08 January 2024 08:13 PM

Elasticity of Demand

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Measure of Responsiveness of Quantity Demand.

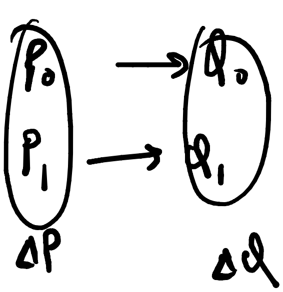
① Own price elasticity of demand

It is defined as the responsiveness of the quantity demanded of good to the change in price.

ie it is the ratio of % change in quantity demanded w.r.t % change in price of that commodity.

Mathematically,

$$e_p = \frac{\%\text{ change in quantity demand}}{\%\text{ change in price}}$$



$$= \frac{\frac{\Delta Q}{Q_0} \times 100}{\frac{\Delta P}{P_0} \times 100} = \frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0}$$

$$\approx \frac{\partial Q}{\partial P} \cdot \frac{P_0}{Q_0}$$

Types of own price elasticity of demand :

Case 1 : perfectly elastic demand , $|e_p| \to \infty$

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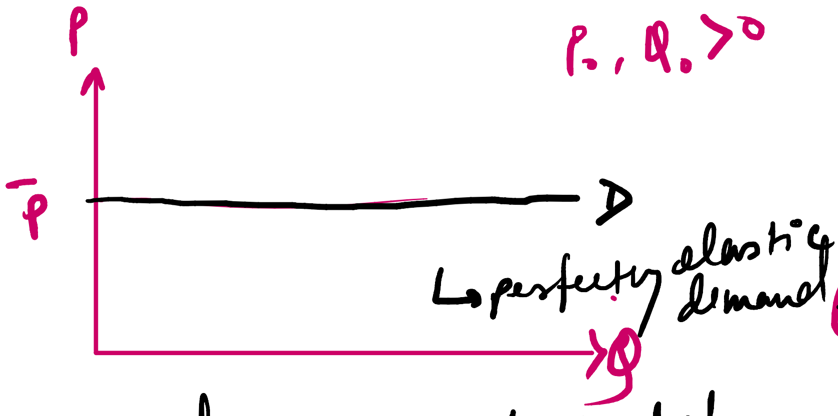
$$\frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0} \to \infty$$

$P_0, Q_0 > 0$

$$\frac{\Delta Q}{\Delta P} \to \infty$$

$\Delta P \to 0$

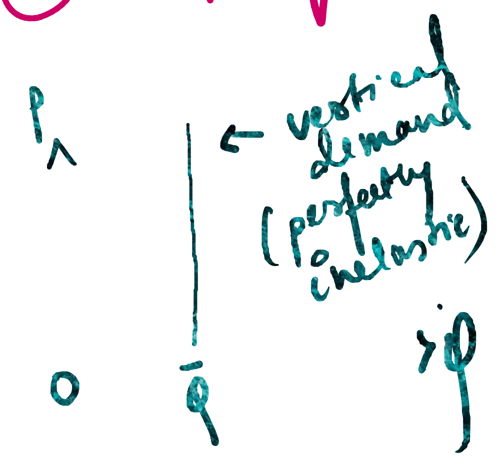


(for a negligible change in price, quantity demand will change drastically.)

Demand curve is horizontal.

② Perfectly inelastic demand curve.

$|e_p| \to 0$



ie $\frac{\Delta Q}{\Delta P} \times \frac{P_0}{Q_0} \to 0$

Again $P_0, Q_0 > 0$

$\therefore \frac{\Delta Q}{\Delta P} \to 0$

$\Rightarrow \Delta Q \to 0$

[change in demand is 0 for any change in price level]

③ Unit elastic demand

$|e_p| = 1.$

$$\frac{\%\text{ change in } Q_x}{\%\text{ change in } P_x} = 1$$

$P = 3 \quad q = 2 \quad 6$

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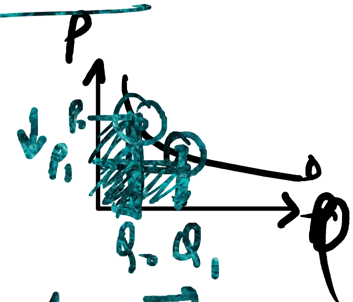
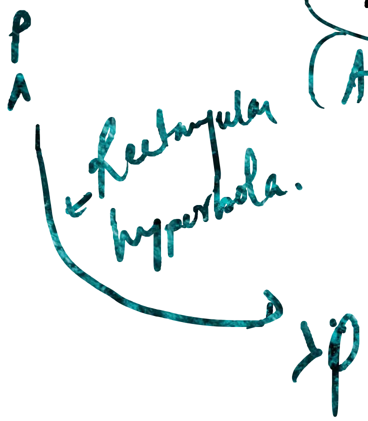
% change in Q_x = % change in P_x

% change in Px

% change in Qx = % change in Px

TR is same (unchanged)
(Area under the demand curve is the same)





∴ DC is a rectangular hyperbola.

Case IV : inelastic demand

$0 < |e_p| < 1$

% change in Q^x < % change in P^x

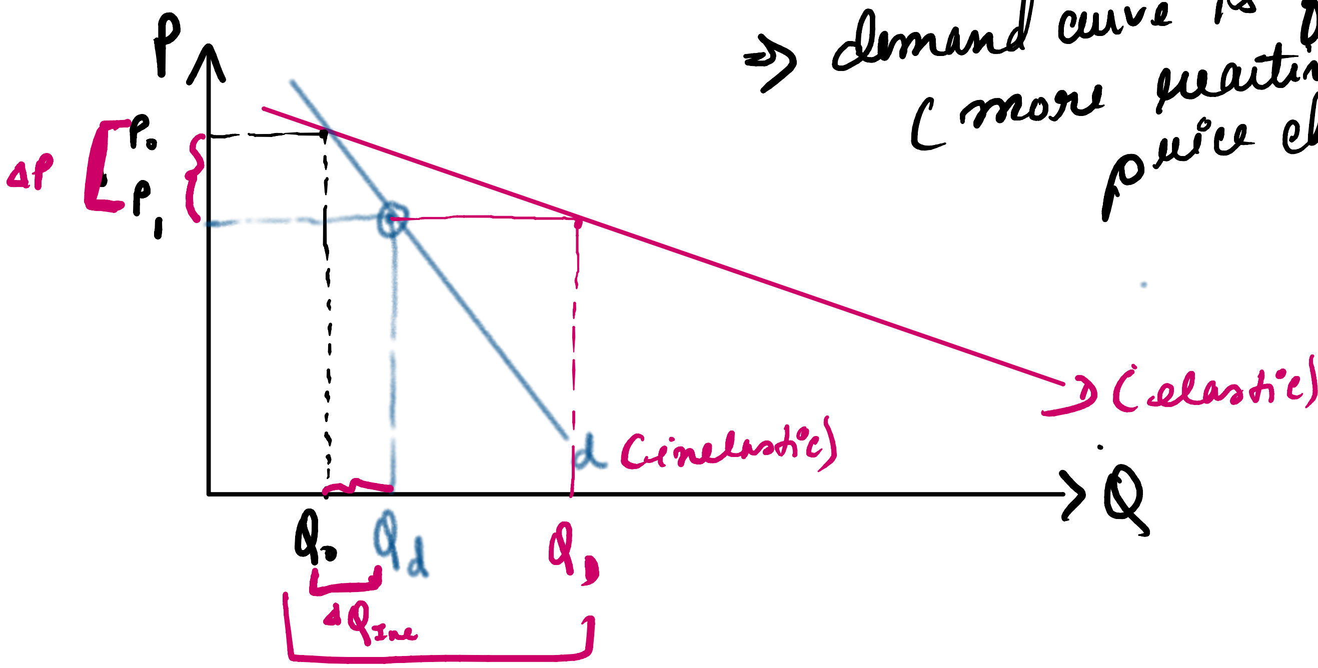
→ demand curve is steeper (less reactive to price change).

Case V: elastic demand

$|e_p| > 1$

% change in Q^x > % change in P^x

⇒ demand curve is flatter (more reactive to price change).



dQ elastic

elasticity	Demand	Change in Total Revenue.
1. perfectly inelastic $\lvert e_p \rvert \to 0$	vertical	a) P↑ → TR will increase (because Q is fixed) b) P↓ → TR will decrease
2. inelastic demand $\lvert e_p \rvert < 1$	Steeper	a) P↑ → TR = (P) × Q increases b) P↓ → TR will decrease
3. unit elastic $\lvert e_p \rvert = 1$	rectangular hyperbola	a) P↑, b) P↓ } TR is unchanged.
4. elastic demand $\lvert e_p \rvert > 1$	flatter demand	a) P↓ → TR increase b) P↑ → TR decrease
5. Perfect elastic $\lvert e_p \rvert \to \infty$	horizontal	–

Q For the demand function $X = \frac{20}{P+1}$

find the elasticity of demand with respect to price at a point $P = 3$. What is the nature of elasticity?

$$e_p = \frac{\partial x}{\partial p} \times \frac{P}{x}$$

at $P = 3$

$$x = \frac{20}{3+1} = 5$$

$$x = 20/(p+1)$$

taking derivatives on both side,

$$\frac{\partial x}{\partial p} = -\frac{20}{(p+1)^2}$$

$$\therefore e_p = -\frac{20}{(p+1)^2} \times \frac{3}{5} = -\frac{\cancel{20}\,4}{144} \times \frac{3}{\cancel{5}}$$

$$= -3/4 = -0.75$$

$|e_p| = 0.75 < 1 \Rightarrow$ inelastic in nature.

② If $y = \frac{2x+1}{3x+2}$ then obtain the value of elasticity at $x = 1$.

$$e_p = \frac{\partial y}{\partial x} \cdot \frac{x}{y}$$

at $x = 1$
$\Rightarrow y = \frac{3}{5}$

$$\frac{\partial y}{\partial x} = \frac{(3x+2)(2) - (2x+1)(3)}{(3x+2)^2} = \frac{\cancel{6x} + 4 - \cancel{6x} - 3}{(3x+2)^2}$$

$$= 1/(3x+2)^2$$

$$\therefore e_p = \frac{1}{(3x+2)^2} \times \frac{1}{\frac{3}{5}} = \frac{1}{\cancel{25}\,5} \times \frac{\cancel{5}}{3} = \frac{1}{15}$$

(ans)

Cross price Elasticity of demand

Ratio of % change in quantity demanded of commodity x w.r.t change in price of related commodity y.

i.e.
$$e_c^{xy} = \frac{\frac{\Delta Q_x}{Q_x}}{\frac{\Delta P_y}{P_y}} = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x} \approx \frac{\partial Q_x}{\partial P_y} \cdot \frac{P_y}{Q_x}.$$

$\uparrow x$, $y \downarrow$

P_x $P_y \uparrow$

Cases:

(1) If x and y are positively related.

$\Rightarrow \frac{\partial Q_x}{\partial P_y} > 0 \Rightarrow e_c^{xy} > 0 \Rightarrow$ substitute goods.

(2) If x and y are negatively related

$\Rightarrow \frac{\partial Q_x}{\partial P_y} < 0 \Rightarrow e_c^{xy} < 0 \Rightarrow$ complementary goods

(3) If x and y are unrelated

$e_c^{xy} = 0$.

(3) Income Elasticity of Demand.

Ratio of % change in Quantity Demand of ... % change in income x w.r.t % change in income

ie $e_m^x = \dfrac{\text{\% change in } Q^x}{\text{\% change in } m}$

$= \dfrac{\Delta q}{\Delta m} \times \dfrac{m}{q} \approx \dfrac{\partial q}{\partial m} \times \dfrac{m}{x}$.

① if $e_m^x < 0 \Rightarrow \dfrac{\partial q}{\partial m} < 0 \rightarrow$ inferior goods

② if $e_m^x = 0 \Rightarrow \dfrac{\partial q}{\partial m} = 0 \Rightarrow$ neutral goods.

③ if $e_m^x > 0 \Rightarrow \dfrac{\partial q}{\partial m} > 0 \Rightarrow$ Normal goods.

a) $0 < e_m^x < 1 \Rightarrow \dfrac{\partial q}{\partial m} < 1 \Rightarrow$ Necessary Normal goods

b) $e_m^x > 1 \Rightarrow \dfrac{\partial q}{\partial m} > 1 \Rightarrow$ Luxury Goods.

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