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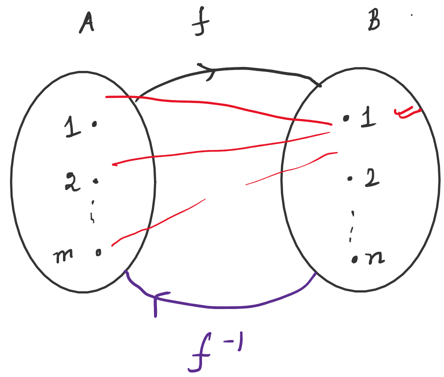
Q. Consider 2 sets A & B having 'm' & 'n' elements respectively. How many functions $f: A \to B$ are possible?

[Surjective + Injective]



$B_1 \Rightarrow m$ possibilities.

$|B| = n$.

$\therefore$ Total no. of fns $= n^m$.

Bijective Functions: [Surjective + Injective].

Inverse of a function: $f: A \to B$.

$f^{-1}: B \to A$.

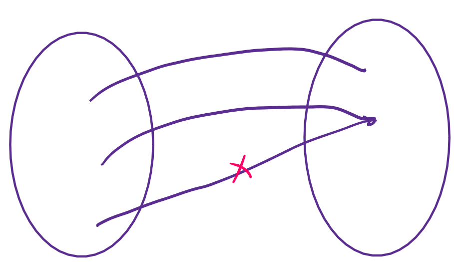
If $y = f(x) \Rightarrow x = f^{-1}(y)$

Note: Inverse of a function exists only when the function is Bijective.

Q. Let $A = \{1, 2, \ldots, m\}$ $B = \{1, 2, \ldots, n\}$.

Find the number of bijective functions $f: A \to B$ when $m = n$.

Bijective $\Rightarrow$ one-one functions.



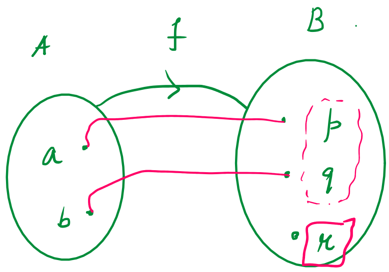
B

n	$(n-1)$	$(n-2)$	......	1
1	2	3	.....	n

$\therefore$ Total no. of Bijective fns $= n(n-1)(n-2)\ldots 1 = n!$

Q. $A = \{a, b\}$ $B = \{p, q, r\}$ $f: A \to B$.

Find the no. of bijective fns = 0.



${}^{3}P_{2}$.

$\left.\begin{array}{l} a \to p \\ b \to q \end{array}\right\}$ Fn is injective

But is not surjective.

Q. $A = \{1, 2, \cdots, n\}$ $B = \{1, 2, \cdots, n\}$. Find the no. of bijective fns $f: A \to B$ such that $f(x) \neq x \ \forall x \in A$.

$|A| = |B| = n$. Hence Bijective fns can be constructed.

$\therefore$ No. of Bijective fns $= n! - [\text{No. of fns s.t } f(x) = x.]$

$f:$ $f(1) = 1 \to n$ cases

$f(2) \neq 2$

$\vdots$

$f(1) = 1$

$f(2) = 2$

$f(3) \neq 3$

$\vdots$

. . . . .

$f(1) = 1$

$f(2) = 2$

$f(3) = 3$

$\vdots$

$f(n) = n$.

$\to f(x) = x \ \forall x$

${}^{n}C_{1} \ (n-1)!$ $\qquad {}^{n}C_{2} \ (n-2)!$ . . . . ${}^{n}C_{n} \cdot 0!$

Total no. of fns where at only 1 pt $f(x) = x$.

Total no. of bijective fns $= n! - {}^{n}C_{1}(n-1)! + {}^{n}C_{2}(n-2)! + \cdots$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

Generalize: $n\left(\bigcup_{i=1}^{n} A_i\right) = \sum n(A_i) - \sum\sum n(A_i \cap A_j) + \sum\sum\sum n(A_i \cap A_j \cap A_k) + \cdots$

Let A_i = set of ways in which $f(i) = i$

$n(A_1 \cup A_2 \cup \ldots \cup A_n)$ = Total no. of ways in which $\exists$ atleast one $i \in A$ s.t $f(i) = i$

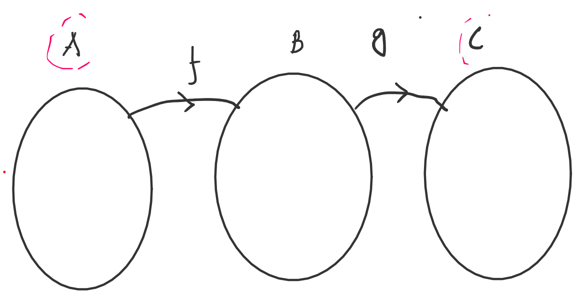
$= {}^nC_1 (n-1)! - {}^nC_2 (n-2)! + {}^nC_3 (n-3)! + \cdots$

Total no. of Bijective fns $= n! - \left[{}^nC_1 (n-1)! - {}^nC_2 (n-2)! + {}^nC_3 (n-3)! + \cdots\right]$

$= n! - {}^nC_1 (n-1)! + {}^nC_2 (n-2)! + \cdots + (-1)^n \, {}^nC_n \, 0!$

Composite Functions:

$f : A \to B \quad g : B \to C$.



$\left.\begin{array}{l} f \circ g : A \to C . \\ g \circ f : C \to A \end{array}\right\} \Rightarrow$ Composite fns

Composite fn $f \circ g$ exists iff $R_f \subseteq D_g$.

Q. Let $f_1 : \{a_1, a_2, a_3\} \to \{b_1, b_2, b_3, b_4\}$.

$f_2 : \{b_1, b_2, b_3, b_4\} \to \{c_1, c_2, c_3\}$

s.t $f_1(a_1) = b_1 \quad f_1(a_2) = b_2 \quad f_1(a_3) = b_3$.

s.t $f_1(a_1) = b_1 \quad f_1(a_2) = b_2 \quad f_1(a_3) = b_3$.

$f_2(b_1) = c_1 \quad f_2(b_2) = c_2 \quad f_2(b_3) = f_2(b_4) = c_3$

Then $f_2 \circ f_1$ is :

(a) one-one & onto

(b) one-one & not onto

(c) not one-one & onto

(d) not one-one & not onto .