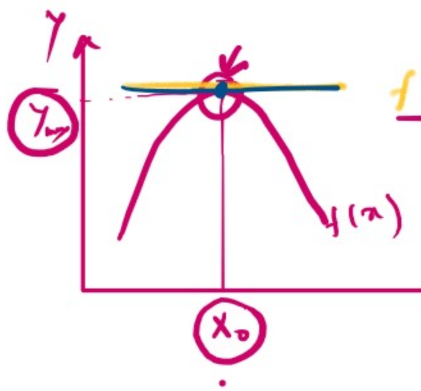
06 November 2023 11:02 PM

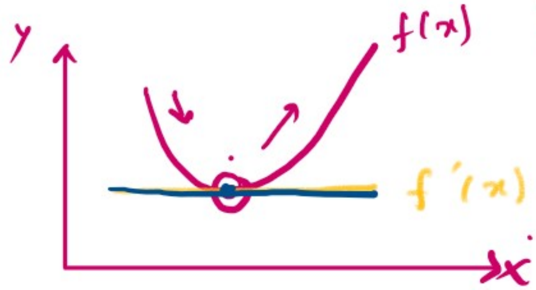
Maxima, Minima and point of inflexion.

$y = f(x)$



$f'(x) = 0$ → $\frac{dy}{dx}$ (stationary) Necessary condition

Average cost curve.



$f'(x) = 0$ (stationary)

$\frac{dy}{dx} = 0$

For Maxima $Y = f(x)$

a) $f'(x) = 0$ (Necessary)

ie $\frac{dy}{dx} = 0$

b) s.o.c (Sufficiency condition)

$f''(x) < 0$

$\boxed{\frac{d^2y}{dx^2} < 0}$

(concavity)

for minima

a) $f'(x) = 0$ (Necessary)

ie $\frac{dy}{dx} = 0$

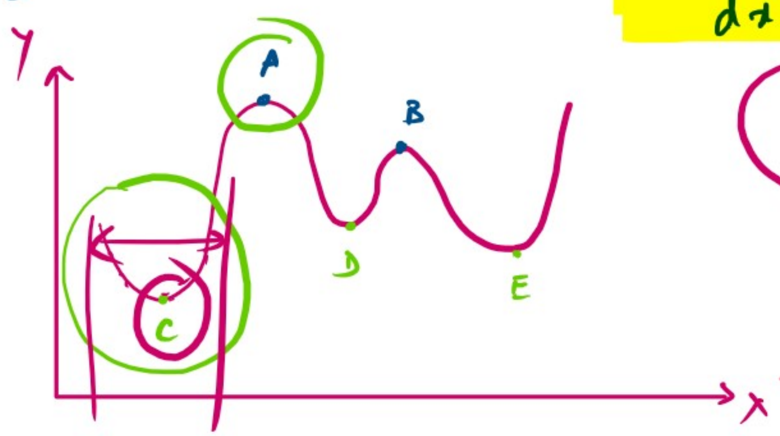
b) S.O.C (Sufficiency)

ie $f''(x) > 0$

or, $\frac{d^2y}{dx^2} > 0$

(convexity)

$Y = f(x)$



Point of Inflection: (i) $f'(x) = 0$ ✓ } At a Stationary point

Point of Inflution:

(A)
(i) $f'(x) = 0$
(ii) $f''(x) = 0$
(iii) $f'''(x) \neq 0$

Stationary point ↳ pt of inflution

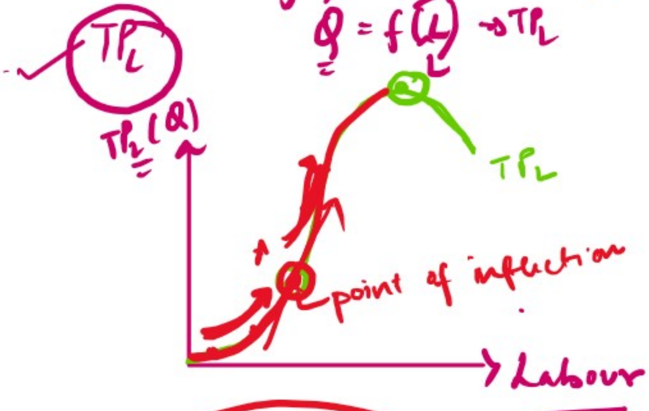
$y = f(x)$

(B)
(i) $f'(x) \neq 0$
(ii) $f''(x) = 0$
(iii) $f'''(x) \neq 0$

point of inflution and Non Stationary.

Total product curve of labour

$Q = f(L) \rightarrow TP_L$



pt of influxion is the point where a curve changes it's curvature
↳ (second-order derivative)

pt A : $\frac{dQ}{dL} \neq 0$ >0 (increasing)
Non-stationary.

$\frac{d^2Q}{dL^2} = 0 \Rightarrow$ point of inflexion

$\frac{d^3Q}{dL^3} \neq 0$

[Ex of TP_L]

Note:
First Derivative ↳ slope → >0, <0, =0
Second Derivative ↳ curvature → convex, concave, straight lt.

Summary: $y = f(x)$

① For maxima → $f'(x) = 0$, $f''(x) < 0$

② For minima → $f'(x) = 0$, $f''(x) > 0$

③ Point of inflexion (Stationary) → $f'(x) = 0$, $f''(x) = 0$

③ Pt of inflexion (Stationary) → $f'(x) = 0$

$f''(x) = 0$

$f'''(x) \neq 0$

④ Pt of inflexion (Non-stationary) → $f'(x) \neq 0$

$f''(x) = 0$

$f'''(x) \neq 0$

Note: $TP_L = Q$ (total product)

$AP_L = \frac{TP_L}{L} = \frac{Q}{L}$

$MP_L = \frac{\partial Q}{\partial L}$ (also slope of TP_L)

Revenue.

$TR = P \times Q$

$AR = \frac{TR}{Q}$

$MR = \frac{dTR}{dQ}$ (slope of TR)

Cost: TC = total cost

$AC = \frac{TC}{Q}$

$mc = \frac{\partial TC}{\partial Q}$ (slope of TC)

Total cost

TVC → $\frac{TVC}{Q} = AVC$

TFC → $\frac{TFC}{Q} = AFC$

Suppose, $\boxed{C = 500 + 100Q}$ Total cost funct

$TC = TFC + TVC$

$AC = \frac{C}{Q} = \frac{500}{Q} + \frac{100\cancel{Q}}{\cancel{Q}} = \frac{500}{Q} + 100$

$AFC = TFC = 500$ and $AVC = \frac{TVC}{} = \frac{100Q}{Q}$

$$AFC = \frac{TFC}{Q} = \frac{500}{Q} \quad \text{and} \quad AVC = \frac{TVC}{Q} = \frac{100Q}{Q}$$

$$AVC = 100$$

Profit function of firm $\pi = TR - TC$

———— * ————

Q. Obtain the extrema for the Total cost function

$$C = q^3 + 2q^2 - 4q + 4$$

For cost minimisation

F.O.C is $\frac{dc}{dq} = 0$

or, $3q^2 + 4q - 4 = 0$

$(3q - 2)(2 + q) = 0$

$\therefore q = 2/3$ or $q = -2$

Since q = production. q cannot -ve

$\therefore q = 2/3$

S.O.C $\frac{d^2c}{dq^2}$ at $q = 2/3$ is

$$\frac{d^2c}{dq^2} = 6q + 4 = \cancel{6}^{2} \times \frac{2}{\cancel{3}} + 4$$

$$= 4 + 4 = 8 > 0$$

$\therefore$ S.O.C for minimisation is satisfied

$\therefore$ at $q = 2$ units c is minimised.

$\therefore$ at $q = \frac{2}{3}$ units c is minimised.

What is the minimum cost?

$$C = q^3 + 2q^2 - 4q + 4$$

$$\text{at } q = 2/3 \Rightarrow C_{min} = (2/3)^3 + 2(2/3)^2 - 4 \times \frac{2}{3} + 4$$

2: Obtain the inflexion point for

$$y = x^3 + 6x^2 + 5x - 30.$$

Examine whether this point is a stationary or non-stationary.

Ans. $y = x^3 + 6x^2 + 5x - 30$

$$\frac{dy}{dx} = 3x^2 + 12x + 5$$

$$\frac{d^2y}{dx^2} = 6x + 12$$

$$\frac{d^3y}{dx^3} = 6 \neq 0$$

At point of inflexion

$$\frac{d^2y}{dx^2} = 0$$

$$6x + 12 = 0$$

$$\boxed{x = -2}$$

Now $\frac{dy}{dx}$ at $x = -2$

$\frac{dy}{dx}$ at $x=-2$

$= 3(-2)^2 + 12(-2) + 5$

$= 3\times 4 - 24 + 5$

$= 12 - 24 + 5$

$= -12 + 5$

$= -7 < 0$

$\therefore \frac{dy}{dx} \neq 0 \Rightarrow$ Non-stationary.

3: The total cost (C) and total Revenue (R) function of a firm by $C = 4q^2 + 10$ ✓
and $R = -2q^2 + 6q$ ✓

Find the output level (q) at which profit of the firm is maximum.

Ans

Profit function, $\pi = TR - TC$

$= (-2q^2 + 6q) - (4q^2 + 10)$

or, $\pi = -2q^2 - 4q^2 + 6q - 10$

$$\boxed{\pi = -6q^2 + 6q - 10}$$

F.O.C for profit maximisation, $\frac{d\pi}{dq} = 0$

$-12q + 6 = 0$

$$\boxed{q = \tfrac{1}{2} \text{ units}}$$

S.O.C $\Rightarrow \frac{d^2\pi}{dq^2} = -12 < 0$

$S.O.C \Rightarrow \frac{d^2\pi}{dq^2} = -12 < 0$

$\therefore$ at $q = \frac{1}{2}$ units firm's profit is maximised.

3. $f(x) = \frac{x}{x^2+4}$

F.O.C

$\Rightarrow f'(x) = 0$

$x = \text{value}$

S.O.C

$x = 5, -3$

$\frac{d^2y}{dx^2} = x+2$

at $x=5 \Rightarrow \frac{d^2y}{dx^2} = 7 > 0$ (minimum extreme)

at $x=-3 \Rightarrow \frac{d^2y}{dx^2} = -1 < 0$ (maximum extreme)

4. The demand curve faced by a firm is

$\boxed{p = 20-3q}$

If the firm's average cost (AC) function is

$\boxed{AC = (\ldots)}$ find the optimum

$\boxed{AC = (10q - 5)}$, find the optimum output level of firm assuming the objective is to maximum profit.

① TR?   TC = ?

$TR = P \times Q$ and $AR = \frac{TR}{Q} = \frac{PQ}{Q} = P$

$AR = 20 - 3q$ $\therefore TR = (20 - 3q)q$

$TR = 20q - 3q^2$

② $AC = \frac{TC}{q}$ $\therefore TC = AC \cdot q \Rightarrow TC = 10q^2 - 5q$

$\pi = TR - TC$

f.o.c $\frac{d\pi}{dq} = 0$

s.o.c $\frac{d^2\pi}{dq^2} < 0$