08 November 2023 10:34 PM

1. maximisation $\Rightarrow$ $f'(x) = 0$
   $f''(x) < 0$

2. minimisation $\Rightarrow$ $f'(x) = 0$
   $f''(x) > 0$

3. at point of inflection $f'(x) = 0$ (stationary)
   or $f'(x) \neq 0$ (non-stationary)
   and $f''(x) = 0$
   $f'''(x) \neq 0$

$\boxed{y = f(x)}$

4. Slope of $y = \frac{dy}{dx} = f'(x)$ (first order derivative)

and to find the curvature $\Rightarrow$ second order derivative

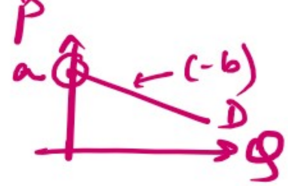
ie if $\underline{\underline{f''(x)}} = \frac{d^2y}{dx^2} > \underline{\underline{0}} \Rightarrow$ <u>convex</u>

if $\underline{\underline{f''(x)}} = \frac{d^2y}{dx^2} < \underline{\underline{0}} \Rightarrow$ <u>concave</u>

Q Suppose the consumer will demand 40 units of a product when the price is Rs 12 per unit and 25 units when price is Rs 18 each. Find the demand function assuming that it is linear. Also determine the TR, AR and MR functions.

Note: $y = mx + c \Rightarrow$ straight line / linear

Linear Demand: $\boxed{P = -bQ + a}$



when $Q = 40$ units $\quad P = 12$

and $Q = 25$ units $\quad P = 18$

$\therefore$ we get:

$$\cancel{a} - 40b = 12$$
$$\cancel{a} - 25b = 18$$
$$- \quad - \quad -$$
$$-15b = -6$$
$$b = \frac{6}{15} = \frac{2}{5}$$

$$\therefore a = 18 + 25 \times \frac{2}{5} = 18 + 10 = 28$$

$$\therefore a = 28 \text{ and } b = \frac{2}{5}$$

$\therefore$ Linear demand fn is $\boxed{P = 28 - \frac{2}{5}Q}$

$$\therefore TR = P \times Q = \left(28 - \frac{2}{5}Q\right) \cdot Q = 28Q - \frac{2}{5}Q^2$$

and $AR = \frac{TR}{Q} = P = 28 - \frac{2}{5}Q$

$$MR = \frac{dTR}{dQ} = 28 - \frac{4}{5}Q \quad \text{(ans)}$$

$\boxed{\frac{d^2TR}{dQ^2} = -\frac{4}{5} < 0 \quad \text{concave}}$

↳ slope of TR

↳ slope of TR

slope of AR : $\frac{dAR}{dQ} = -\frac{2}{5} < 0$

AR curve is a downward sloping straight line.

slope of MR : $\frac{dMR}{dQ} = \frac{-4}{5}$

if asked find the slope
if asked what is the shape ⇒ FO & SO

slope of TR $= 28 - \frac{4}{5}Q$ → MR → curvature $\frac{d^2TR}{dQ^2} < 0$ concave

slope of AR $= -\frac{2}{5}$

slope of MR $= -\frac{4}{5}$ } const ↳ straight line.

Q2 Show that the demand curve $\boxed{p = \left(\frac{a}{x+b}\right) - c}$ is downward sloping and convex to the origin/below.

... MR

-sloping and convex [illegible]

Also check the same for MR.

Slope of demand is

$$\frac{dP}{dx} = \frac{-a}{(x+b)^2}$$

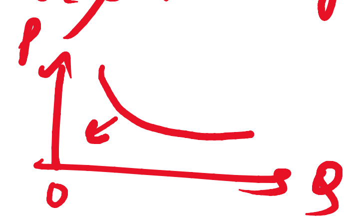
Since $a>0$ and $(x+b)^2>0$

$\therefore \frac{dP}{dx} < 0$

$\therefore$ demand curve is downward sloping.

$$\frac{d^2P}{dx^2} = -a\left[\frac{0 - 2(x+b)}{(x+b)^4}\right] = \frac{2a}{(x+b)^3} > 0$$

and convex to origin



$$AR = P = \left(\frac{a}{x+b}\right) - c$$

$$TR = P \cdot x = \frac{ax}{(x+b)} - cx$$

$$MR = \frac{dTR}{dx} = a\left[\frac{(x+b)1 - x\cdot 1}{(x+b)^2}\right] - c$$

$$= \frac{a(x+b) - ax}{(x+b)^2} - c$$

$$\boxed{MR = \frac{ab}{(x+b)^2} - c}$$

Slope of MR: $\dfrac{dMR}{dx} = ab\left[\dfrac{-2(x+b)}{(x+b)^4}\right]$

$= \dfrac{-2ab}{(x+b)^3} < 0$

MR is downward sloping.

Again $\dfrac{d^2MR}{dx^2} = -2ab\left[\dfrac{0-3(x+b)^2}{(x+b)^6}\right]$

$= \dfrac{6ab}{(x+b)^4} > 0$

Convex to the origin.

Note:

$TP_L = Q$ $\quad AP_L = Q/L$ $\quad MP_L = \dfrac{\partial Q}{\partial L}$

$TP_K = Q$ $\quad AP_K = \dfrac{Q}{K}$ $\quad MP_K = \dfrac{\partial Q}{\partial K}$

$MRTS_{LK} = \dfrac{MP_L}{MP_K} = \dfrac{\partial Q/\partial L}{\partial Q/\partial K}$

(Marginal Rate of technical Substitution)

Let the production function be $\boxed{Q = A\cdot L^{\alpha}K^{\beta}}$

output, Labour, capital

Calculate (i) MP_L, MP_K, $MRTS_{L,K}$

(ii) Slope of TP_L, AP_L, MP_L

(iii) curvature TP_L, AP_L, MP_L

Calculate (i) MP_L, MP_K, $MRTS_{L,K}$ Labour   cap.

(ii) Slope of TP_L, AP_L, MP_L

(iii) curvature TP_L, AP_L, MP_L

$$Q = A\, L^{\alpha} K^{\beta} \qquad (A, \alpha, \beta)$$

(i) partial derivative of Q w.r.t L

$$MP_L = \frac{\partial Q}{\partial L} = A\,\alpha L^{\alpha-1} K^{\beta}$$

$$MP_K = \frac{\partial Q}{\partial K} = A L^{\alpha} \beta K^{\beta-1}$$

$$MRTS_{L,K} = \left|\frac{MP_L}{MP_K}\right| = \frac{\cancel{A}\,\alpha L^{\alpha-1} K^{\beta}}{\cancel{A}\,\beta L^{\alpha} K^{\beta-1}} = \frac{\alpha}{\beta}\cdot\frac{K}{L}$$

slope

(ii) Slope of TP_L MP_L / MP_K

$$Q = A\, \underset{u}{L^{\alpha}}\, \underset{v}{K^{\beta}}$$

u.v rule
u.dv + v.du

Total derivative we get

$$dQ = A\left[\alpha L^{\alpha-1}\cdot dL \cdot K^{\beta} + L^{\alpha} \beta K^{\beta-1} dK\right]$$

because Q = 0 along the production

$$0 = \alpha L^{\alpha-1}\cdot dL\, K^{\beta} + L^{\alpha}\beta K^{\beta-1} dK$$

$$-\alpha L^{\alpha-1} K^{\beta}\cdot dL = \beta L^{\alpha} K^{\beta-1} dK$$

$$\frac{dK}{dL} = -\frac{\alpha L^{\alpha-1} K^{\beta}}{\beta L^{\alpha} K^{\beta-1}}$$

$$\frac{dQ}{dL} = \qquad \beta L^{\alpha} K^{\beta-1}$$

$$\boxed{\frac{dK}{dL} = -\frac{\alpha}{\beta}\,\frac{K}{\beta}}$$

$$\boxed{\text{Slope of } TP_L = \frac{dK}{dL} = -\frac{MP_L}{MP_K} = -\frac{\partial Q/\partial L}{\partial Q/\partial K}}$$

$$AP_L = \frac{TP_L}{L} = \frac{A L^{\alpha} K^{\beta}}{L} = A L^{\alpha-1} K^{\beta}$$

Slope of AP_L : $\frac{\partial AP_L}{\partial L} =$

$$\frac{\partial^2 AP_L}{\partial L^2} =$$

$$MP_L = \frac{\partial Q}{\partial L} = A \alpha L^{\alpha-1} K^{\beta}$$

slope of $MP_L \Rightarrow \frac{\partial MP_L}{\partial L} =$

and $\frac{\partial^2 MP_L}{\partial L^2} =$

and $\frac{\partial MP_L}{\partial L^2} =$

1. Homogeneous & Homothetic
2. Unconstrained optimisation (two variables)
3. Lagrange → Constrained optimisation & Bordered Hessian Matrix.
4. elasticity, few other examples of production fn

partials