16 July 2023 13:52

$f(x) \geq P(x) \geq a.$

$f(x)=a$

$\Downarrow$

$P(x)=a$

Functions

① Algebraic $\rightarrow$ Polynomials.

② Trigonometric.

③ Exponential.

④ logarithmic.

If x, y ∈ [0, 10], then number of solutions (x,y) of inequation $3^{\sec^2 x-1}\sqrt{9y^2-6y+2} \leq 1$ is

(A) 2 (B) 4

(C) 6 (D) infinite

$\cos\alpha = \cos\theta$

$\sec^2 x - 1 = \tan^2 x.$

$3^{\tan^2 x}\sqrt{9y^2-6y+2} \leq 1$

$$\boxed{1 \leq \sqrt{9y^2-6y+2} \leq \frac{1}{3^{\tan^2 x}}.}$$

$\text{LHS} = \sqrt{9y^2-6y+1+1}$

$= \sqrt{(3y-1)^2+1}$

$\geq 0.$

LHS ≥ 1

$\boxed{3^{-\tan^2 x} \geq 1}$

$\tan^2 x \geq 0$

$-\tan^2 x \leq 0.$

$\boxed{0 \leq 3^{-\tan^2 x} \leq 1}$

$\boxed{x = [0,10]}$

$3^{-\tan^2 x} = 1$

$\Rightarrow \tan x = 0 \Rightarrow \boxed{x = n\pi}$

n	x
0	0
1	$\pi = 3.14.$
2	$2\pi = 6.28.$
3	$3\pi = 9.42$

$9y^2-6y+2=1$

$\Rightarrow (3y-1)^2 = 0. \Rightarrow \boxed{y = \frac{1}{3}}$

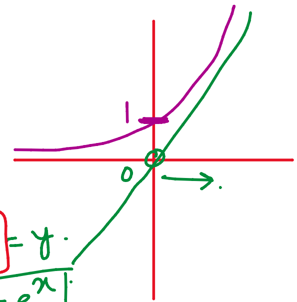
$x^3 = (x-1) = y$

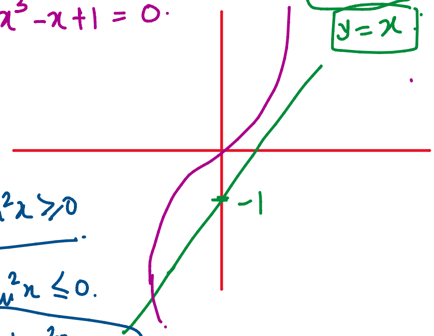
$x^3 - x + 1 = 0.$

$\boxed{e^x = x} = y.$

$\boxed{y = e^x}$

$\boxed{y = x}$





$f(x) = x^3 - x + 1 = 0$

+ve roots $= 2, 0$

$f(-x) = -x^3 + x + 1$

$\boxed{-\text{ve roots} = 1}$

The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60} = 1$ is: (-1) ③

$(x^2-5x+5)^{x^2+4x-60.} = 1$

$a^b = 1$

① $a=1$, $b \in R.$ ② $a \in R$ except 0. $b=0$ ③ $a=-1$, $b=$ even

① $x^2-5x+5=1$

$x^2-5x+4=0.$

$(x-4)(x-1)=0.$

$\boxed{x=1,4}$ ✓

② $x^2+4x-60=0$

$(x+10)(x-6)=0$

$\boxed{x=-10,6}$ ✓

$a = x^2-5x+5$

③ $x^2-5x+5=-1$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$\boxed{x=2,\cancel{3}}$

$x^2+4x-60.$

Sum of $x = -10+6+1+4+2$

$= 3$

Find the first digit from left of 3^{62} where $\log 3 = 0.4771$

Let $(\log^2 x)$ ...

No of digit of 3^{62} = 30. Squares of nos from 1 to 32 and cubes from 1 to 15

$\log(a^b) = b\log a$

$3^6 = 729$ → 3 digits

$\log(3^6) = 6\log 3 = 6 \times 0.4771 = 2.\boxed{8626}$

Integer part + 1 = No of digits

$\log 2 = 0.301$
$\log 3 = 0.4771$
$\log 7 = 0.8541$

$\log 7 < 0.8626 < \log 8$

Starting digit of $3^6 = 8-1 = 7$

$4-1=3$

$\log(3^{62}) = 62\log 3 = 29.\boxed{5802}$

$\log 2 = 0.301$
$\log 3 = 0.4771$
$\log 4 = 0.602$
$\log 5 = 0.699$
$\log 6 = 0.7781$
$\log 7 = 0.8541$
$\log 8 = 0.903$
$\log 9 = 0.9542$

Let x,y, z be real numbers with $x \geqslant y \geqslant z \geqslant \frac{\pi}{8}$ such that $x+y+z = \frac{\pi}{2}$ and let $P = \cos x . \sin y . \cos z$ then which is/are CORRECT?

$\frac{\pi}{4}$, $\frac{\pi}{2} - (x+y)$

(A) Minimum value of P is $\frac{1}{8}$

(B) Minimum value of P is $\frac{1}{4}$

(C) Maximum value of P is $\frac{2+\sqrt{3}}{4}$

(D) Maximum value of P is $\frac{\sqrt{2}+1}{4\sqrt{2}}$

$P = \cos x \sin y \cos z$.

$P \le 8$.

$P_{max} = r$

$0 - \frac{\pi}{2}$

sin →

cos ←

$y_{min} = z_{min} = \frac{\pi}{8}$

$x \ldots = \frac{\pi}{\ldots} - 2\times\frac{\pi}{8}$

$P = \frac{1}{2} \cdot 2\cos x \sin y \cos z = \frac{1}{2}\cos x \cdot (2\sin y \cos z)$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$P = \frac{1}{2}\cos x\,[\sin(y+z) + \sin(y-z)]$

+ve

$y \geqslant z$

$y - z \geqslant 0$

$\sin(y-z) \geqslant 0$

$P = \frac{1}{2}\cos x\,[\sin(\frac{\pi}{2} - x) + \text{+ve}]$

$P = \frac{1}{2}\cos x\,[\cos x + \text{+ve}]$

$\boxed{P \geqslant \frac{1}{2}\cos^2 x}$

$x = \frac{\pi}{2} - (y+z)$

$y_{min} = z_{min} = \frac{\pi}{8}$ $\qquad P = \frac{1}{2}\cos x\,[\ \dots\ ]$

$x_{max} = \frac{\pi}{2} - 2\times\frac{\pi}{8}$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$P = \frac{1}{2}\cos x\,[\cos x + \text{+ve}]$

$\boxed{P \geq \frac{1}{2}\cos^2 x}$

$x = \frac{\pi}{2} - (y+z)$

$P_{min} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ $\qquad P_{min} = \frac{1}{2}\cos^2 x$

max $= \frac{\pi}{4}$ $\qquad$ min

$\sin(-\theta) = -\sin\theta$

$P = \frac{1}{2}(2\cos x\sin y)\cos z = \frac{1}{2}\cos z\,[\sin(y+x) + \sin(y-x)]$

$P = \frac{1}{2}\cos z\,[\sin(\frac{\pi}{2} - z) - \underbrace{\sin(x-y)}_{\text{+ve}}]$

$z_{min} = \frac{\pi}{8}$

$P \leq \frac{1}{2}\cos^2 z$ $\qquad P_{max} = \frac{1}{2}\cos^2 z$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = \cos 2\theta + 1$

$= \frac{1}{4}\cdot 2\cos^2 z = \frac{1}{4}(\cos 2z + 1)$

$P_{max} = \frac{1}{4}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$ $\qquad = \frac{1}{4}(\cos\frac{\pi}{4} + 1) = \frac{1}{4}(\frac{1}{\sqrt{2}} + 1)$

P(x) is a polynomial of degree 98 such that P (K) = $\frac{1}{K}$ for K = 1, 2, 3, ... , 99. The value of P(100) is

(A) $\lfloor 100 + 1$ (B) $\frac{1}{100}$ (C) $\frac{1}{50}$ (D) $\frac{1}{\lfloor 100}$

Fundamental definition of a polynomial.

$P(x) = a_n x^{n} + a_{n-1}x^{n-1} + \dots + a_0$

n degree

$f(x) = 0$ for $x = a \Rightarrow (x-a)$ is a factor of $P(x)$

$P(x) = \frac{1}{x}\ \forall\ x = 1, 2, 3 \dots, 99$

$f(x) = xP(x) - 1 = 0\ \forall\ x = 1, 2, 3, \dots 99$

$(x-1), (x-2), (x-3) \dots (x-99)$ are factors of $f(x)$

$f(x) = A(x-1)(x-2)(x-3)\dots(x-99)$

$xP(x) - 1 = A(x-1)(x-2)\dots(x-99)$

$f(x) = A(x-1)(x-2)(x-3)\cdots(x-99)$.

$xP(x) - 1 = A(x-1)(x-2)\cdots(x-99)$

$\underline{\underline{x=0}}$. $\quad -1 = A(-1)(-2)\cdots(-99)$

$-1 = A(-1)\,99!$. $\qquad \boxed{A = \frac{1}{99!}}$.

$xP(x) - 1 = \frac{1}{99!}(x-1)(x-2)\cdots(x-99)$

$\underline{x=100}$ $\quad 100P(100) - 1 = \frac{1}{99!}(99)(98)(97)\cdots(1) = 1$

$100P(100) = 2$. $\qquad P(100) = \frac{2}{100} = \frac{1}{50}$.

If x and y both are non-negative integral values for which $(xy-7)^2 = x^2 + y^2$, then find the sum of all possible value of x

(A) 7 (B) 10 (C) 13 (D) 14

Identity the correct statements:

S_1 : Number of values of x, between 0 to 2π which satisfy $e^x \cot x = 1$ is four

S_2: If $\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = 2+\sqrt{3}$ then there exist two values of θ between 0 to π which satisfy the abo equation

S_3: If $\alpha^2+\beta^2+\gamma^2 = \alpha\beta + \beta\gamma + \alpha\gamma$ then $4\cos(60^\circ-\alpha)\cos\beta\ \cos(60^\circ+\gamma) = \cos(\alpha+\beta+\gamma)$

(A) TFT (B) TTF (C) FFT (D) FFF

Let P(x) be a polynomial such that $x.P(x-1) = (x-4)P(x)\ \forall x \in R$. Then the degree of P(x) cannot be

(A) 2 (B) 3 (C) 4 (D) 5