$ax^2+by^2+cxy+dx+ey+f=0.$

2 lines

If the equation $6x^2 - 11xy - 10y^2 - 19y + c = 0$ represents a pair of lines, find their equations. Also find the angle between the two lines.

L1 : $a_1x+b_1y+c_1=0$ L2 : $a_2x+b_2y+c_2=0.$

$L1 \cdot L2 = 0$

$(a_1a_2)x^2+(b_1b_2)y^2+c_1c_2+(a_1b_2+a_2b_1)xy+(a_1c_2+a_2c_1)x+(b_1c_2+b_2c_1)y=0.$

$6x^2-10y^2+c-11xy-19y=0.$

$a_1a_2=6.$ $a_1b_2+a_2b_1=-11$

$b_1b_2=-10$ $a_1c_2+a_2c_1=0.$

$c_1c_2=c$ $b_1c_2+b_2c_1=-19.$

$6=1\times6$

$=3\times2.$

$-10=-1\times10$

$=2\times-5$

$-5\times3+2\times2$

$a_1=3$ $a_2=2$ $b_1=2.$ $b_2=-5$

$a_1c_2=-a_2c_1$

$3\times-2=-2\times3.$

$c_1=3$ $c_2=-2$

$$\boxed{\begin{aligned}L1:\ & 3x+2y+3=0.\\ L2:\ & 2x-5y-2=0.\end{aligned}}$$

$m_1=-\frac{3}{2}$

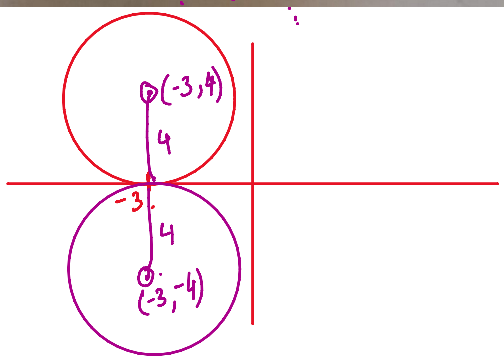
$m_2=\frac{2}{5}.$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-3/2-2/5}{1+(-3/2)(2/5)}\right|=\left|\frac{\frac{19}{10}}{\frac{4}{10}}\right|=\frac{19}{4}$$

$\theta=\tan^{-1}\left(\frac{19}{4}\right)$

The equation to the circle whose radius is 4 and which touches the negative x-axis at a distance 3 units from the origin is -

(A) $x^2+y^2-6x+8y-9=0$
(B) $x^2+y^2\pm6x-8y+9=0$
(C) $x^2+y^2+6x\pm8y+9=0$
(D) $x^2+y^2\pm6x-8y-9=0$



$(x-h)^2+(y-k)^2=r^2.$

$(x+3)^2+(y\mp4)^2=4^2.$

$x^2+y^2+6x\mp8y+9=0.$

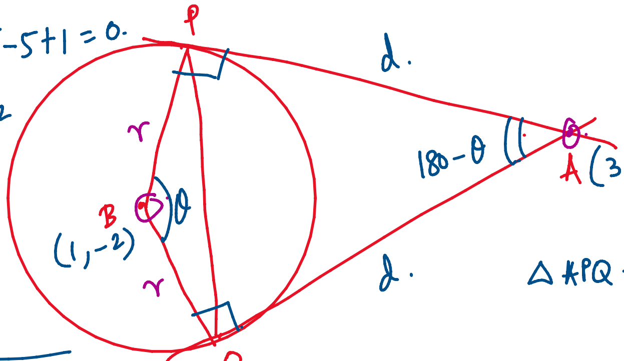
Let B be the centre of the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8 \cdot \left(\frac{\text{area}\,\Delta APQ}{\text{area}\,\Delta BPQ}\right)$ is equal to 18. [JEE MAINS - ONLINE - 2021]

$(x-1)^2 + (y+2)^2 - 5 + 1 = 0.$

$(x-1)^2 + (y+2)^2 = 2^2$

$B(1,-2)$ ✓

$\boxed{r = 2.}$



Area of a $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times ab \sin C$ ✓

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= r \cdot s$

(r = inradius, s = semiperimeter)

$\Delta APQ = \frac{1}{2} \cdot d^2 \sin(180 - \theta)$

$= \frac{1}{2} d^2 \sin\theta.$

$\Delta BPQ = \frac{1}{2} r^2 \sin\theta.$

$AB = \sqrt{4+9}$

$= \sqrt{13}.$

$AB^2 = r^2 + d^2$

$13 = 4 + d^2$

$\boxed{d = 3}.$

$\frac{\Delta APQ}{\Delta BPQ} = \left(\frac{d}{r}\right)^2 = \left(\frac{3}{2}\right)^2$

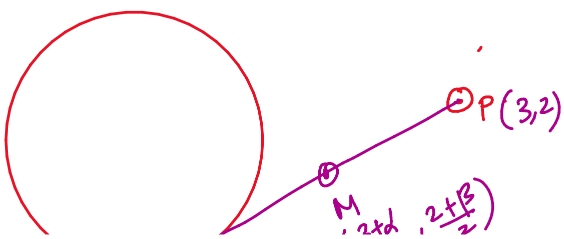
$= \frac{9}{4}$

$8\left(\frac{\Delta APQ}{\Delta BPQ}\right) = \frac{9}{4} \times 8 = 18.$

If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to : [JEE MAINS - ONLINE - 2021]

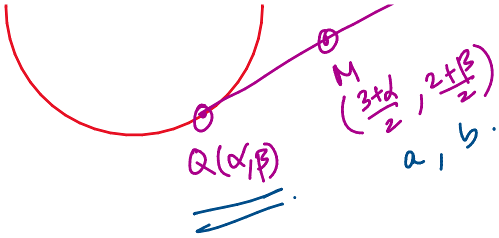
(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

To find the locus of M we have to find a relation between (a,b).



$\boxed{\alpha^2 + \beta^2 = 1}$

$3 + \alpha$



$\boxed{\alpha+\beta=1}$

$a = \frac{3+\alpha}{2}$

$b = \frac{2+\beta}{2}$

$\boxed{2a-3=\alpha}$ $\boxed{2b-2=\beta}$

$(2a-3)^2 + (2b-2)^2 = 1$

$4(a-3/2)^2 + 4(b-1)^2 = 1$

$\boxed{(a-3/2)^2 + (b-1)^2 = \frac{1}{4} = \left(\frac{1}{2}\right)^2}$

$r = 1/2.$

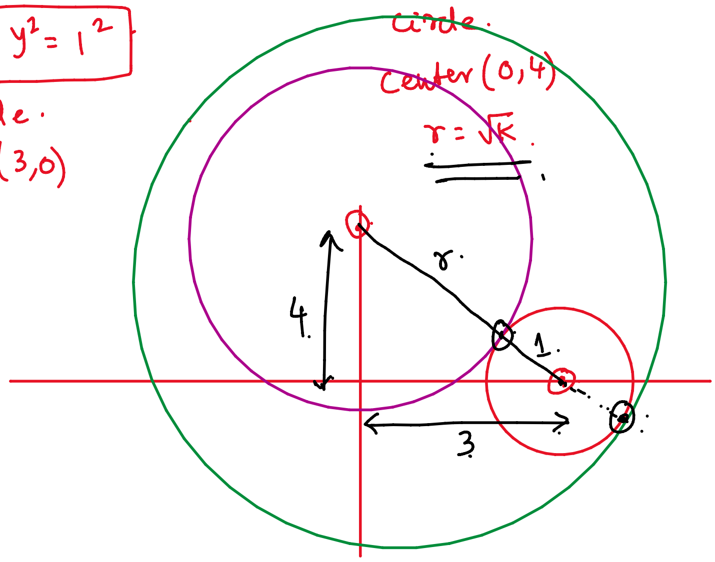
If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, $(k > 0)$ touch each other at a point, then the largest value of k is ______. **[JEE MAINS - ONLINE - 2020]**

$(x-3)^2 + y^2 - 9 + 8 = 0.$

$\boxed{(x-3)^2 + y^2 = 1^2}$

circle.
center $(3,0)$
$r = 1$

$\boxed{x^2 + (y-4)^2 = (\sqrt{k})^2}$



$r+1 = 5$

$\boxed{r=4} = \sqrt{k}$

$\boxed{R=6} = \sqrt{k}.$

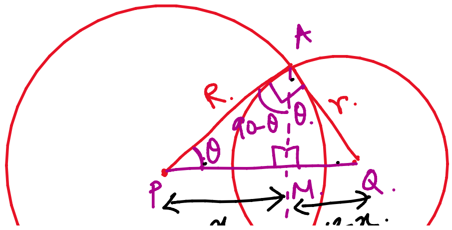
$K = 16, 36$

If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is : **[JEE-MAIN ONLINE-2019]**

(1) $\frac{60}{13}$ (2) $\frac{120}{13}$ (3) $\frac{13}{2}$ (4) $\frac{13}{5}$

$\frac{d}{x} = \frac{5}{12}.$

$\boxed{d = \frac{5}{12}x}$



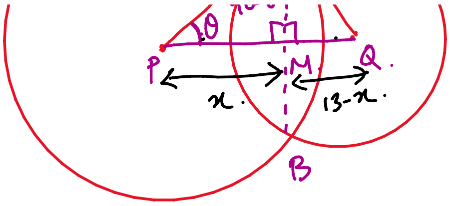
$R = 12$ $r = 5.$

$AM = \frac{AB}{2} = d.$

$PQ = 13.$

$\tan\theta = \frac{r}{R} = \frac{5}{12}$

$$\boxed{d = \frac{5}{12}x}$$



$PQ = 13.$

$\tan\theta = \frac{r}{R} = \frac{5}{12}.$

$\tan\theta = \frac{d}{x} = \frac{13-x}{d} = \frac{5}{12}.$

$13 - x = \frac{5}{12}d.$

$13 - x = \frac{25}{144}x.$

$\cancel{13} = \left(1 + \frac{25}{144}\right)x = \frac{\cancel{169}^{13}}{144}x.$

$x = \frac{144}{13}.$

$d = \frac{5}{\cancel{12}} \times \frac{\cancel{144}^{12}}{13} = \frac{60}{13}.$

$AB = \frac{120}{13}.$

Tips. to solve questions on circles

① Arrange the equation in the standard form of $(x-h)^2 + (y-k)^2 = r^2$

center (h, k) radius $= r.$

② apply geometrical & trigonometrical properties to identify unknowns

(length of chords / length of tangents from a pt)

③. If the question is on the equation of tangent / normal at a pt

then find $\frac{dy}{dx}$ using the equation of the circle;

find $\left(\frac{dy}{dx}\right)$ at the given pt. $= \left(\frac{dy}{dx}\right)_{P(\alpha,\beta)}$

equ of the tangent: $(y-\beta) = \left(\frac{dy}{dx}\right)_{(\alpha,\beta)}(x-\alpha)$

equ of the Normal: $y - \beta = \left[-\frac{1}{\left(\frac{dy}{dx}\right)_{\alpha,\beta}}\right](x-\alpha)$

If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2, 1), then its radius is ______.

Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y - 3x = 5\sqrt{77}/3 + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ____.

The set of values of k for which the circle $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point (1, - 1/3) lies on or inside the circle C is :

(A) An empty set

(B) (6,95/9]

(C) [80/9,10)

(D) (9, 92/9]

Let a circle C of radius 5 lie below the x-axis. The line $L_1 = 4x + 3y - 2$ passes through the center P of the circle C and intersects the line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x - 12y + 51 = 0$ is

If the tangents drawn at the point O(0, 0) and P(1 + √5,2) on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) 3 + √5/2

(B) 4 + 2√5//2

(C) 5 + 3√5/2

(D) 7 + 3√5/2