Classical Linear Regression Model (CLRM)

Multivariate extension of SLRM:

SLRM: $Y_i = \alpha + \beta X_i + u_i$

k- variable Model:-

PRF : $Y_i = \breve{\beta}_1 + \breve{\beta}_2 X_{2i} + \breve{\beta}_3 X_{3i} + \cdots + \breve{\beta}_k X_{ki} + u_i$, $i = 1, 2, \ldots, n$. $[n > k]$

Parameters: $\beta_1, \beta_2, \ldots, \beta_k$ where

β_1 is the intercept (1)

$(\beta_2, \beta_3, \ldots, \beta_k)$ are the slope coeffecients (k-1)

Explanatory variables: $(X_2, X_3, \ldots, X_k)$ (k-1)

We collected a r.s of size n for $(Y_i, X_{2i}, X_{3i}, \ldots, X_{ki})_{i=1}^{n}$

X_{2i} = i^{th} obs for explanatory variable X_2.

$$\left.\begin{aligned} E(\hat{\beta}_1) &= \beta_1 \\ E(\hat{\beta}_2) &= \beta_2 \\ &\vdots \\ E(\hat{\beta}_k) &= \beta_k \end{aligned}\right\} \rightarrow \quad E\underbrace{\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}}_{\hat{\beta}} = \begin{bmatrix} E(\hat{\beta}_1) \\ \vdots \\ E(\hat{\beta}_k) \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}$$

PRF : $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$, $i = 1, 2, \ldots, n$ $[n > k]$.

$i = 1$, $Y_1 = \beta_1 + \beta_2 X_{21} + \beta_3 X_{31} + \cdots + \beta_k X_{k1} + u_1$

$i = 2$, $Y_2 = \beta_1 + \beta_2 X_{22} + \beta_3 X_{32} + \cdots + \beta_k X_{k2} + u_2$.

$\cdots$

$i = n$, $Y_n = \beta_1 + \beta_2 X_{2n} + \beta_3 X_{3n} + \cdots + \beta_k X_{kn} + u_n$

Define: $Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}_{n\times 1}$ $\quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix}_{k\times 1} = \begin{bmatrix} \beta_1 \\ b \end{bmatrix}$ $\quad U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}_{n\times 1}$

b → vector of slope coeff (k-1)

$X = \begin{bmatrix} 1 & X_{21} & X_{31} & \cdots & X_{k1} \\ 1 & X_{22} & X_{3 2} & \cdots \end{bmatrix}$

$$X = \begin{bmatrix} 1 & x_{21} & x_{31} & \cdots & x_{k1} \\ 1 & x_{22} & x_{32} & & x_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{2n} & x_{3n} & & x_{kn} \end{bmatrix}_{n \times k} = \begin{bmatrix} \vdots & \vdots & & \vdots \\ 1 & X_2 & \cdots & X_k \\ \vdots & \vdots & & \vdots \end{bmatrix} = \begin{bmatrix} \vdots & \vdots \\ 1 & X_{(2)} \\ \vdots & \vdots \end{bmatrix}$$

$X_{(2)}$

column of X_2

$X_{(2)}$: collection of all values of explanatory variables.

Model in Matrix Form: $\boxed{Y = X\beta + U}$