, n > 0$ Conv

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}$$

$$B(3/2, 2) = \int_0^1 x^{3/2-1}(1-x)^{2-1}\,dx$$

$$B(m,n) = \frac{(m-1)!}{n(n+1)\cdots(n+m-1)}$$

$$B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$$

Gamma functions:

the integral $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$

Conv $\boxed{n > 0}$

$\Gamma(n+1) = n\Gamma(n)$

$B(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

$$\boxed{\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}}$$

$\Gamma(1-m)\Gamma(m) = \dfrac{\pi}{\sin m\pi}$

$\int_0^\infty e^{-x^2} x^9 dx = ?$

Put $x^2 = z$, $2x\,dx = dz$ $\quad dx = \dfrac{x^{-1} dz}{2}$

at $x = 0$, $z = 0$

$x = \infty$, $z = \infty$

$\therefore \int_0^\infty e^{-x^2} x^9 dx = \int_0^\infty e^{-z} . z^{9/2} . \frac{1}{2} z^{-\frac{1}{2}} dz$

$= \frac{1}{2}\int_0^\infty e^{-z} . z^4 dz = \frac{1}{2}\int_0^\infty e^{-z} z^{5-1} dz$

$= \frac{1}{2}\Gamma(5) = \frac{1}{2} \times 4! = 12$

① China
② Japan
③ Korea
④ India
⑤ Uzbekistan

4th

India → olympics → ① → 3/4/5/6/7/8 ...

Negative Binomial → success

$X \to$ NBD (k, p)

$P(X \geq m) = ?$

$X \sim NB(k, p) \to$ pmf $\to$

$f(x) = {}^{-k}C_x \, p^x (-q)^x$ $\quad x = 0, 1, 2 \ldots$

$= {}^{-k}C_x \, Q^{-k} (-P/Q)^x$ $\quad x = 0 \,\&\, N$

When, $p = \frac{1}{Q}$, $q = \frac{P}{Q}$ $\quad p + q = 1$

$Q - P = 1$

discrete → Pascal's trials

$NB(r, p)$ $\quad r > 0$ no of success until an exp is stopped.

$p \in (0, 1)$

$k =$ failures $\in \{0, 1, 2, 3 \ldots\}$

$K = $ failure $\in \{0, 1, 2, 3 - -\}$

pmf $\rightarrow$ ${}^{K+r-1}C_K (1-p)^K p^r$

mean $\rightarrow$ $\frac{r(1-p)}{p}$

var $\rightarrow$ $\frac{r(1-p)}{p} \cdot \frac{1}{p}$

** Moment Methods $r = \frac{E[X]^2}{V[X] - E[X]}$

$p = 1 - \frac{E[X]}{V[X]}$

Negative $\rightarrow$ Related to the concept of waiting for a fixed number of success in Bernoulli Trials

$\rightarrow$ failure before the success ...

(-ve) $\rightarrow$ counting of failures