Simple Linear Reg Model (SLRM)

PRF (True Model) : $Y_i = \alpha + \beta X_i + u_i$

α, β are unknown popln parameters.

u_i : random disturbance terms.

SRE (Estimated Model) : $\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$, $i = 1, 2, \dots, n$, $n > 2$.

$\hat{\alpha}, \hat{\beta}$ = estimates of α, β / estimators of α, β.

$e_i = Y_i - \hat{Y}_i$, error in estimation

Assumptions of SLRM: (Assumption on u_i's)

(i) $E(u_i) = 0 \ \forall i$ ... [Zero mean assumption]

From Gujarati : $\underbrace{E(u/X)}_{\text{conditional mean}} = 0 \Rightarrow \underbrace{E(u)}_{\text{unconditional mean}} = E_X[E(u/X)] = E(0) = 0$

Note: for any 2 r.vs X_1; X_2

$E_{X_1}[X_1 / X_2 = x_2]$ = conditional exp of X_1 for any given value of X_2.

$\to \int_{x_1} x_1 \cdot f(x_1/x_2) \cdot dx_1$.

$E(X_1) = E_{X_2}\left[E_{X_1}(X_1 / X_2) \right]$ = unconditional exp of X_1.

(ii) $Var(u_i) = \sigma^2 \ \forall i$ [Homoskedasticity].

(iii) $cov(u_i, u_j) = 0$ [No Autocorrelation].

↳ [Any two random disturbance terms are

uncorrelated] .

(iv) $\mathrm{COV}(X, u) = 0$ [No endogeneity]

explanatory variable (X), Random Error (u) ... Both are uncorrelated

(v) SLRM is linear in parameters.

Estimation Process of the SLRM:

Estimation method to be used: Ordinary Least Squares (OLS).

e_i = error in estimation

∴ In the process, we want to find $\hat{\alpha}, \hat{\beta}$.

Objective: $\min\limits_{\hat{\alpha}, \hat{\beta}} \sum_{i=1}^{n} e_i^2$ ... [OLS Technique]

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta} X_i)^2 \quad (2)$$

~~$\min\limits_{\hat{\alpha}, \hat{\beta}} \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)$ True value, $= 0$ Expected value.~~

~~$x_1, x_2, \ldots, x_n$~~

~~$\sum_{i=1}^{n} (x_i - \bar{x}) = 0$ [by defn]~~

∴ For Minimization:

$$\frac{\partial \sum e_i^2}{\partial \hat{\alpha}} = 0 \Rightarrow \sum_{i=1}^{n} (2)(Y_i - \hat{\alpha} - \hat{\beta} X_i)(-1) = 0 \Rightarrow \sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta} X_i) = 0 \quad \ldots (i)$$

$$\frac{\partial \sum e_i^2}{\partial \hat{\beta}} = 0 \Rightarrow \sum_{i=1}^{n} (2)(Y_i - \hat{\alpha} - \hat{\beta} X_i)(-X_i) = 0 \Rightarrow \sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta} X_i) X_i = 0 \quad \ldots (ii)$$

∴ We obtain 2 eqns (i), (ii) to solve for 2 unknowns $\hat{\alpha}, \hat{\beta}$.

(i): $\sum_{i=1}^{n} (Y_i - \hat{\alpha} - \hat{\beta} X_i) = 0$.

$$\sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} \hat{\alpha} - \hat{\beta} \sum_{i=1}^{n} X_i = 0.$$

$a_1 x + b_2 y = c_1$

$a_2 x + b_2 y = c_2$.

$x =$

$$\sum_{i=1} Y_i - \sum_{i=1} \alpha - \beta \sum_{i=1} X_i = 0.$$

$$x =$$

$$\frac{n\hat{\alpha}}{n} + \frac{\hat{\beta}\sum_{i=1}^{n} X_i}{n} = \frac{\sum_{i=1}^{n} Y_i}{n}.$$

$$\hat{\alpha} + \hat{\beta}\bar{X} = \bar{Y} \quad \ldots\ldots (i\,a).$$

(ii): $$\sum_{i=1}^{n}(Y_i - \hat{\alpha} - \hat{\beta}X_i)X_i = 0.$$

$$\sum_{i=1}^{n} Y_i X_i - \hat{\alpha}\sum_{i=1}^{n} X_i - \hat{\beta}\sum_{i=1}^{n} X_i^2 = 0.$$

$$\left(\sum_{i=1}^{n} X_i\right)\hat{\alpha} + \left(\sum_{i=1}^{n} X_i^2\right)\hat{\beta} = \sum_{i=1}^{n} Y_i X_i \quad \ldots\ldots (ii\,a).$$

∴ Summarizing:

$$\left.\begin{array}{l} \hat{\alpha} + \bar{X}\hat{\beta} = \bar{Y} \quad \ldots\ldots (i\,a) \\ (\Sigma X_i)\hat{\alpha} + (\Sigma X_i^2)\hat{\beta} = \Sigma Y_i X_i \quad \ldots (ii\,a) \end{array}\right\}$$ solve for $\hat{\alpha}, \hat{\beta}$.

HW. $$\hat{\beta} = \frac{\begin{vmatrix} 1 & \bar{Y} \\ \Sigma X_i & \Sigma X_i Y_i \end{vmatrix}}{\begin{vmatrix} 1 & \bar{X} \\ \Sigma X_i & \Sigma X_i^2 \end{vmatrix}} \ldots.$$

Expand to show: $$\hat{\beta} = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}$$