CI IN STATISTICS 11/2/2024

11 February 2024 12:11

Advanced Statistical Analysis

Zone — Smaller Data set

Larger Data Set

How to choose ??

Poisson → Parameter

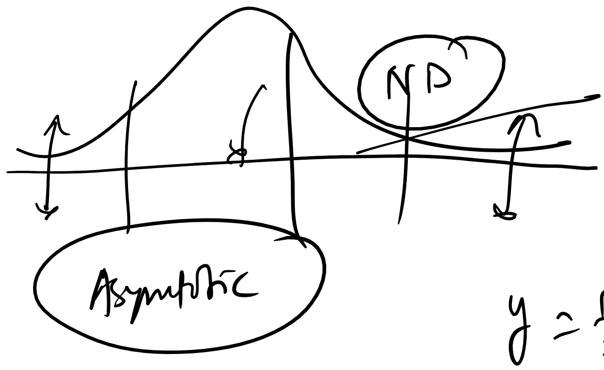
λ → Parameter, $e^{-\lambda}$

| Parameter | [common parts] | Errors |

95% — 5%

99% — 1%

100% ??



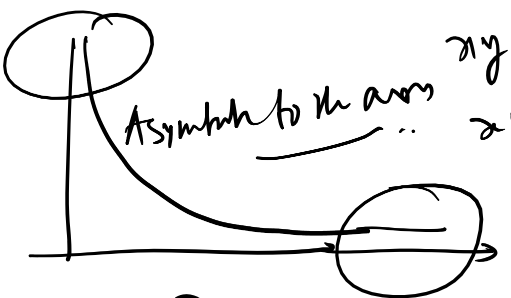
$y = \frac{1}{x}$

$xy = Const$

$xy = 7063.21$

Elementary

$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots$



Case I: $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ (3)

Case II: $y = \alpha_2 + \beta_1' x_1' + \beta_2 x_2' + \dots + \beta_n' x_n'$ (n), $n > 3$

Number of independent variables may not be the dependent.

Number of Independent variables

DRI

Professor

Govt Employee

Unemployed

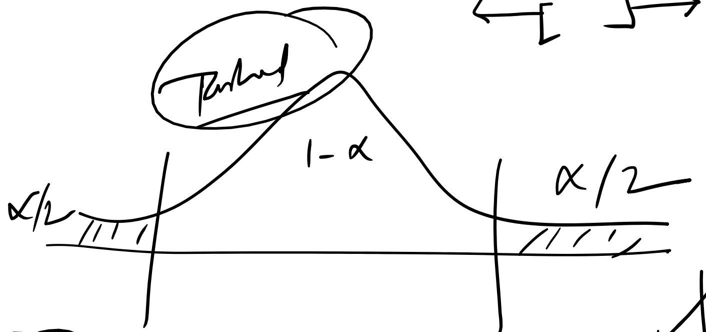
Time x

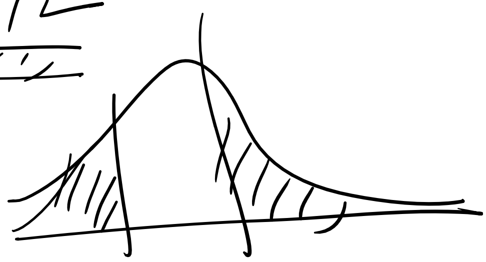
Ability ✓

Correct value ← on What Basis??

4 yr → 1 → 18yrs in 2023

[ ]





CLT

ND

$$\mu \in \left(\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}\right)$$

Sample Size → Larger is better X

→ More Relevant is Better ..

Boxing →

Person's Strength ??

weight ?? → weight lifted

42
50
60

Weight ?? → Weight eg: 50, 60, 80, 100, 120, 140

Arm wrestling (kg)

Warm up

Q. $x_1, x_2 \dots x_n$ → r.v. (n)

$N(\mu, 16)$ .. Population

If a 95% CI for μ is $[\bar{X} \pm 0.98]$

$n = ?$

ISI 2012

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

width → ± 0.98 — (i)

width → $\pm Z_{0.025} \frac{\sigma}{\sqrt{n}}$ — (ii)

Expanding (i) & (ii)

$$\pm 0.98 = \pm 1.96 \frac{4}{\sqrt{n}}$$

$$\sqrt{n} = \frac{1.96}{0.98} \times 4 = 8$$

$n = \boxed{64}$

(Z) $n \geq 30$ | (t) $n < 30$ | (n) | (Z^2) → χ^2

$n = 9$, $\mu = 5$,

Observed Statistics $\frac{1}{8} \sum X_i^2 = 39.125$ → $\sum X_i^2 = 313$

$\sum X_i = 45$

$\sum x_i = 45$

95% CI for σ^2 ?

Ans

$$\sum (x_i - \mu)^2 = \sum x_i^2 - 2\mu \sum x_i + 9\mu^2$$

$$= 313 - 450 + 225$$

$$= 88$$

Only to be given on the CI

95% CI

$$\sigma^2 \in \left[\frac{\sum (x_i-\mu)^2}{\chi^2_{\frac{\alpha}{2}, n-1}} , \frac{\sum (x_i-\mu)^2}{\chi^2_{1-\alpha/2, n-1}} \right]$$

$$\sigma^2 \in \left(\frac{88}{17.53}, \frac{88}{2.180} \right)$$

→ 5 to 41

ISS 2018

$x_1, x_2 \dots x_n$ → PDF

$f(x|\theta) = \theta e^{-\theta x}$ $x>0, \theta>0$

$= 0$ otherwise

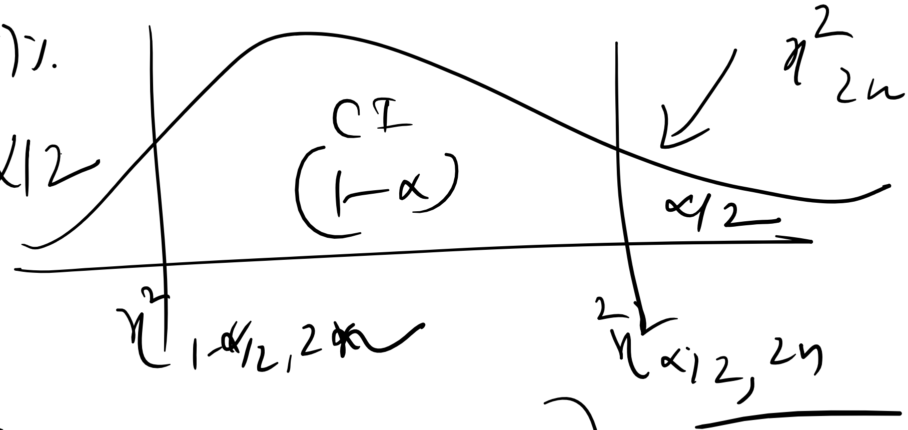
Which of the following is/are $100(1-\alpha)\%$ CI for θ ?

Ans: P.Q. : $-2 \sum \ln \left(1 - F(x_i, \theta)\right) \sim \chi^2_{2n}$

Here: $-2 \sum^n \ln \left[1 - F(x_i, \theta)\right] = 2\theta \sum x_i \sim \chi^2_{2n}$

So, formally, P.Q. : $2\theta \sum_{i=1} x_i \sim \chi^2_{2n}$

Then $(1-\alpha)\%$



~~Now~~ Ford

$$(1-\alpha) = P\left[0 < 2\theta \sum x_i < \chi^2_{2n,\alpha}\right]$$

$$= P\left[0 < \theta < \frac{\chi^2_{2n,\alpha}}{2\sum x_i}\right]$$

$$\Rightarrow \theta \in \left[0, \frac{\chi^2_{2n,\alpha}}{2\sum x_i}\right]$$

Case of 2 variables

Question Let. $x_1, x_2 \longrightarrow N(0, \theta)$ $\theta > 0$

Then the value of K, for which

$$\left(0, \frac{x_1^2 + x_2^2}{K}\right)$$ is a 95% CI for θ

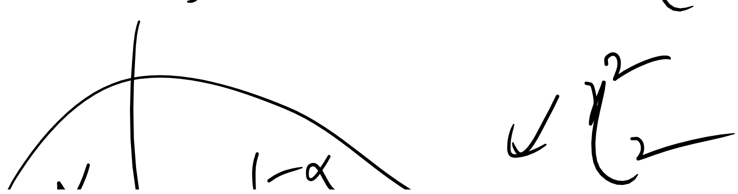
Approach

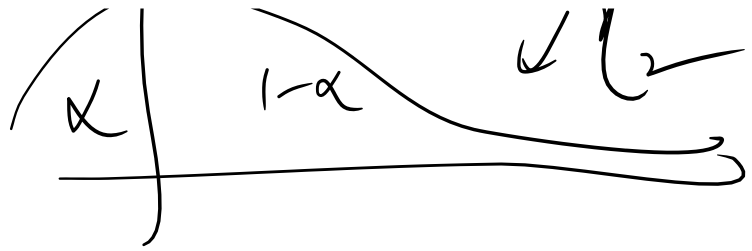
$x_1 \sim N(0, \theta) \Rightarrow \frac{x_1}{\sqrt{\theta}} \sim N(0,1) \Rightarrow \frac{x_1^2}{\theta} \sim \chi^2_1$

x_2 - - - - - - $\frac{x_2^2}{\theta} \sim \chi^2_1$

Let,
$$Y = \frac{x_1^2 + x_2^2}{\theta} \sim \chi^2_2$$

$$95\%\ CI \to \theta \to \left(0, \frac{x_1^2 + x_2^2}{K}\right)$$





$1-\alpha = 95\%$ $\alpha = 5\%$ $\chi^2_{0.95,\,2} = 0.1026$ (Table)

So, $0.95 = P(0.1026 < \chi^2_2 < \infty)$

$= P\left(0.1026 < \frac{X_1^2 + X_2^2}{\theta} < \infty\right)$

$= P\left(\frac{1}{\infty} < \frac{\theta}{X_1^2 + X_2^2} < \frac{1}{0.1026}\right)$

$= P\left(0 < \theta < \frac{X_1^2 + X_2^2}{0.1026}\right)$

$\Rightarrow \theta \in \left(0, \frac{X_1^2 + X_2^2}{0.1026}\right)$

$K = 0.1026$

Short cut → $2 \ln(0.95$ ~~$n \ln($~~

Type ⑤

⑧ **Approximation Types** .. (Normal app to Binomial Distn)

A poll is taken to UNI students before election.

(78) → 33 voters for ... Population 2200

95% CI for the proportion of votes in favour of

(7b) ...

95% CI for the proportion of votes in favour of Mr Smith ??

* (Cambridge Analytica)

Ans: The sample proportion $\hat{p} = \frac{x}{n} = \frac{37}{78} = 0.4231$

$(1-\alpha) = 95\%$

$$p \in \left[\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$$p \in \left[0.4231 \pm 1.96\sqrt{\frac{0.4231(1-0.4231)}{78}}\right]$$

$$p \in (0.3135,\ 0.5327)$$

Alternative Solutions

$\hat{p} = 37/78 = 0.4231$

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.4231 \times 0.5769}{78}} = 0.559$$

The 2.5th Percentile of N.D is

$z_{0.025} = 1.96$ (table)

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$= 0.4231 - (1.96)(0.559)$

$0.4231 - 0.1096$

$= 0.4231 - (1.96)(0.0\ldots)$

$= 0.4231 - 0.1096$

$= 0.3135$

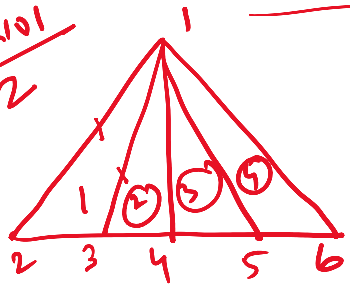
& $\hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$= 0.5327$

QUANT TRICKS (Applicable TO All levels)

1 . . . 100

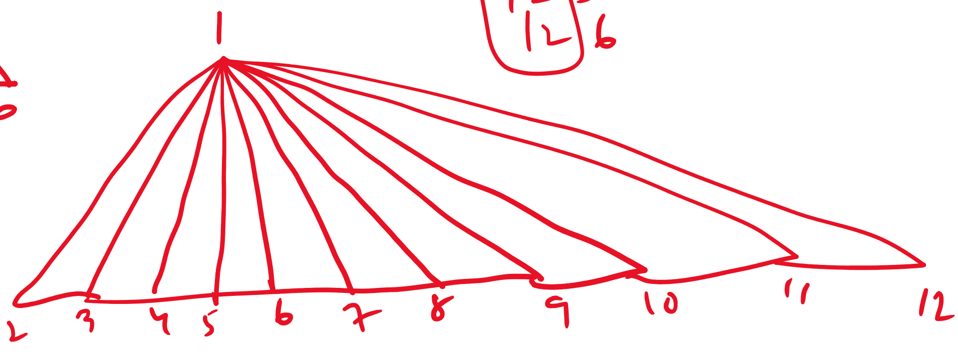
$\dfrac{100 \times 101}{2}$



4+3+2+1

12 3
12 4
12 5
12 6

13

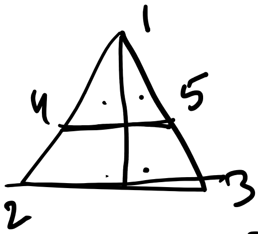


11+10+9+8+ . . . 1

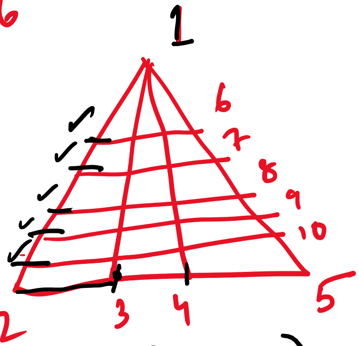
$\Rightarrow \dfrac{11 \times 12}{2}$

$\Rightarrow \dfrac{132}{2}$

$\Rightarrow 66$



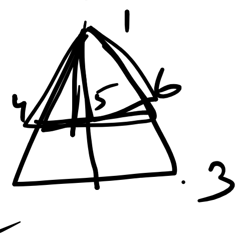
1+2 = 3 × 2 ⇒ 6

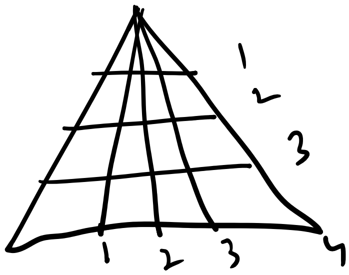


(1+2+3) × 5

⇒ 30

Doubt

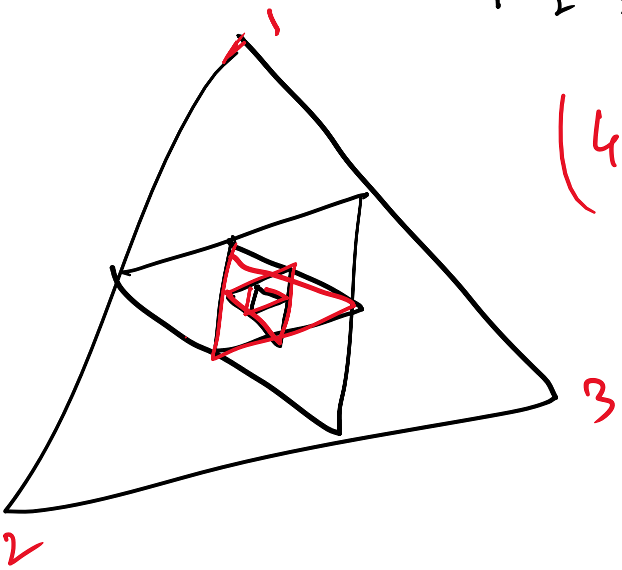




$(1+2+3+4) \times 3$

10

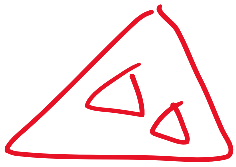
$\Rightarrow 30$

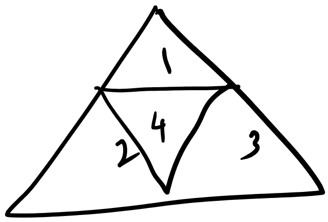


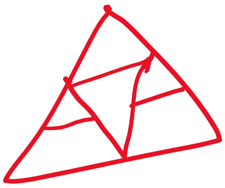
$(4 \times 4) + 1$

$1+4 \Rightarrow 5$

$\Rightarrow 17$



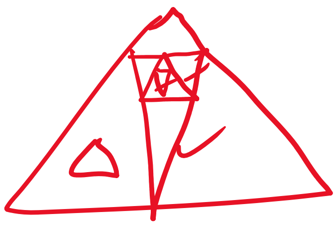


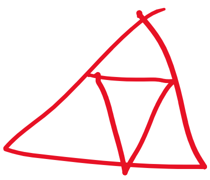


$10 \times 4 + 1$

[illegible] (V)

$(3 \times 4) + 1$

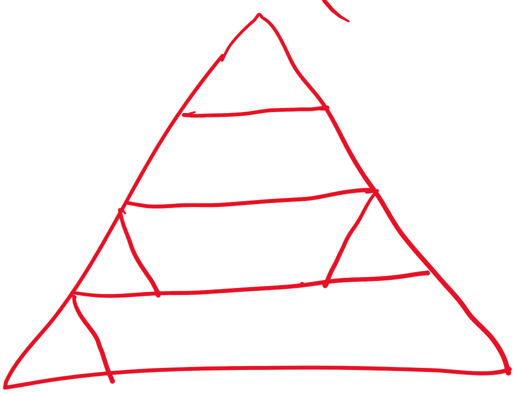




another 7

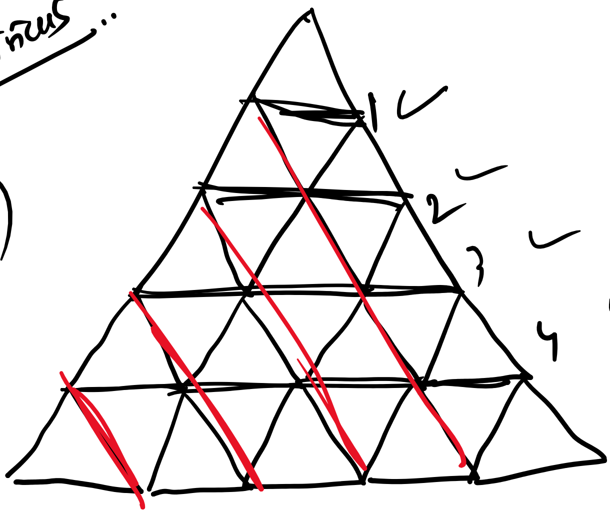
$(8 \times 4) + 1 = 33$

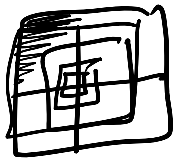
CAT



Advanced Tricks...

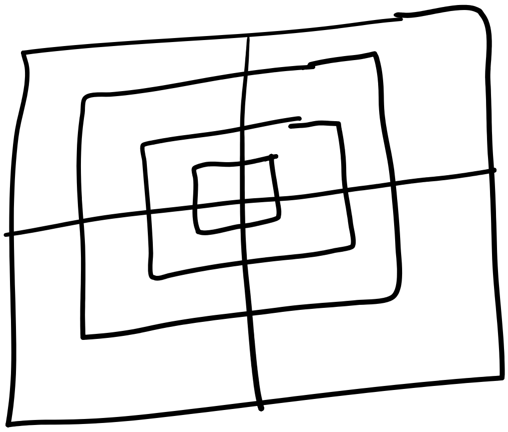
$(2^n - 5)$



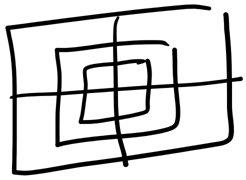


(5)

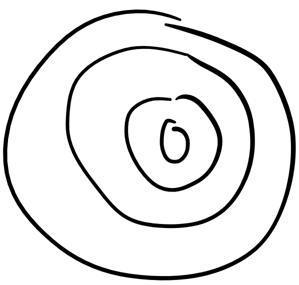
Squares



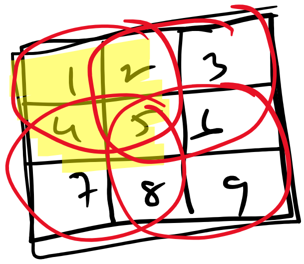
$5 \times 3 = 15$



5×2
$\times 3$



Chen Bond..

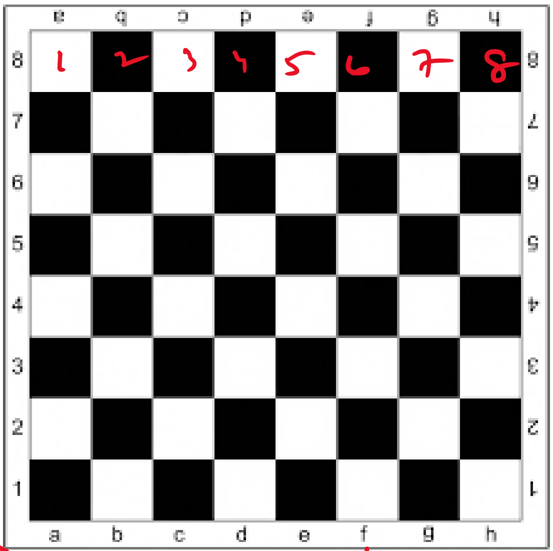


$9 + 1 + 4 \Rightarrow 14$

$1^4 + 2^4 + \ldots$

$1^5 + 2^5 + \ldots n^5$

$1^n + 2^n + \ldots + n^n$



$1^2 + 2^2 + 3^2 + \ldots 8^2$

$1 + 4 + 9 + 16 + \ldots + 64$

$$\frac{8 \times 9 \times 17}{6}$$

11

10–12

2870

$\frac{1}{40}$

870

40

CSAT

GS I

GS II

CAT

100

26

80

$$\frac{8 \times 9 \times 17}{6}$$

$$\frac{7 \times 8 \times 15}{6}$$

$n(n+1)(2n+1)$

$$\frac{n(n+1)(2n+1)}{6}$$

$$\frac{12 \times 13 \times 25}{6}$$

$a^2 + 2ab + b^2$

A

$1^3 = 1$

$2^3 = 8$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$7^3 = 343$

$8^3 = 512$

$9^3 = 729$

$10^3 = 1000$

421

$704969 \rightarrow 89^3$

421875

$\Rightarrow 75^3$

778688

$\Rightarrow 92^3$

175616

$\Rightarrow 56^3$

57^3

185193

$10^3 = 1000$

185 | 19 3

405 | 224

2 | 744

74^3

1+3
2+3
3+3

14^3

(schuul technique

2025

2025
16
425
425
0

45

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$7^2 = 9$
$8^2 = 64$
$9^2 = 81$

$4^2 = 6$
$6^2 = 6$

0,5

8 5 T

2025

$(\ )^{1/4}$ $(\ )^{1/5}$

$\sqrt{2025}$

14×15

45^2

2601

51^2

$5 9^2$

145^2

210 | 25 ⇒

5

21199 | 36

3056
2056
1456
1656

5×6 = 30

30

14
15
0

14 × 15 = 210

211 99

1236
1326
1456
1566

210 | 25

145^2

1 2 3

212 | 235 | 376 | 61

0 1 2

$6^2 = 36$

$7^2 = 49 \leftrightarrow 57$

$8^2 = 64 \leftrightarrow 57$

1456

9

1

1456 × 1437

59757

59 | 2 | 9

7 × 8 = 56

3

77

4×5

20

23 | 04

48^2

42^2

$\sqrt{30}$

$\sqrt[3]{124}$

$\sqrt{78}$ $5 + \frac{5}{10}$ $\frac{21}{8}$ $\sqrt{92}$

$\sqrt{78}$ $5 + \frac{}{10}$ $= 8$ $\sqrt{92}$

$= 8 + \frac{14}{16}$

$\rightarrow 5 + \frac{1}{2}$

$9 - \frac{3}{18}$ $\rightarrow$ 5.5 $= 9$

$= 8.86$

$9^2 = 81$ $10^2 = 100$

$10 - \frac{}{20}$

$= 10 - \frac{4}{10}$ 1724

$= 10 - 0.4$ 41 42

$= 9.6$

Cube Root

$\sqrt{30} + \sqrt{27} + \sqrt{15}$

$= 5\frac{5}{10} + 5\frac{2}{10} + \left(4 - \frac{1}{8}\right)$

$\frac{1}{4} = 0.25$

$\frac{1}{8} = 0.125$

$= 5.5 + 5.2 + 3.875$

$= 5.500 + 5.200 + 3.875$

$=$

Cube Root

$$(A^3 \pm B)^{\frac{1}{3}} = A \pm \frac{B}{3A^2}$$

$126^{\frac{1}{3}} \Rightarrow (5^3 + 1)^{\frac{1}{3}} =$

$126^{\frac{1}{3}} \Rightarrow (5+1)^{} =$

$126^{\frac{1}{3}} = 5 + \frac{1}{75} \Rightarrow 5 + \frac{1}{75} \Rightarrow \frac{376}{75}$

$218^{\frac{1}{3}} = 6 + \frac{2}{108}\ \frac{1}{54}$

$= \frac{325}{54}$ | 325 |

$349^{\frac{1}{3}} \rightarrow 7 + \frac{6\ 2}{147\ 49} \Rightarrow 7\frac{2}{49}$

$\checkmark + \frac{\text{Gap}}{3(\text{dyn})^2}$

$\frac{78}{82}$ $\frac{94}{98}$ $\frac{1066678}{1066682}$ $\frac{99.3}{97.3}$

$\frac{99.3}{99.37}$

$\frac{25}{27}$ | $\frac{31}{312}$ | $\frac{76}{}$ | $\frac{100}{}$

27 | 34 | 81 | 110

$\frac{25}{2} = 12.5$ (ii) | $\frac{31}{3} = 10.33$ (iii) | $\frac{76}{5} = 15.2$ (i) | 10 (iv)

5 10 15 20 25

EJOTY

A → 1

B → 2

⋮

MVJIB

21 2

68^2

(12) → 12000

13 10 9

(55)

25^2 = 625

68

72

45

44

17×17

9

=

17 × 17 = 289

4913

17

$a^2 + 2ab + b^2$

$a^3 + 3a^2b + 3ab^2 + b^3$

1.7 × 17

1 | 14 | 4 | 9

289

1.

17

1 | 21 | 147 | 343

(1) (6)

186

186

1^3

×

4.30 5.15

186

24596

8