Real Analysis

Colleges → They They They

Real Analysis
~~Rai-Singhania~~

$\sqrt{2}$ Irr

Sup → ~~max~~ → Superior → Bara

Inf → ~~mini~~ → Inferior → chota

~~max~~ ~~min~~

GLB LUB

$\frac{x+4}{x} = \boxed{1} + \boxed{\frac{4}{x}}$ $1+\frac{1}{\infty}$

$\boxed{5} \rightarrow$ Sup

$\boxed{3}$

2.33

$\boxed{1} \rightarrow$ Inf

$u_x = \boxed{x} + \frac{4}{x}$

x ~ natural

$-\infty, \ldots 0, \boxed{1, 2 \ldots}$

$u_1 = 1 + \frac{4}{1} = 1+4 = 5$

$u_2 = 2 + \frac{4}{2} = 4$

$u_3 = 3 + 4/3 = 4.33$

$u_\infty = \boxed{\infty}$

$\infty + \infty = 2\infty$

$\boxed{\infty}$

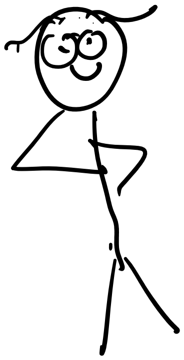
$\infty + \infty = \boxed{\infty}$

$\infty - \infty \neq \boxed{0}$ $\boxed{\infty}$ $\boxed{UD}$

finite $\boxed{UD}$

$4\infty = \boxed{\infty}$

$\boxed{-\infty}$ $\boxed{\infty}$



$-4 < x < \boxed{4}$ LUB

Rich ka Garib

Garib ka Rich

$-3 < x < 10$

$-2 < x < 100$

$\boxed{1.33} < x < 97.33$

$-e^{t} < x < \log 10^{57}$

GLB

maxi in min

mini in max

max → Iphone 14 Pro Max 512GB/1024GB

Galaxy Fold 5

Neighbourhood of a point

$x \to 0$

7800/month

Iphone 13

S22

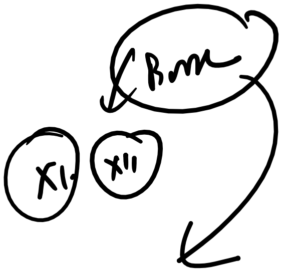
$y = x^2$

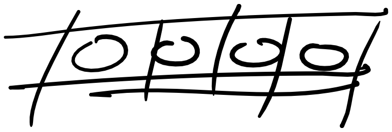
$\frac{dy}{dx} = 2x$

$$L = (2x^2+3x)+4y$$

$$x+y \le 3$$

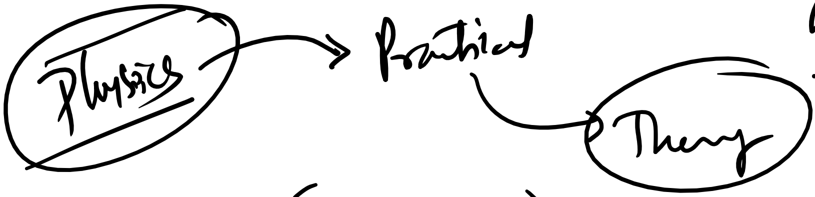
Constraint

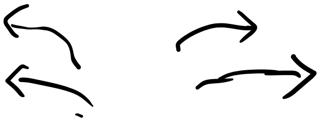




Topology

GRF





$$\sum$$

$$\prod x_i = x_1 \cdot x_2 \cdots x_n$$

$$\boxed{\text{GIF}} \longrightarrow$$

$$2.99 \rightarrow 2$$

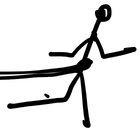
$$2 \rightarrow 2$$

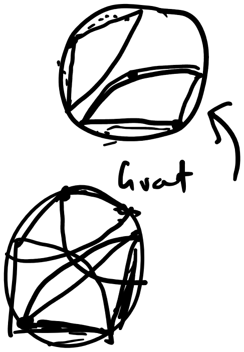
$$2.07 \rightarrow 2$$

Pg class For
14.09.2023

Bounded Convergent ⇒ ①

Bound





1. Let $a_n = \sum_{k=1}^{n} \frac{n}{n^2+k}$, for $n \in \mathbb{N}$. Then, the sequence $\{a_n\}$ is

(a) Convergent
(b) Bounded but not convergent
(c) Diverges to ∞
(d) Neither bounded nor diverges to ∞

∞

$$\frac{n}{n^2+1}+\frac{n}{n^2+2}+\cdots+\frac{n}{n^2+n} \text{ --- (i)}$$

$$\frac{n}{n^2+1} \quad \frac{n^2}{n^2} \text{ --- (ii)}$$

∞

∀ v.g.

$$a_n = \frac{n}{n^2+1}+\frac{n}{n^2+2}+\cdots+\frac{n}{n^2+n}$$

$$< \frac{n}{n^2}+\frac{n}{n^2}+\cdots+\frac{n}{n^2} = \frac{n^2}{n^2} = 1$$

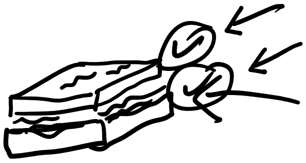
$\boxed{a_n < 1}$ ∀ $n \in N$ $\quad a_n > 0$ ∀ $n \in N$

B/A + B/B

∃ there exists

Now, $k \le n$ $\quad n^2+k \le n^2+n$

$$\frac{n}{n^2+k} \ge \frac{n}{n^2+n}$$



$a_n < 1$, $a_n > 0$ ✓ $\quad a_n = \sum \frac{n}{n^2+k} \ge \sum \frac{n}{n^2+n}$

$$\lim \sum \frac{n}{n^2} \ge a_n \ge \sum \frac{n}{n^2+n}$$

$$\Rightarrow \frac{n \cdot n}{n^2} \ge a_n \ge \frac{n \cdot n}{n^2+n}$$

$$\Rightarrow \frac{n}{n+1} \le a_n \le 1$$

$$\lim_{n\to\infty} \frac{n}{n+1} \le \lim_{n\to\infty} a_n \le \lim_{n\to\infty} 1$$

$$\boxed{1 \le \lim_{n\to\infty} a_n \le 1}$$

2. The number of real roots of the equation $x^3 + x - 1 = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

$\Rightarrow \boxed{l = \lim_{n\to\infty} a_n = 1}$ $n+1$

2. The number of real roots of the equation $x^3 + x - 1 = 0$ is
(a) 0 (b) 1 ✓ (c) 2 (d) 3

$x^3 + x - 1$ ① +ve Real

$-x^3 - x - 1$ ⓪

$3 - 1 = 2$ ② Thumb

$\lim_{n\to\infty} a_n = 1$

f_1 f_2 f_3 Interval

Lt

3. The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2 + kn}}$ is
(a) $2(\sqrt{2} - 1)$ ✓ (b) $2\sqrt{2} - 1$ (c) $2 - \sqrt{2}$ (d) $\frac{1}{2}(\sqrt{2} - 1)$

Riemann's Sum for def int. $\lim_{n\to\infty}$

$$\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\frac{1}{n} = \int_0^1 f(x)\,dx$$

$$= \lim_{n\to\infty} \frac{1}{n\sqrt{1 + \frac{k}{n}}} = \lim_{n\to\infty} \sum \frac{1}{n} \frac{1}{\sqrt{1 + \frac{k}{n}}} = \int_0^1 \frac{dx}{\sqrt{1+x}}$$

$$= \lim_{n\to\infty} \sum \frac{1}{\sqrt{n^2 + kn}} = 2(1+x)^{1/2}\Big|_0^1 = \underline{\underline{2(\sqrt{2} - 1)}}$$

$\underset{n\to\infty}{Lt} \frac{a_{n+1}}{a_n}$

a_{n+1} · Conv

4. *The set of all x at which the power series* $\sum_{n=1}^{\infty} \frac{n}{(2n+1)^2}(x-2)^{3n}$ *converges, is*

(a) [-1,1) (b) [-1,1] ~~(c) [1,3)~~ (d) [1,3]

a_n , a_{n+1}

$\underset{n\to\infty}{Lt} \frac{a_{n+1}}{a_n} = \underset{n\to\infty}{Lt} \frac{(n+1)(2n+1)^2 (x-2)^{3n+3}}{(2n+3)^2 n (x-2)^{3n}} = (x-2)^3$

$1 \le x < 3$

$1 \le x \le 3$

$(|x-2|^3) < 1$

$|x-2| < 1$

$-1 < x-2 < 1$

$1 < x < 3$

$2 \in (1,3)$

$x=2$ Conv

$x=0$ not Conv

$\frac{n}{(2n+1)^2} \to 0 \quad n\to\infty$ monotonic ↓ing sequence of +ve term.

Leibnitz' test → @ $x=1$ Conv

@ $x=3$ $\sum \frac{n}{(2n+1)^2}$ $u_n = \frac{n}{(2n+1)^2}$

$v_n = 1/n$

$\Sigma v_n, \Sigma u_n$

$\underset{n\to\infty}{Lt} \frac{u_n}{v_n} = \underset{n\to\infty}{Lt} \frac{n}{(2n+1)^2} \times \frac{n}{1}$

$= \underset{n\to\infty}{Lt} \frac{1}{(2+\frac{1}{n})^2} = \frac{1}{4}$ finite

$\therefore \Sigma v_n$ div p-test Σu_n also div

At $x=3$ non-Conv

5. *Consider the following subsets of* $\mathbb{R}$*:*

$E = \left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}$, $F = \left\{\frac{1}{1-x} : 0 \le x < 1\right\}$ *Then*

~~(a) Both E and F are closed~~ (b) E is closed and F is not closed

~~(c) E is NOT closed and F is closed~~ (d) Neither E nor F is closed.

$(1+x)^n \to$ - - - - - -

$F = \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ ---- $\Rightarrow 0.5, 0.67, 0.75, 0.80 \to \to 1$

(c) E is NOT closed and F is closed (d) Neither E nor F is closed.

$E = 1/2, 2/3, 3/4, 4/5 \cdots \Rightarrow 0.5, 0.67, 0.75, 0.80 \longrightarrow \longrightarrow \boxed{1}$

$\frac{1}{2} \leq x < 1 \;\forall\, x \in E \rightarrow$ E is Bounded

$1 \notin E$ $\left(\because \text{ if } 1 \in E \Rightarrow \frac{n}{n+1} = 1 \Rightarrow n = n+1 \Rightarrow 1 = 0 \;\times \right]$

But E Contains infinitely many values in the neighborhood of 1

So, E is Not closed.

$F = \frac{1}{1-x}$; $0 \leq x < 1$

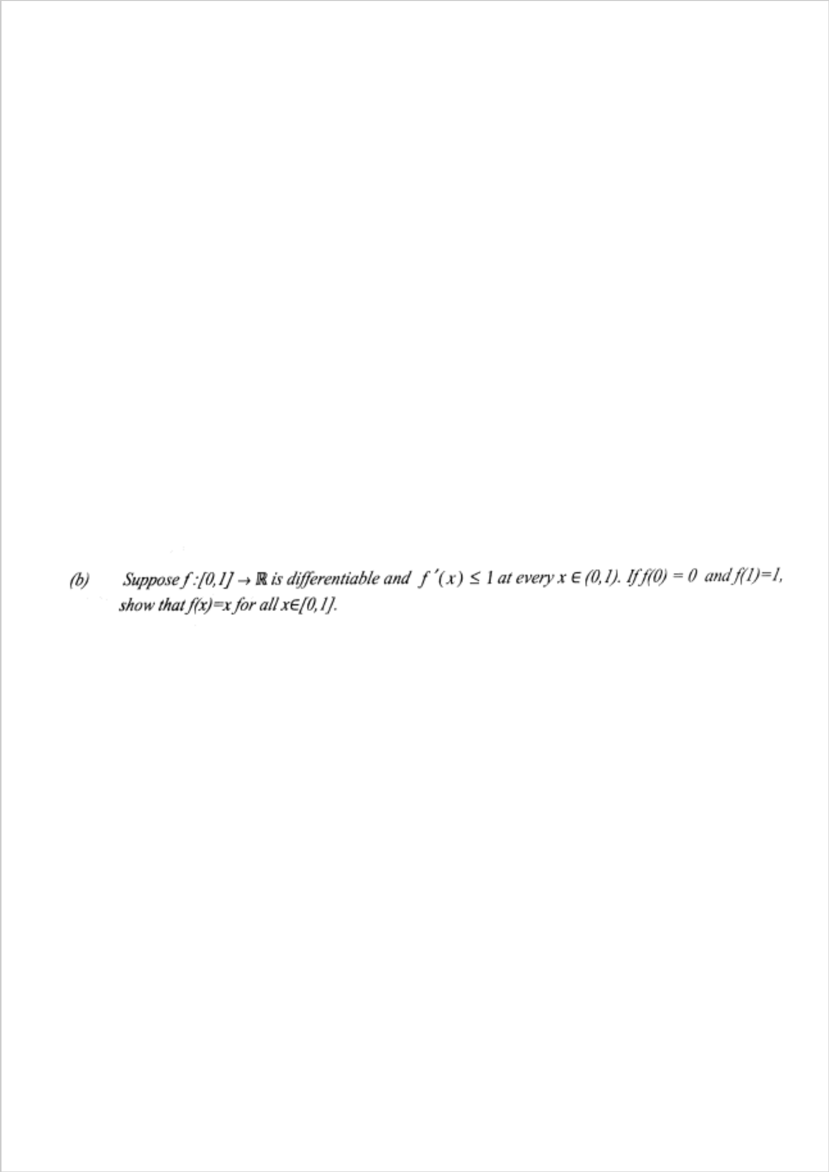
As, $0 \leq x < 1 \Rightarrow -1 < -x \leq 0$

$0 < 1-x \leq 1$

$1-x \leq 1 \Rightarrow \frac{1}{1-x} \geq 1$

$\boxed{\text{F is closed}}$

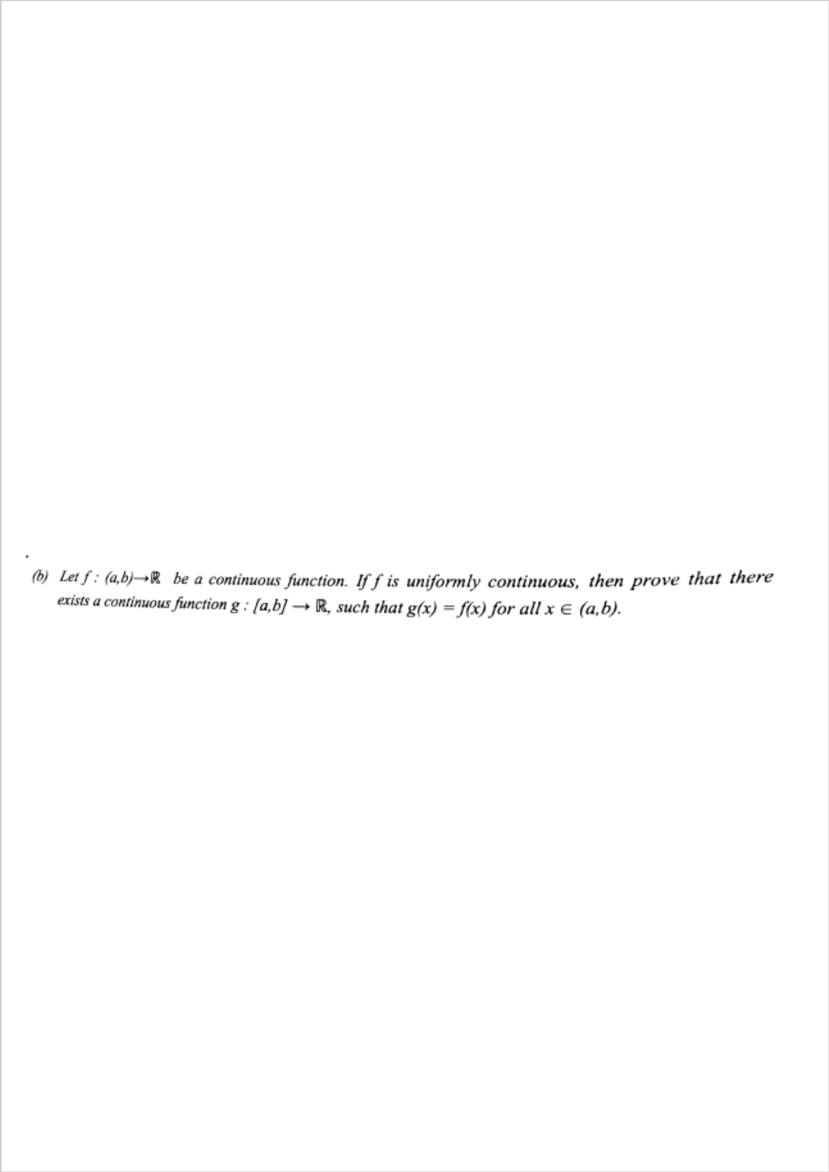
6. (a) Let $\{a_n\}$ be a sequence of non-negative real numbers such that $\sum_{n=1}^{\infty} a_n$ converges, and let $\{k_n\}$ be a strictly increasing sequence of positive integers. Show that $\sum_{n=1}^{\infty} a_{k_n}$ also converges.



(b) Suppose $f:[0,1] \rightarrow \mathbb{R}$ is differentiable and $f'(x) \leq 1$ at every $x \in (0,1)$. If $f(0) = 0$ and $f(1)=1$, show that $f(x)=x$ for all $x \in [0,1]$.

7. *Show that the series* $\sum_{n=1}^{\infty} \frac{x}{\sqrt{n}(1+n^p x^2)}$ *converges on* $\mathbb{R}$ *for* $p > 1$

8. (a) *If E is a subset of $\mathbb{R}$ that does not contain any of its limit points, then prove that E is a countable set.*



(b) Let $f : (a,b) \to \mathbb{R}$ be a continuous function. If f is uniformly continuous, then prove that there exists a continuous function $g : [a,b] \to \mathbb{R}$, such that $g(x) = f(x)$ for all $x \in (a,b)$.

9062395123

9. Let $\{x_n\}$ be the sequence $+\sqrt{1}, -\sqrt{1}, +\sqrt{2}, -\sqrt{2}, +\sqrt{3}, -\sqrt{3}, +\sqrt{4}, -\sqrt{4}, \ldots$ If $y_n = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$ for all $n \in \mathbb{N}$, then the sequence $\{y_n\}$ is Conv.

$$y_n = \frac{x_1 + \cdots x_n}{n}$$

$x_n = -x_{n-1}$ $\forall\, n \in \mathbb{N}$ n is even

$y_n = 0$ $\forall$ even n

If n is odd $y_n = \frac{1}{n}\sqrt{\frac{n \pm 1}{2}}$ $\Rightarrow \lim_{n\to\infty} y_n = 0$

(a) monotonic (b) *not bounded*

(c) bounded *but not convergent* [✓ (d) *convergent*]

+ve > 0

10. The number of distinct real roots of the equation $x^9 + x^7 + x^5 + x^3 + x + 1 = 0$ is

[✓ (a) 1] (b) 3 (c) 5 (d) 9

$- \quad - \quad - \quad - \quad - \curvearrowright +$

11. *If the power series* $\sum_{n=0}^{\infty} a_n x^n$ *converges for* $x = 3$*, then the series* $\sum_{n=0}^{\infty} a_n x^n$.

[✓ *(a) Converges absolutely for x = -2*]

(b) Converges but not absolutely for x = -1

(c) Converges but not absolutely for x = 1

(d) Diverges for x = -2

Radius of Conv $R \geq 3$ $\boxed{R \geq 3}$

The series Converges absolutely inside the region of Converge.

Series Converges absolutely for $x = -1, -2, 1$

$$\left|\frac{x}{1+|x|}\right| = \frac{|x|}{|1+|x||} = \frac{|x|}{1+|x|} = 1 - \frac{1}{1+|x|} < 1$$

$$\left[\frac{1}{1+|x|} > 0\right]$$

12. If $Y = \left\{ \frac{x}{1+|x|} \mid x \in \mathbb{R} \right\}$, then the set of all limit points of Y is

(a) (-1, 1) (b) (-1, 1] (c) [0, 1] [✓ (d) [-1, 1]]

So, $\Rightarrow \left|\frac{x}{1+|x|}\right| < 1 \;\forall\; x \in R$

$\Rightarrow -1 < \frac{x}{1+|x|} < 1 \;\forall\; x \in R$

$Y = \underline{(-1, 1)}$

~~Not A~~

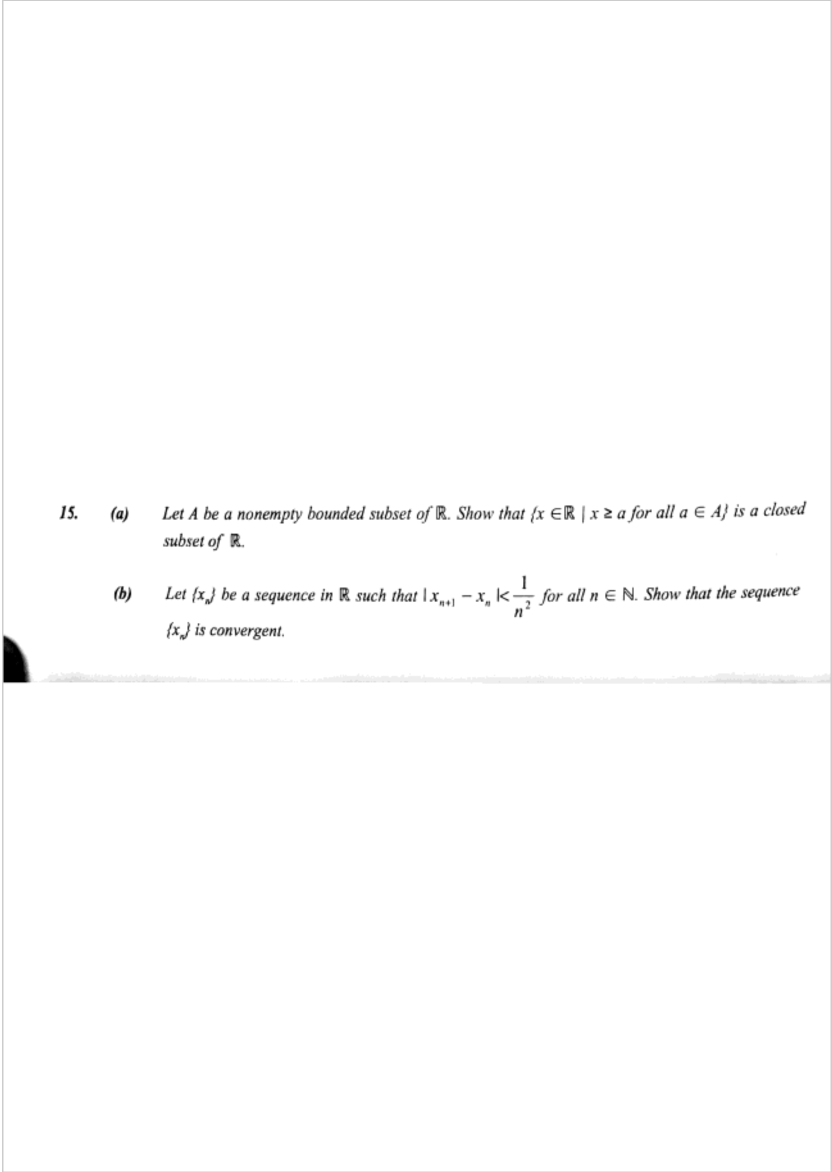
But D

13. **(a)** *Examine whether the following series is convergent* $\sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \ldots (2n-1)}$

(b) *For each* $x \in \mathbb{R}$*, let* $[x]$ *denotes the integer less than or equal to x. Further, for a fixed* $\beta \in (0,1)$*, define* $a_n = \frac{1}{n}[n\beta] + n^2\beta^n$ *for all* $n \in \mathbb{N}$*. Show that the sequence* $\{a_n\}$ *converges to* β.

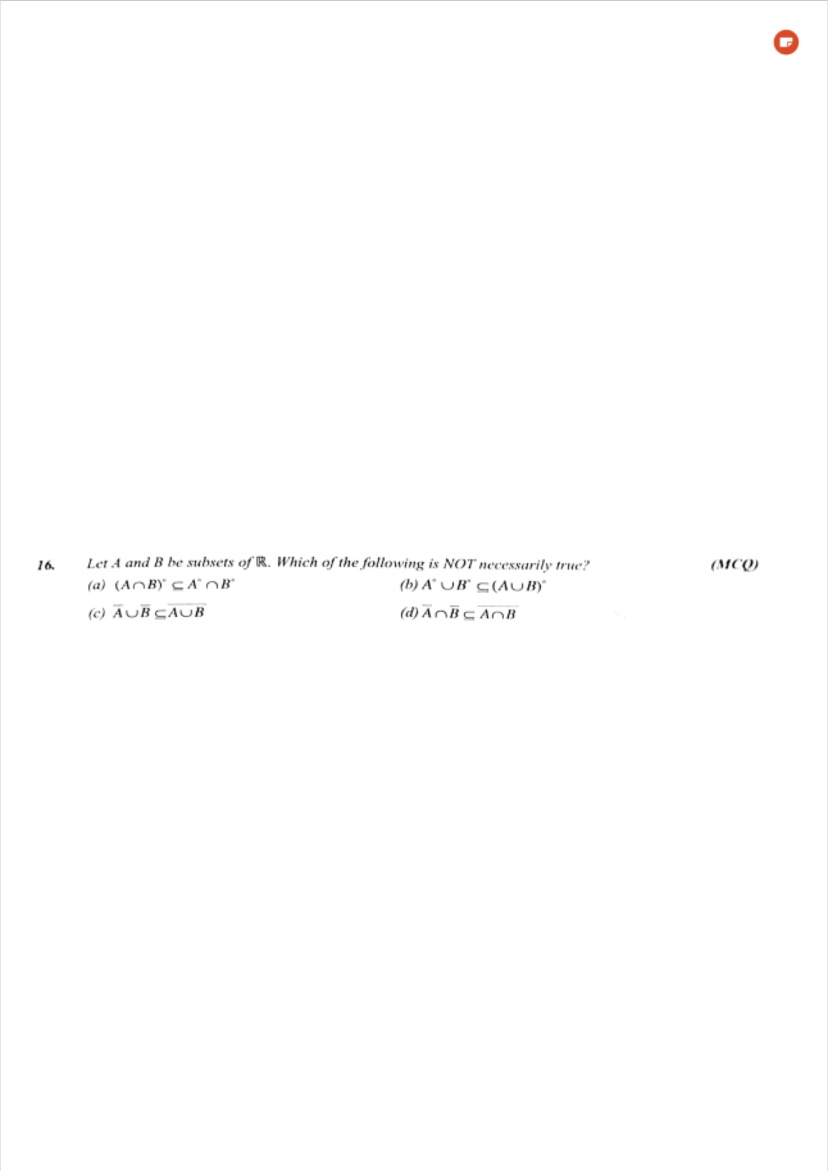
14. **(a)** *Show that the function* $f : \mathbb{R} \to \mathbb{R}$*, defined by* $f(x) = x^2$ *for* $x \in \mathbb{R}$*, is not uniformly continuous.*

(b) *For each* $n \in \mathbb{N}$*, let* $f_n : \mathbb{R} \to \mathbb{R}$ *be a uniformly continuous function. If the sequence* $\{f_n\}$ *converges uniformly on* $\mathbb{R}$ *to a function* $f : \mathbb{R} \to \mathbb{R}$*, then show that* f *is uniformly continuous.*



15. **(a)** *Let A be a nonempty bounded subset of* $\mathbb{R}$. *Show that* $\{x \in \mathbb{R} \mid x \geq a \text{ for all } a \in A\}$ *is a closed subset of* $\mathbb{R}$.

(b) *Let* $\{x_n\}$ *be a sequence in* $\mathbb{R}$ *such that* $|x_{n+1} - x_n| < \frac{1}{n^2}$ *for all* $n \in \mathbb{N}$. *Show that the sequence* $\{x_n\}$ *is convergent.*



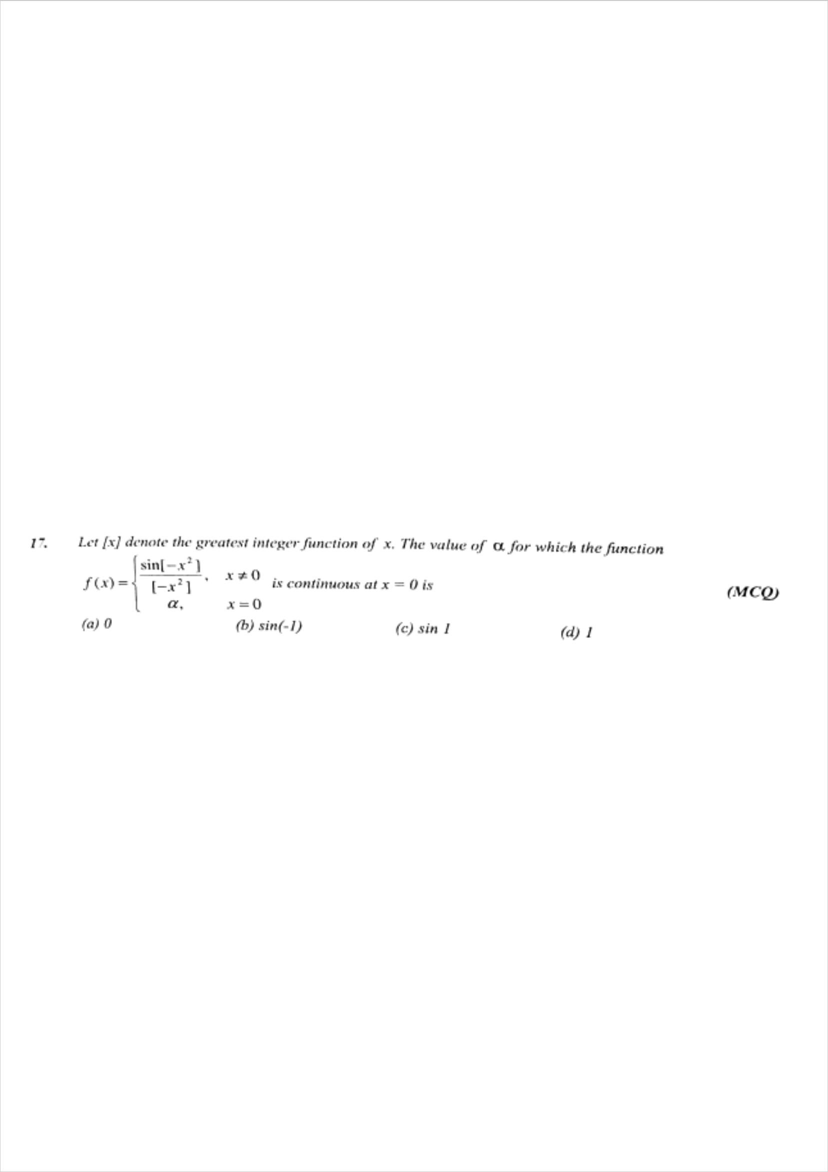
16. *Let A and B be subsets of $\mathbb{R}$. Which of the following is NOT necessarily true?* **(MCQ)**

(a) $(A \cap B)^{\circ} \subseteq A^{\circ} \cap B^{\circ}$

(b) $A^{\circ} \cup B^{\circ} \subseteq (A \cup B)^{\circ}$

(c) $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$

(d) $\overline{A} \cap \overline{B} \subseteq \overline{A \cap B}$



17. *Let [x] denote the greatest integer function of x. The value of* α *for which the function*

$$f(x)=\begin{cases}\dfrac{\sin[-x^2]}{[-x^2]}, & x\neq 0\\ \alpha, & x=0\end{cases}$$ *is continuous at* $x = 0$ *is* **(MCQ)**

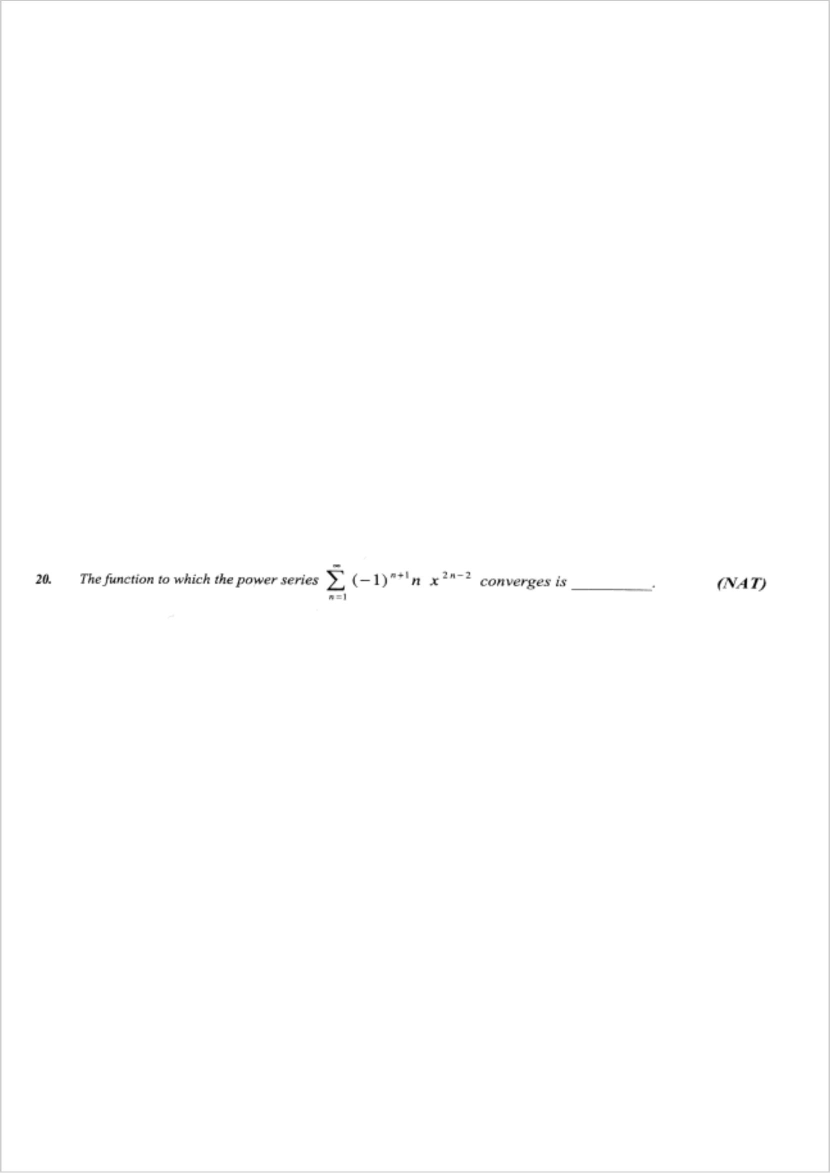
(a) 0 *(b) sin(-1)* *(c) sin 1* *(d) 1*

18. *Let the function f(x) be defined by* $f(x) = \begin{cases} e^{x}, & x \text{ is rational} \\ e^{1-x}, & x \text{ is irrational} \end{cases}$ *for x in (0, 1). Then* ***(MCQ)***

(a) f *is continuous at every point in (0, 1).* *(b)* f *is discontinuous at every point in (0,1).*
(c) f *is discontinuous only at one point in (0,1).* *(d)* f *is continuous only at one point in (0,1).*

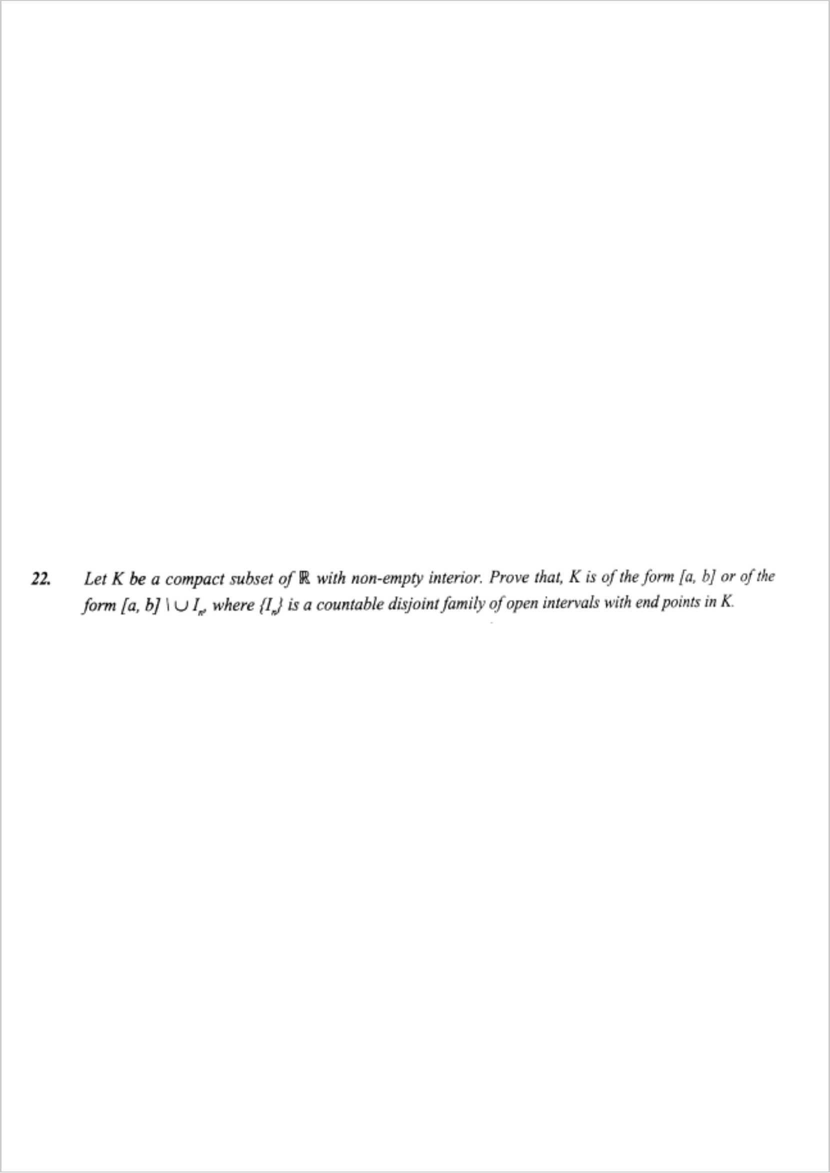
19. Let $x_n = \left(1-\frac{1}{3}\right)^2 \left(1-\frac{1}{6}\right)^2 \left(1-\frac{1}{10}\right)^2 \cdots \left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2$, $n \geq 2$. Then $\lim_{n \to \infty} x_n$ is (MCQ)

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{81}$ (d) 0



20. *The function to which the power series* $\sum_{n=1}^{\infty} (-1)^{n+1} n \; x^{2n-2}$ *converges is* __________. ***(NAT)***

21. *Let* $0 < a \le 1$, $s_1 = \dfrac{a}{2}$ *and for* $n \in \mathbb{N}$, *let* $s_{n+1} = \dfrac{1}{2}(s_n^2 + a)$. *Show that the sequence* $\{s_n\}$ *is convergent and find its limit.*



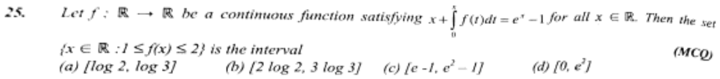
22. *Let K be a compact subset of* $\mathbb{R}$ *with non-empty interior. Prove that, K is of the form* $[a, b]$ *or of the form* $[a, b] \setminus \cup I_n$*, where* $\{I_n\}$ *is a countable disjoint family of open intervals with end points in K.*

23. *The coefficient of $(x-1)^2$ in the Taylor series expansion of $f(x) = xe^x$ $(x \in \mathbb{R})$ about the point $x = 1$ is* ***(MCQ)***

(a) $\frac{e}{2}$ (b) $2e$ (c) $\frac{3e}{2}$ (d) $3e$

3. X. The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^{2n} x^{n^2}$ is **(MCQ)**

1 (a) $\frac{1}{4}$ (b) 1 (c) 2 (d) 4



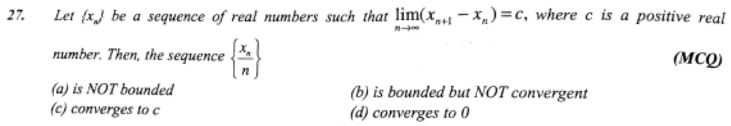
25. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function satisfying $x + \int_0^x f(t)dt = e^x - 1$ for all $x \in \mathbb{R}$. Then the set $\{x \in \mathbb{R} : 1 \leq f(x) \leq 2\}$ is the interval **(MCQ)**

(a) [log 2, log 3] (b) [2 log 2, 3 log 3] (c) $[e - 1, e^2 - 1]$ (d) $[0, e^2]$

2a Let $x_n = 2^{2n}\left(1 - \cos\left(\frac{1}{2^n}\right)\right)$ *for all* $n \in \mathbb{N}$. *Then, the sequence* $\{x_n\}$ **(MCQ)**

(a) does NOT converge *(b) converges to 0*

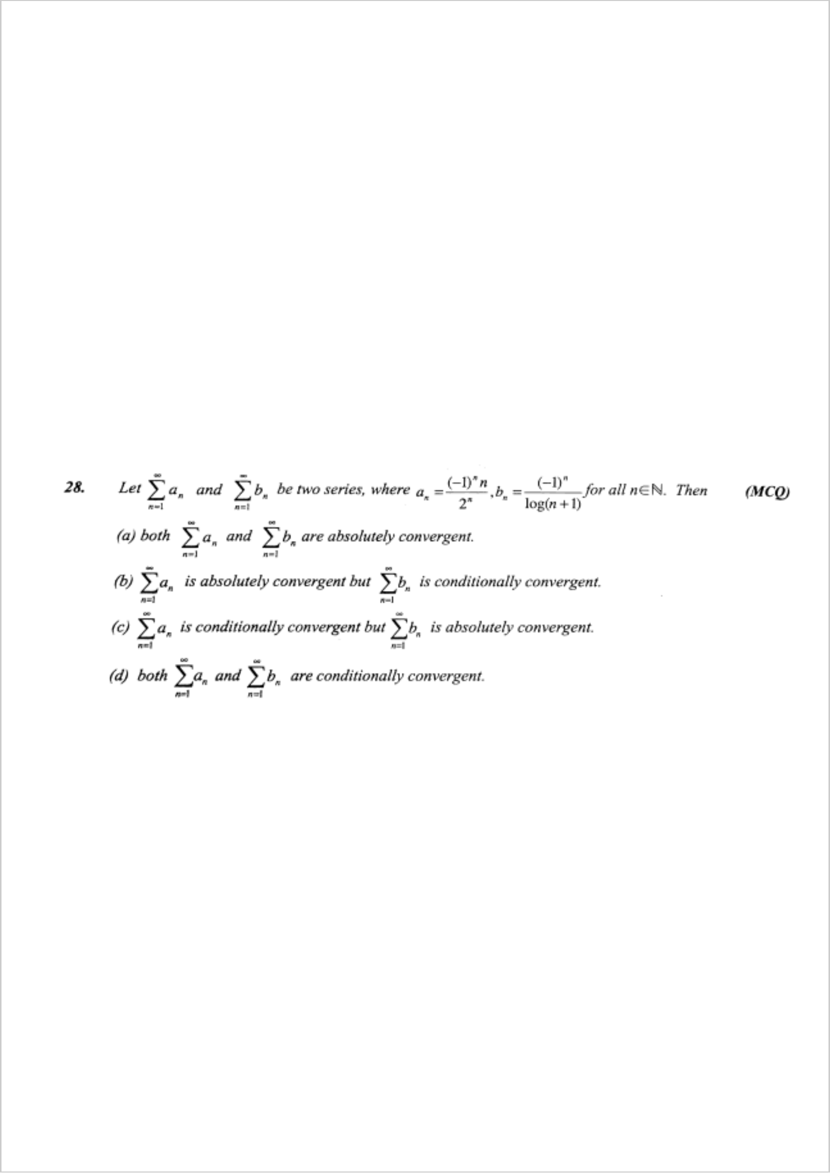
(c) converges to $\frac{1}{2}$ *(d) converges to* $\frac{1}{4}$



27. *Let $\{x_n\}$ be a sequence of real numbers such that* $\lim_{n\to\infty}(x_{n+1} - x_n) = c$, *where c is a positive real number. Then, the sequence* $\left\{\frac{x_n}{n}\right\}$ ***(MCQ)***

(a) is NOT bounded *(b) is bounded but NOT convergent*

(c) converges to c *(d) converges to 0*



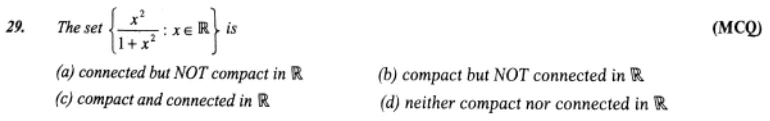
28. *Let* $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *be two series, where* $a_n = \frac{(-1)^n n}{2^n}, b_n = \frac{(-1)^n}{\log(n+1)}$ *for all* $n \in \mathbb{N}$. *Then* ***(MCQ)***

(a) both $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *are absolutely convergent.*

(b) $\sum_{n=1}^{\infty} a_n$ *is absolutely convergent but* $\sum_{n=1}^{\infty} b_n$ *is conditionally convergent.*

(c) $\sum_{n=1}^{\infty} a_n$ *is conditionally convergent but* $\sum_{n=1}^{\infty} b_n$ *is absolutely convergent.*

(d) both $\sum_{n=1}^{\infty} a_n$ *and* $\sum_{n=1}^{\infty} b_n$ *are conditionally convergent.*



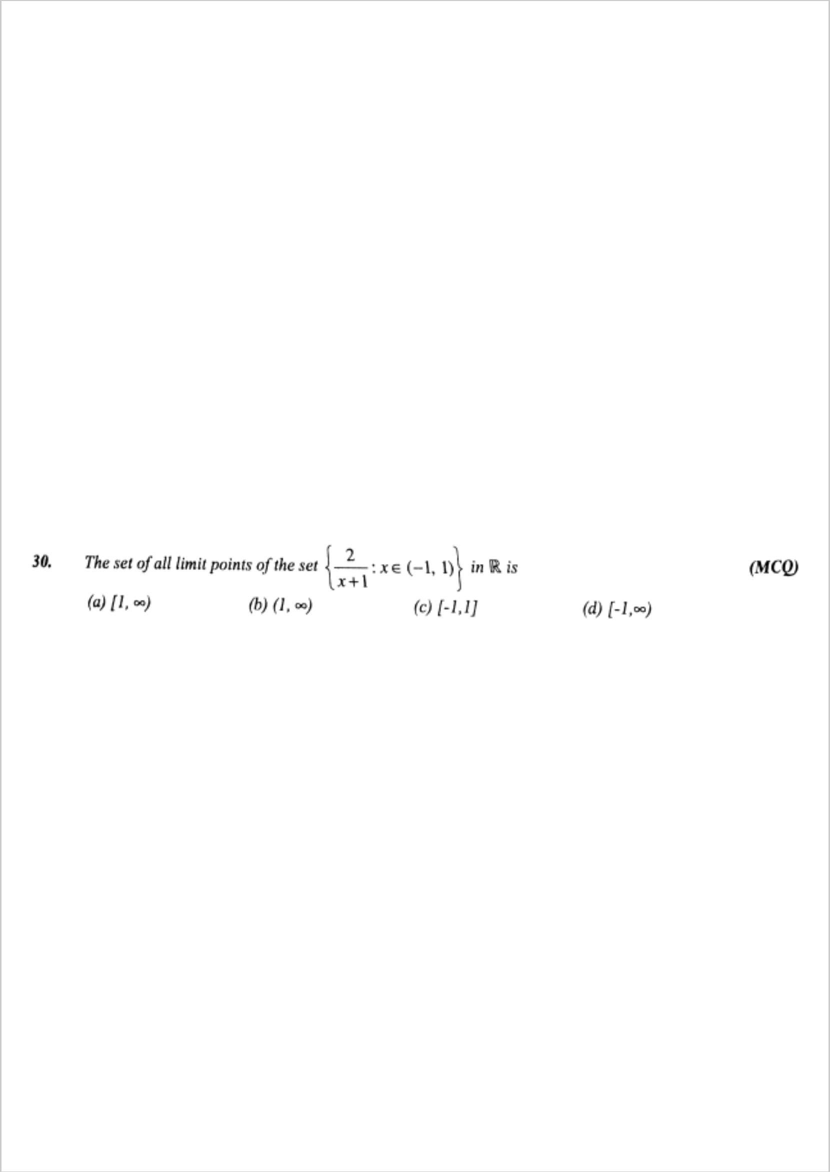
29. *The set* $\left\{ \frac{x^2}{1+x^2} : x \in \mathbb{R} \right\}$ *is* **(MCQ)**

(a) connected but NOT compact in $\mathbb{R}$

(b) compact but NOT connected in $\mathbb{R}$

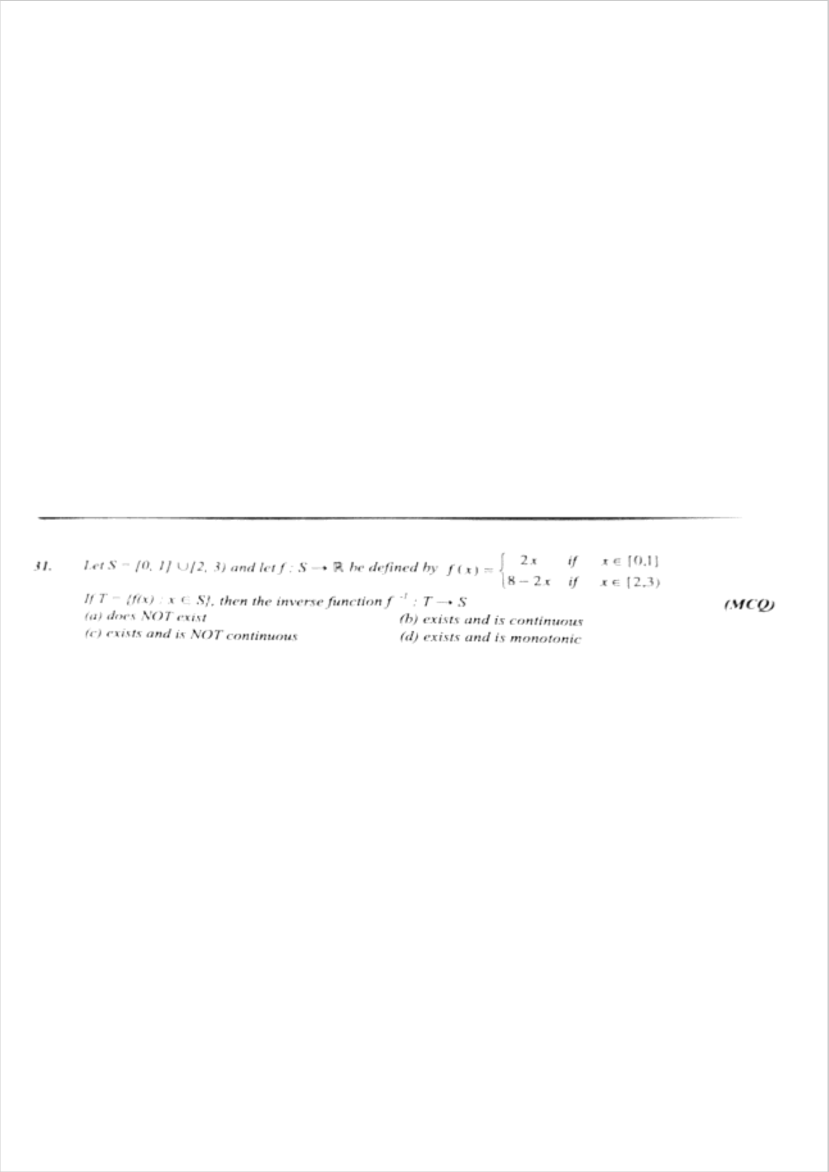
(c) compact and connected in $\mathbb{R}$

(d) neither compact nor connected in $\mathbb{R}$



30. *The set of all limit points of the set* $\left\{\frac{2}{x+1} : x \in (-1, 1)\right\}$ *in* $\mathbb{R}$ *is* **(MCQ)**

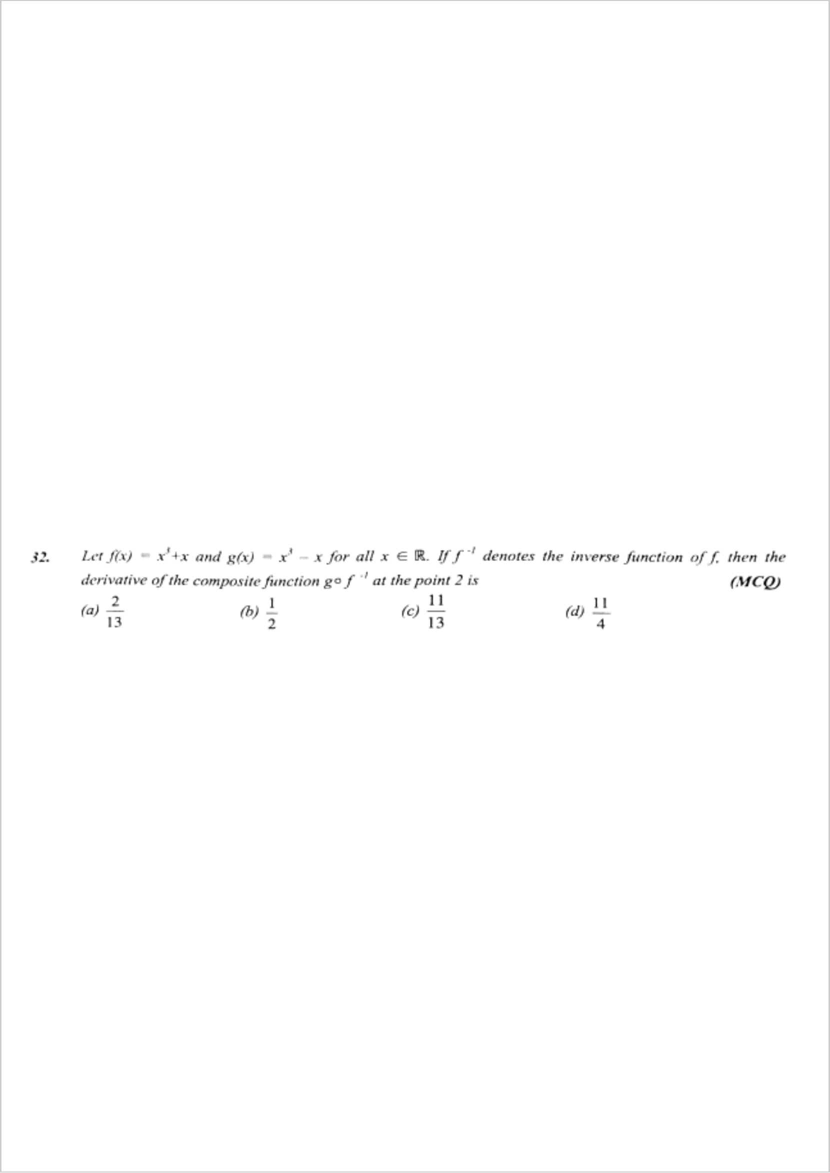
(a) $[1, \infty)$ (b) $(1, \infty)$ (c) $[-1,1]$ (d) $[-1,\infty)$



31. Let $S = [0, 1] \cup [2, 3)$ and let $f : S \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 2x & \text{if} \quad x \in [0,1] \\ 8 - 2x & \text{if} \quad x \in [2,3) \end{cases}$

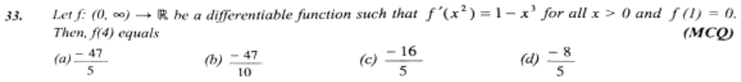
If $T = \{f(x) : x \in S\}$, then the inverse function $f^{-1} : T \to S$ **(MCQ)**

(a) does NOT exist
(b) exists and is continuous
(c) exists and is NOT continuous
(d) exists and is monotonic



32. *Let* $f(x) = x^3 + x$ *and* $g(x) = x^3 - x$ *for all* $x \in \mathbb{R}$. *If* f^{-1} *denotes the inverse function of* f, *then the derivative of the composite function* $g \circ f^{-1}$ *at the point 2 is* ***(MCQ)***

(a) $\frac{2}{13}$ (b) $\frac{1}{2}$ (c) $\frac{11}{13}$ (d) $\frac{11}{4}$



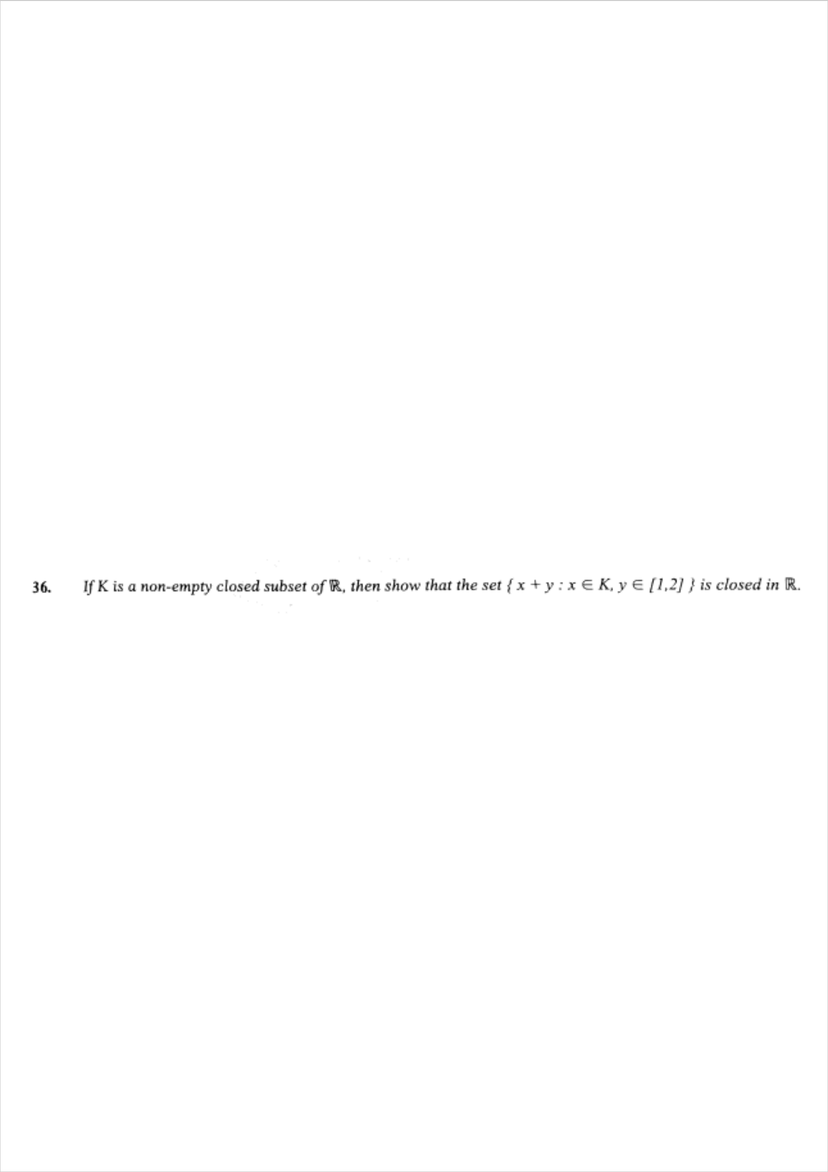
33. *Let f: (0, ∞) → ℝ be a differentiable function such that* $f'(x^2) = 1 - x^3$ *for all* $x > 0$ *and* $f(1) = 0$.
Then, f(4) equals ***(MCQ)***

(a) $\frac{-47}{5}$ (b) $\frac{-47}{10}$ (c) $\frac{-16}{5}$ (d) $\frac{-8}{5}$

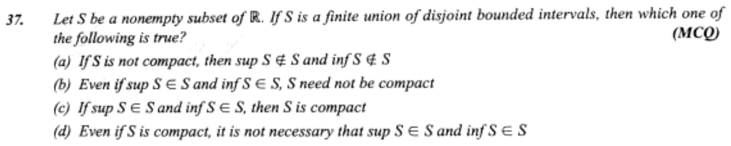
34. Let $S = \{x \in \mathbb{R} : x^6 - x^5 \leq 100\}$ and $T = \{x^2 - 2x : x \in (0, \infty)\}$. The set $S \cap T$ is **(MCQ)**

(a) closed and bounded in $\mathbb{R}$.
(b) closed but NOT bounded in $\mathbb{R}$.
(c) bounded but NOT closed in $\mathbb{R}$.
(d) neither closed nor bounded in $\mathbb{R}$.

35. Let $f : (0,1) \to \mathbb{R}$ be a differentiable function such that $|f'(x)| \leq 5$, for all $x \in (0,1)$. Show that the sequence $\left\{ f\left(\frac{1}{n+1} \right) \right\}$ converges in $\mathbb{R}$.



36. *If K is a non-empty closed subset of* $\mathbb{R}$, *then show that the set* $\{ x + y : x \in K, y \in [1,2] \}$ *is closed in* $\mathbb{R}$.



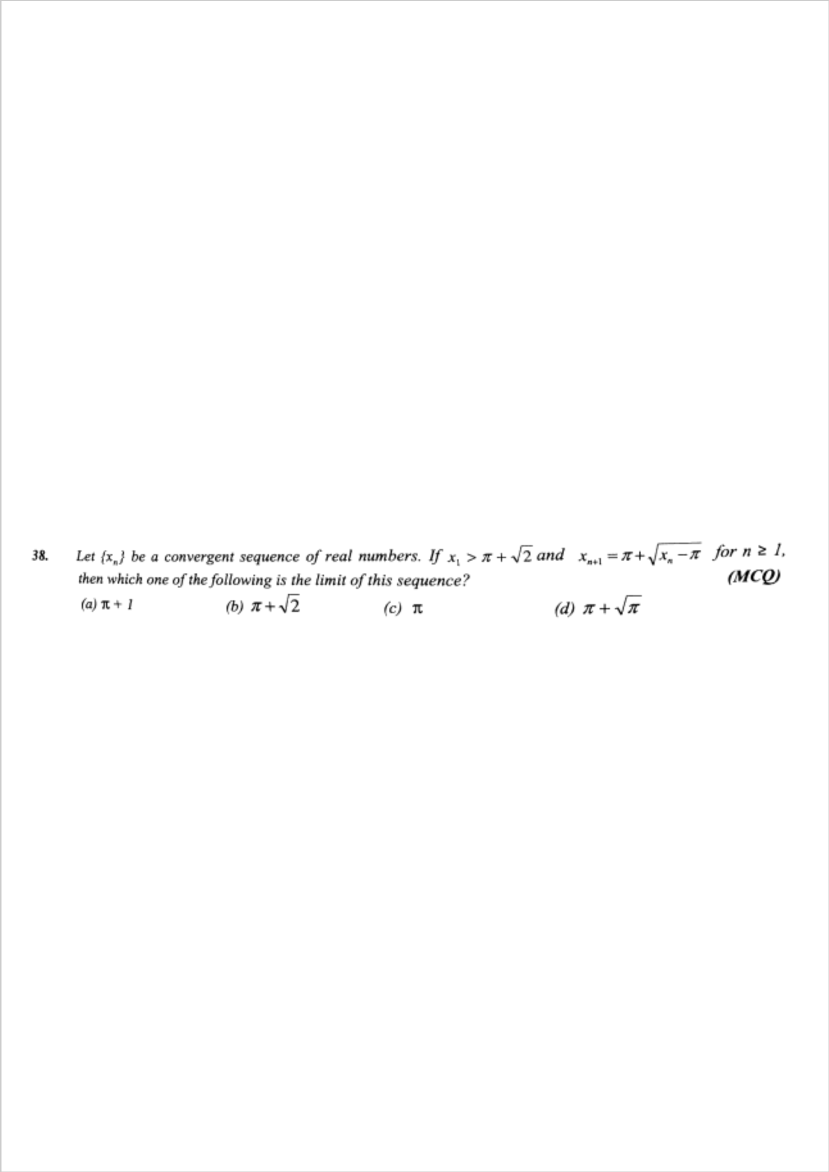
37. Let S be a nonempty subset of $\mathbb{R}$. If S is a finite union of disjoint bounded intervals, then which one of the following is true? **(MCQ)**

(a) If S is not compact, then $\sup S \notin S$ and $\inf S \notin S$

(b) Even if $\sup S \in S$ and $\inf S \in S$, S need not be compact

(c) If $\sup S \in S$ and $\inf S \in S$, then S is compact

(d) Even if S is compact, it is not necessary that $\sup S \in S$ and $\inf S \in S$



38. Let $\{x_n\}$ be a convergent sequence of real numbers. If $x_1 > \pi + \sqrt{2}$ and $x_{n+1} = \pi + \sqrt{x_n - \pi}$ for $n \geq 1$, then which one of the following is the limit of this sequence? ***(MCQ)***

(a) $\pi + 1$ (b) $\pi + \sqrt{2}$ (c) π (d) $\pi + \sqrt{\pi}$

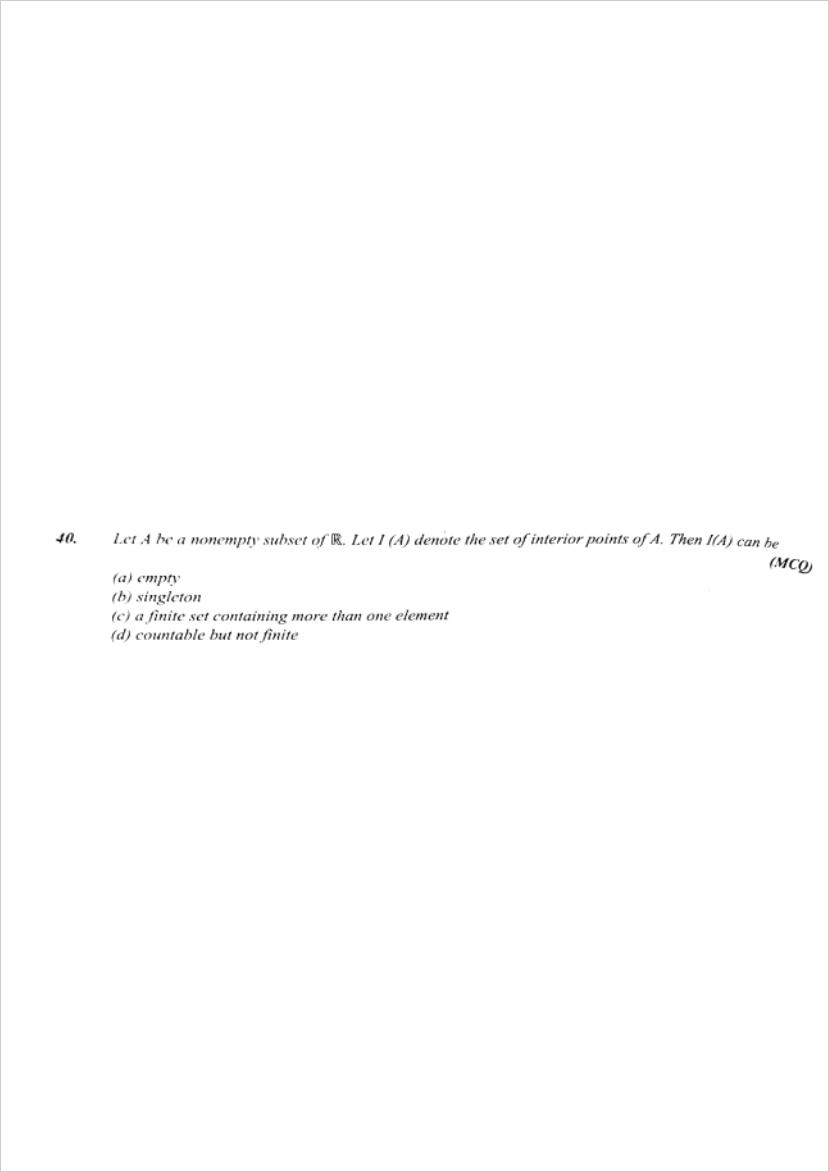
39. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If for all $x \in \mathbb{R}$, $1 < f'(x) < 2$, then which one of the following statements is true on $(0, \infty)$? **(MCQ)**

(a) f is unbounded

(b) f is increasing and bounded

(c) f has at least one zero

(d) f is periodic



40. *Let A be a nonempty subset of ℝ. Let I (A) denote the set of interior points of A. Then I(A) can be*

(MCQ)

(a) empty
(b) singleton
(c) a finite set containing more than one element
(d) countable but not finite

41. *The limit* $\lim_{x\to 0^+} \frac{1}{\sin^2 x}\int_{\frac{x}{2}}^{x} \sin^{-1} t\, dt$ *is equal to* ***(MCQ)***

(a) 0 *(b)* $\frac{1}{8}$ *(c)* $\frac{1}{4}$ *(d)* $\frac{3}{8}$

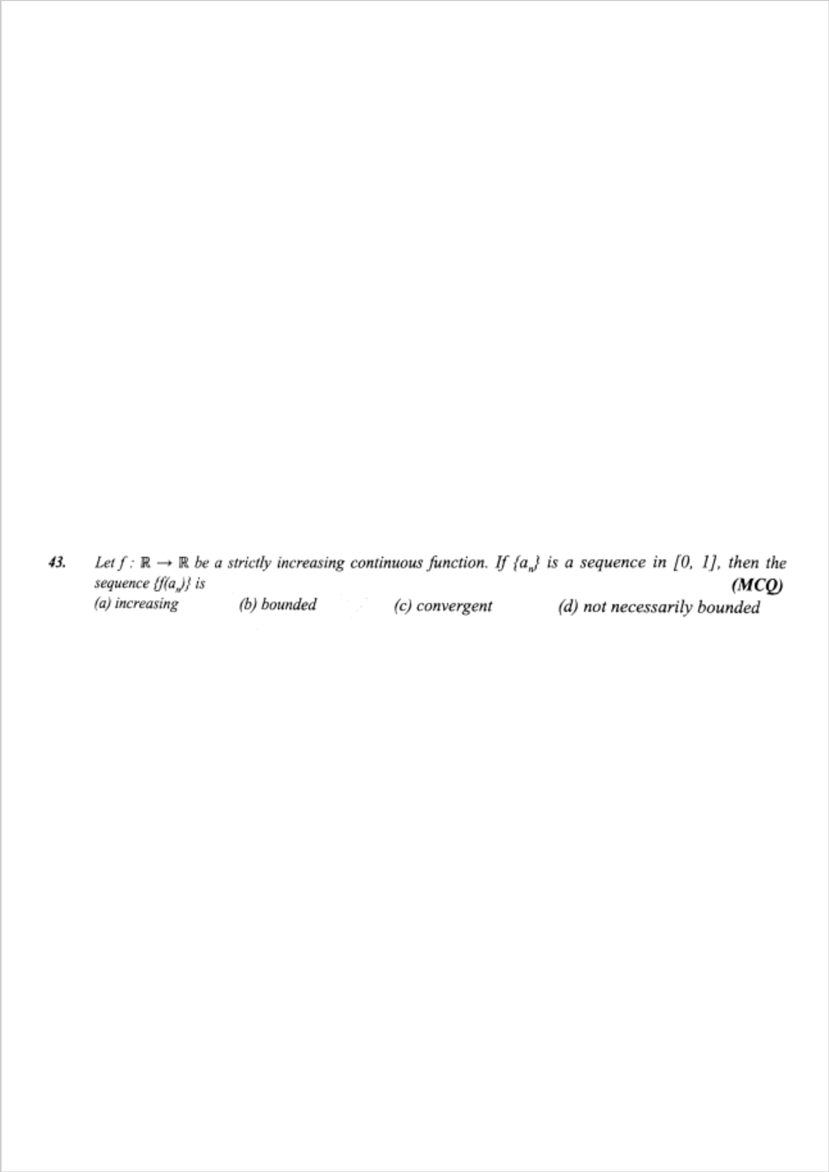
42. *Let* $S = \bigcap_{n=1}^{\infty}\left(\left[0, \frac{1}{2n+1}\right] \bigcup \left[\frac{1}{2n}, 1\right]\right)$. *Which one of the following statements is FALSE?* **(MCQ)**

(a) There exist sequences $\{a_n\}$ *and* $\{b_n\}$ *in* $[0, 1]$ *such that* $S = [0,1] \setminus \bigcup_{n=1}^{\infty} (a_n, b_n)$

(b) $[0, 1] \setminus S$ *is an open set*

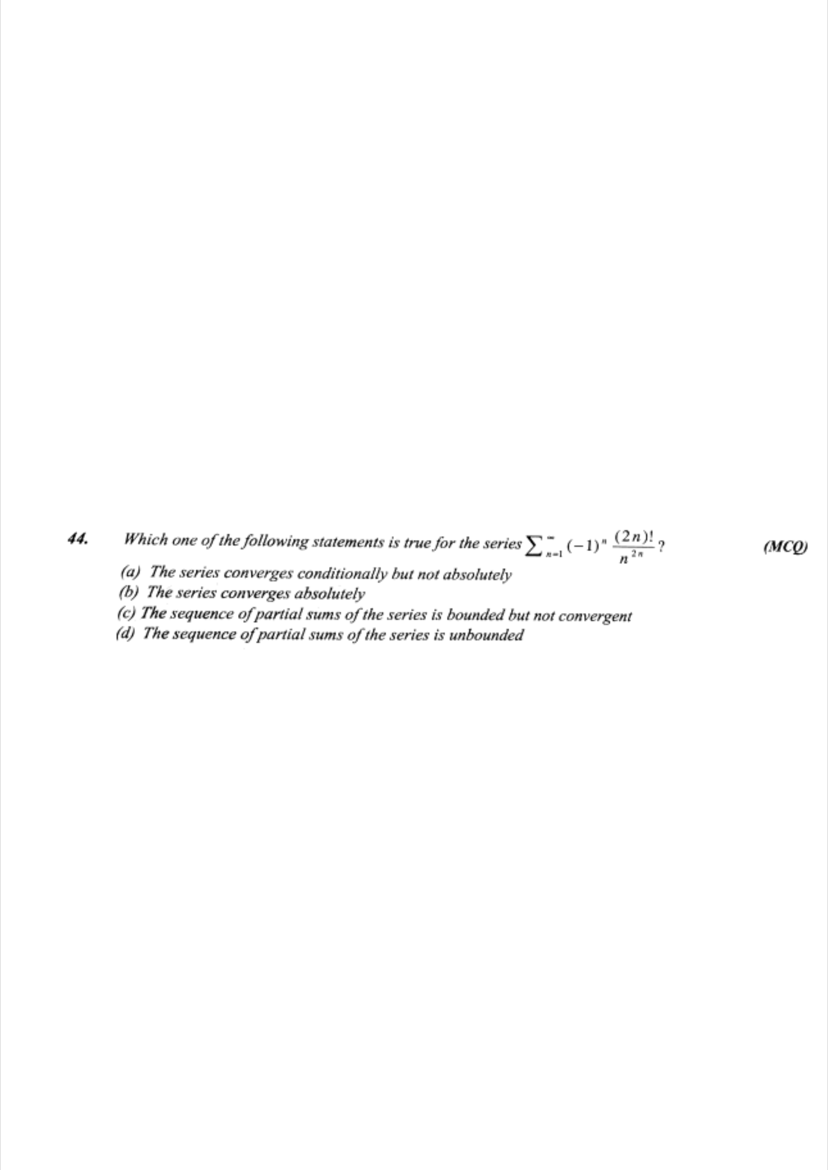
(c) If A is an infinite subset of S, then A has a limit point

(d) There exists an infinite subset of S having no limit points



43. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in $[0, 1]$, then the sequence $\{f(a_n)\}$ is **(MCQ)**

(a) increasing (b) bounded (c) convergent (d) not necessarily bounded



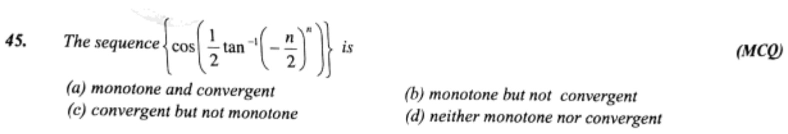
44. *Which one of the following statements is true for the series* $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{n^{2n}}$? ***(MCQ)***

(a) The series converges conditionally but not absolutely

(b) The series converges absolutely

(c) The sequence of partial sums of the series is bounded but not convergent

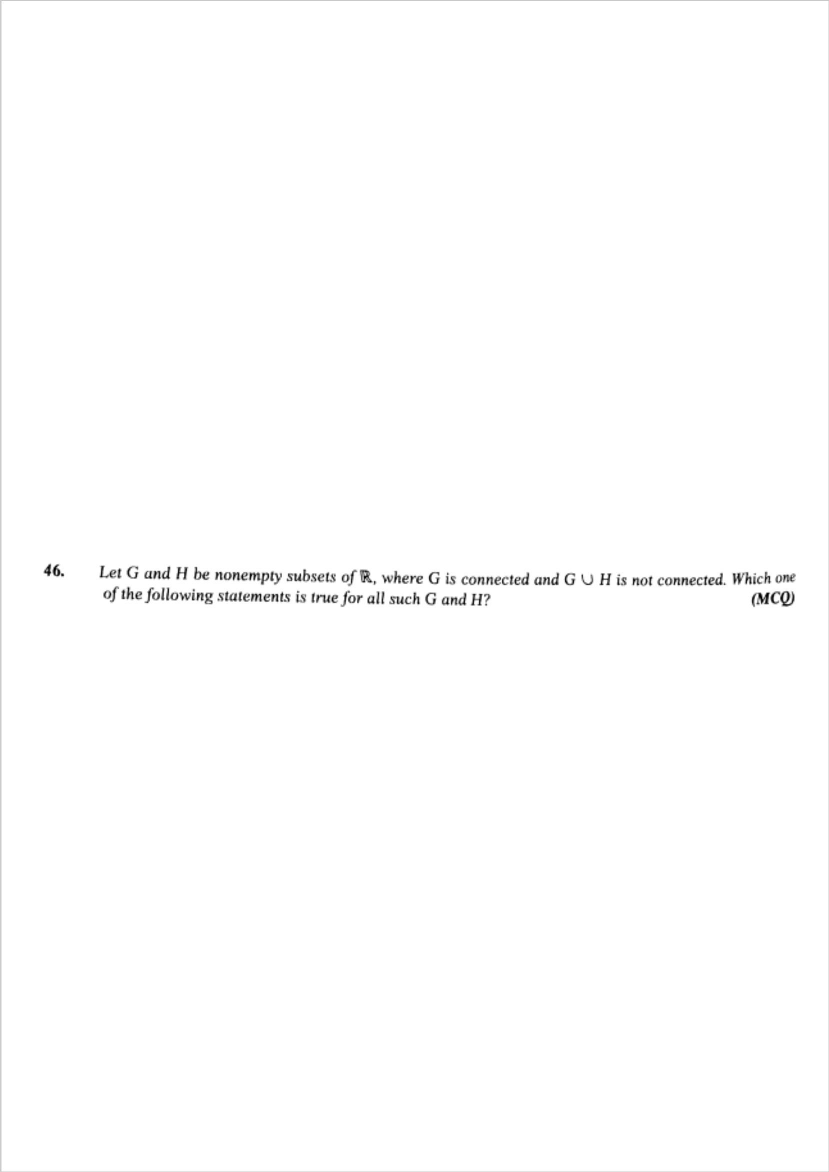
(d) The sequence of partial sums of the series is unbounded



45. *The sequence* $\left\{\cos\left(\frac{1}{2}\tan^{-1}\left(-\frac{n}{2}\right)^{n}\right)\right\}$ *is* ***(MCQ)***

(a) monotone and convergent *(b) monotone but not convergent*

(c) convergent but not monotone *(d) neither monotone nor convergent*



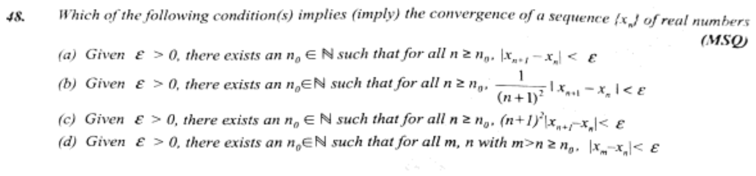
46. *Let G and H be nonempty subsets of $\mathbb{R}$, where G is connected and $G \cup H$ is not connected. Which one of the following statements is true for all such G and H?* ***(MCQ)***

(a) If $G \cap H = \phi$, then H is connected
(b) If $G \cap H = \phi$, then H is not connected
(c) If $G \cap H \neq \phi$, then H is connected
(d) If $G \cap H \neq \phi$, then H is not connected

47. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \int_{-5}^{x} (t-1)^3 \, dt$. *In which of the following interval(s),* f *takes the value 1*

(MSQ)

(a) [-6, 0]
(b) [-2, 4]
(c) [2, 8]
(d) [6, 12]



48. *Which of the following condition(s) implies (imply) the convergence of a sequence $\{x_n\}$ of real numbers*

(MSQ)

(a) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $|x_{n+1} - x_n| < \varepsilon$

(b) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $\frac{1}{(n+1)^2} | x_{n+1} - x_n | < \varepsilon$

(c) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $(n+1)^2 |x_{n+1} - x_n| < \varepsilon$

(d) Given $\varepsilon > 0$, there exists an $n_0 \in \mathbb{N}$ such that for all m, n with $m > n \geq n_0$, $|x_m - x_n| < \varepsilon$

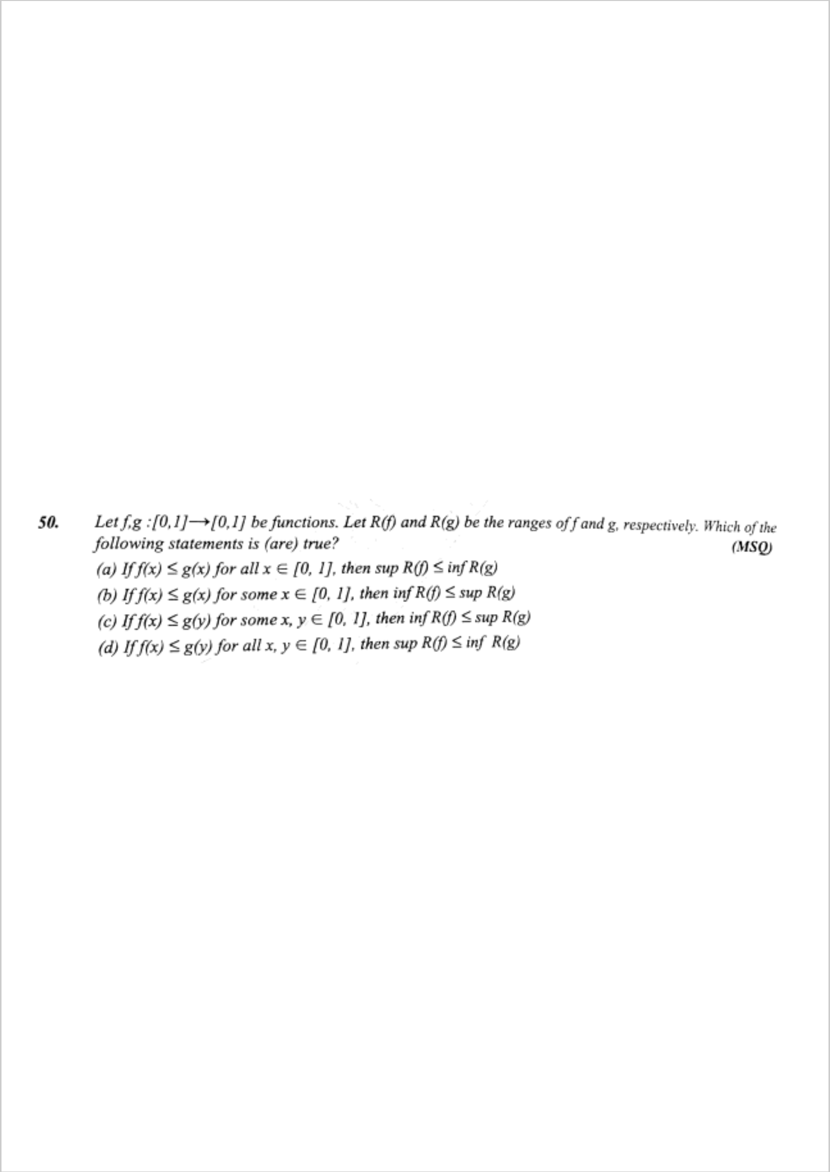
49. *Which of the following statements is (are) true on the interval* $\left(0, \frac{\pi}{2}\right)$? ***(MSQ)***

(a) $\cos x < \cos(\sin x)$

(b) $\tan x < x$

(c) $\sqrt{1+x} < 1 + \frac{x}{2} - \frac{x^2}{8}$

(d) $\frac{1-x^2}{2} < \ell n(2+x)$



50. *Let $f,g:[0,1]\longrightarrow[0,1]$ be functions. Let $R(f)$ and $R(g)$ be the ranges of f and g, respectively. Which of the following statements is (are) true?* ***(MSQ)***

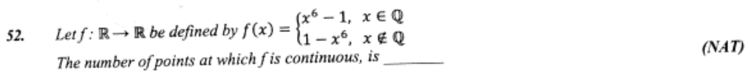
(a) If $f(x) \leq g(x)$ for all $x \in [0, 1]$, then $\sup R(f) \leq \inf R(g)$

(b) If $f(x) \leq g(x)$ for some $x \in [0, 1]$, then $\inf R(f) \leq \sup R(g)$

(c) If $f(x) \leq g(y)$ for some $x, y \in [0, 1]$, then $\inf R(f) \leq \sup R(g)$

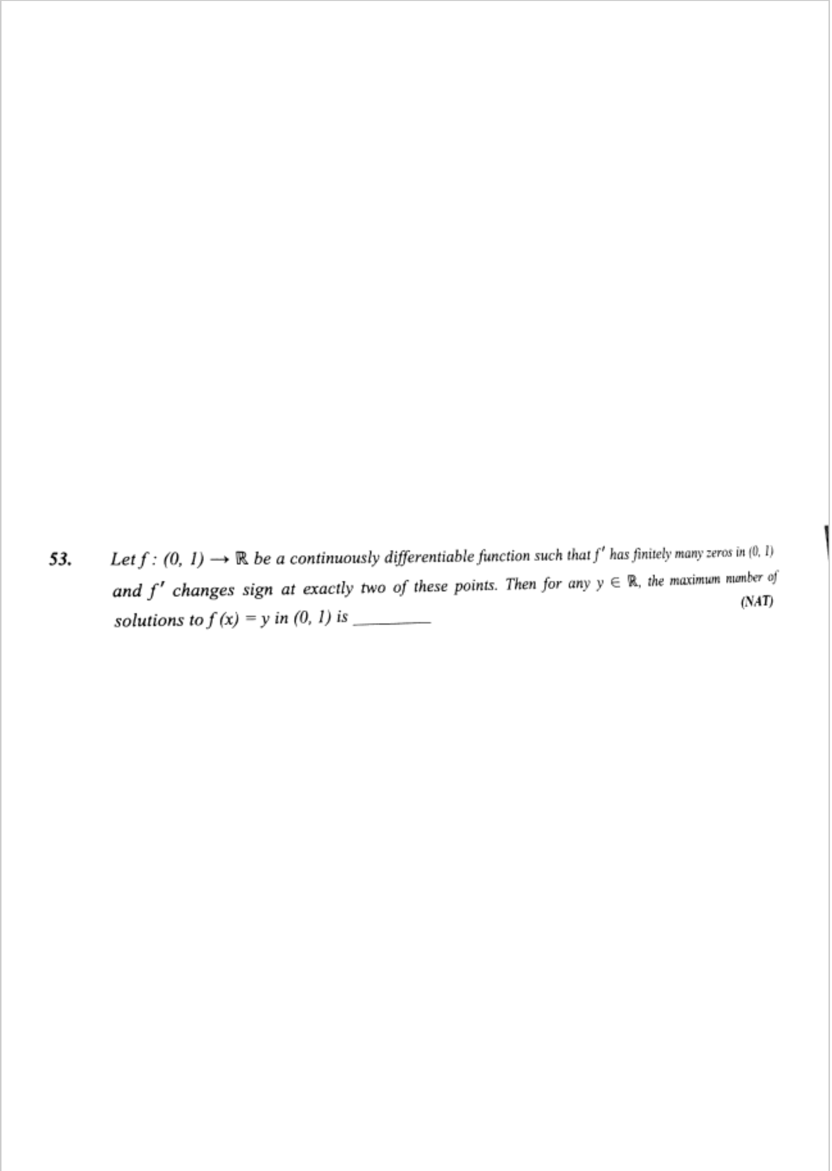
(d) If $f(x) \leq g(y)$ for all $x, y \in [0, 1]$, then $\sup R(f) \leq \inf R(g)$

51. *If the power series* $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^{2n}$ *converges for* $|x| < c$ *and diverges for* $|x| > c$*, then the value of* c*, correct upto three decimal places, is* _________ *(NAT)*



52. *Let* $f:\mathbb{R}\to\mathbb{R}$ *be defined by* $f(x)=\begin{cases}x^6-1, & x\in\mathbb{Q}\\ 1-x^6, & x\notin\mathbb{Q}\end{cases}$ ***(NAT)***

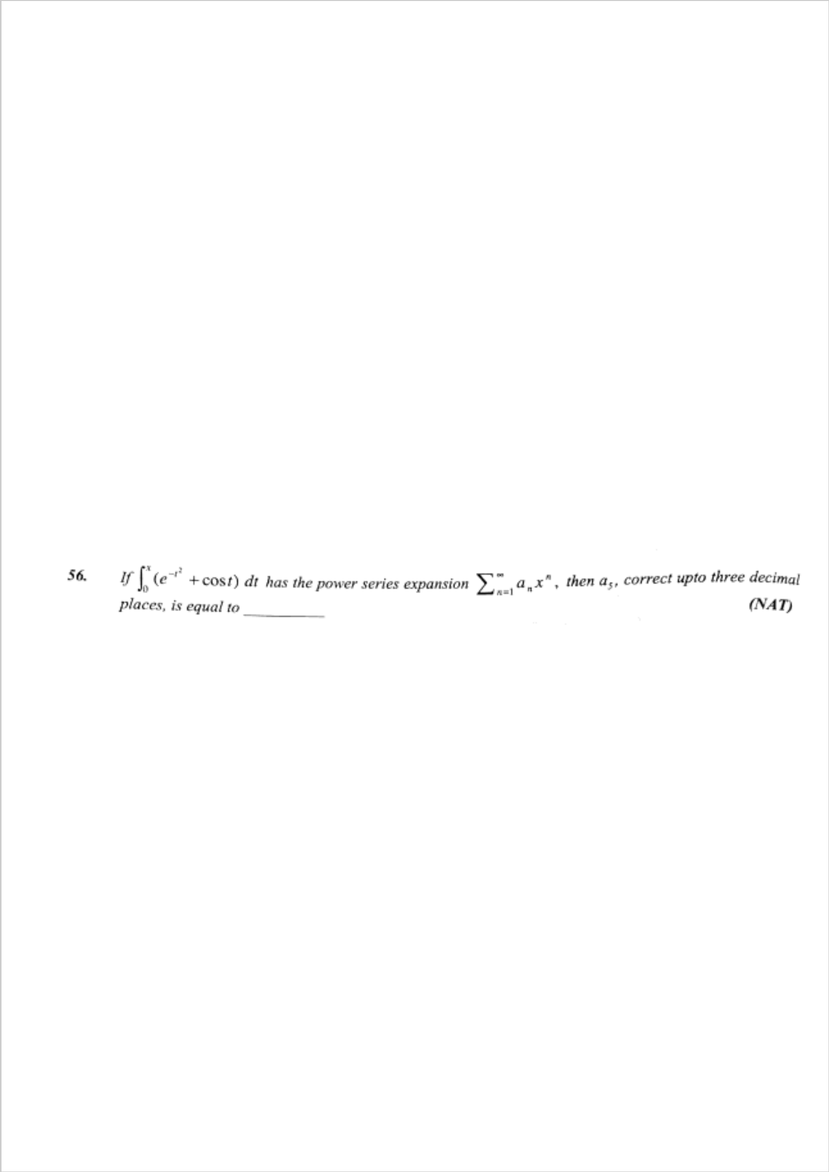
The number of points at which f is continuous, is ________



53. Let $f : (0, 1) \to \mathbb{R}$ be a continuously differentiable function such that f' has finitely many zeros in $(0, 1)$ and f' changes sign at exactly two of these points. Then for any $y \in \mathbb{R}$, the maximum number of solutions to $f(x) = y$ in $(0, 1)$ is _________ **(NAT)**

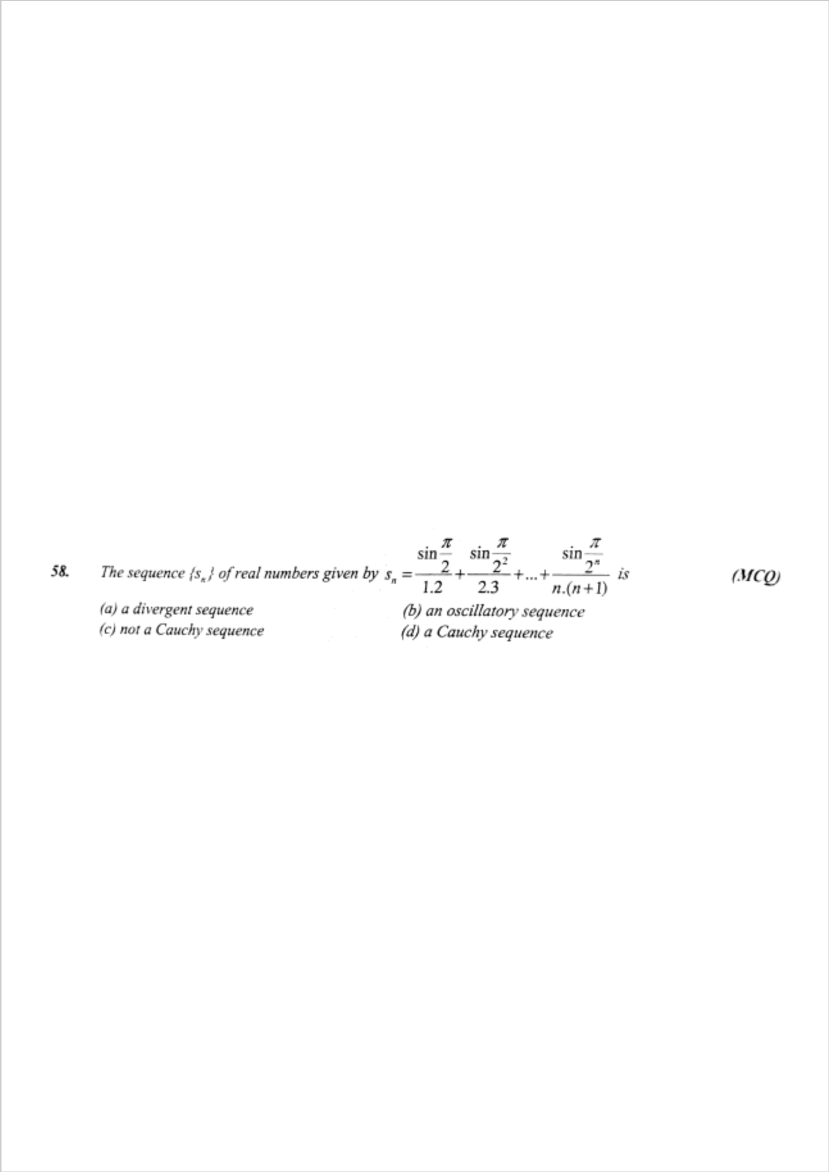
54. *The limit* $\lim_{n \to \infty} \sum_{k=2}^{n} \frac{1}{k^3 - k}$ *is equal to* ________ ***(NAT)***

55. *The coefficient of* $\left(x-\frac{\pi}{4}\right)^3$ *in the Taylor series expansion of the function* $f(x)=3\sin x\cos\left(x+\frac{\pi}{4}\right)$, $x\in\mathbb{R}$ *about the point* $\frac{\pi}{4}$, *correct upto three decimal places, is* ______ ***(NAT)***



56. *If* $\int_0^x (e^{-t^2} + \cos t)\, dt$ *has the power series expansion* $\sum_{n=1}^{\infty} a_n x^n$, *then* a_5, *correct upto three decimal places, is equal to* __________ ***(NAT)***

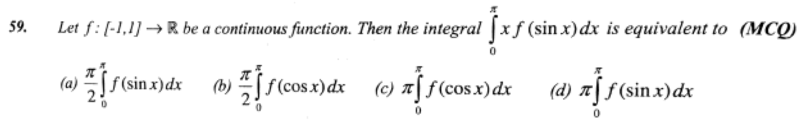
57. *The limit* $\lim\limits_{x \to 0^+} \frac{9}{x}\left(\frac{1}{\tan^{-1} x} - \frac{1}{x}\right)$ *is equal to* __________ ***(NAT)***



58. *The sequence* $\{s_n\}$ *of real numbers given by* $s_n = \frac{\sin\frac{\pi}{2}}{1.2} + \frac{\sin\frac{\pi}{2^2}}{2.3} + \dots + \frac{\sin\frac{\pi}{2^n}}{n.(n+1)}$ *is* ***(MCQ)***

(a) a divergent sequence *(b) an oscillatory sequence*

(c) not a Cauchy sequence *(d) a Cauchy sequence*

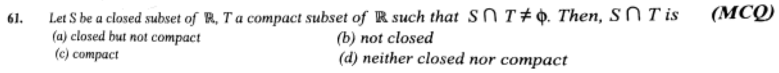


59. *Let* $f : [-1,1] \to \mathbb{R}$ *be a continuous function. Then the integral* $\int_0^{\pi} x f(\sin x)\,dx$ *is equivalent to* ***(MCQ)***

(a) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$ *(b)* $\frac{\pi}{2}\int_0^{\pi} f(\cos x)\,dx$ *(c)* $\pi\int_0^{\pi} f(\cos x)\,dx$ *(d)* $\pi\int_0^{\pi} f(\sin x)\,dx$

60. *The value of* $\lim\limits_{(x,y)\to(2,-2)} \dfrac{\sqrt{(x-y)}-2}{x-y-4}$ *is* **(MCQ)**

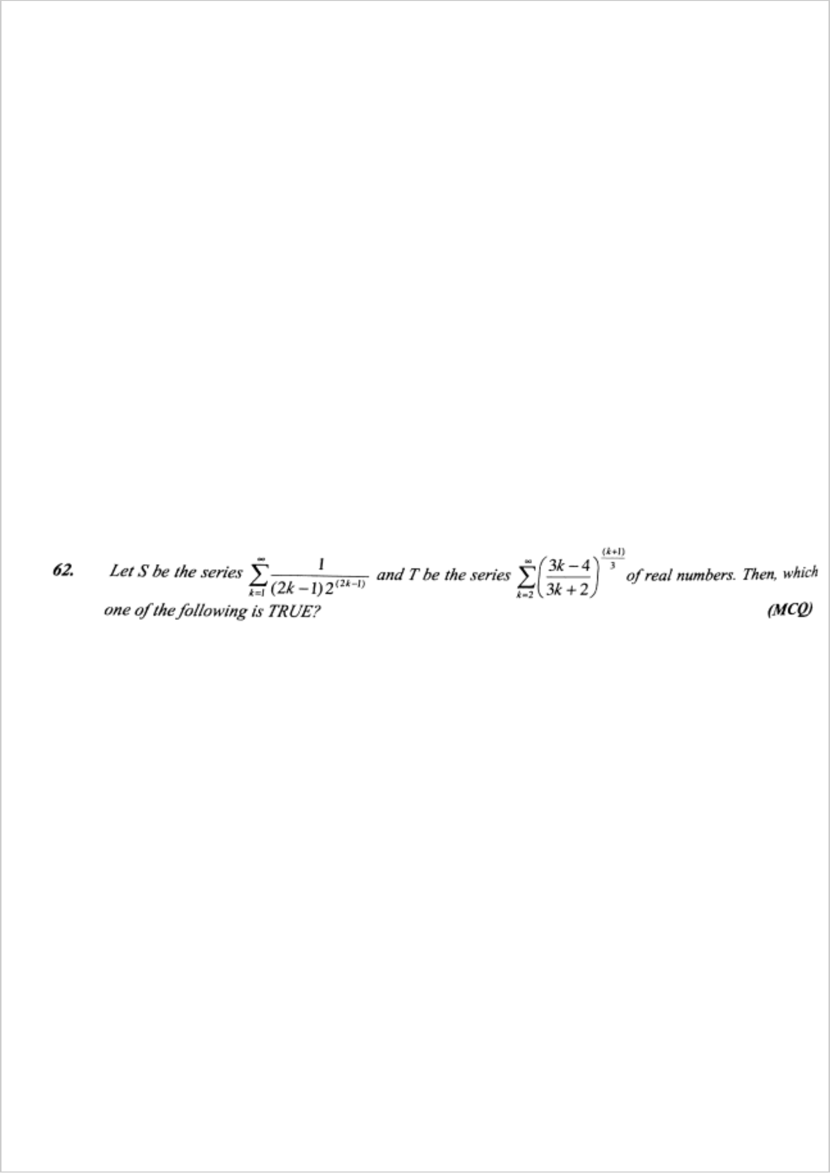
(a) 0 *(b)* $\dfrac{1}{4}$ *(c)* $\dfrac{1}{3}$ *(d)* $\dfrac{1}{2}$



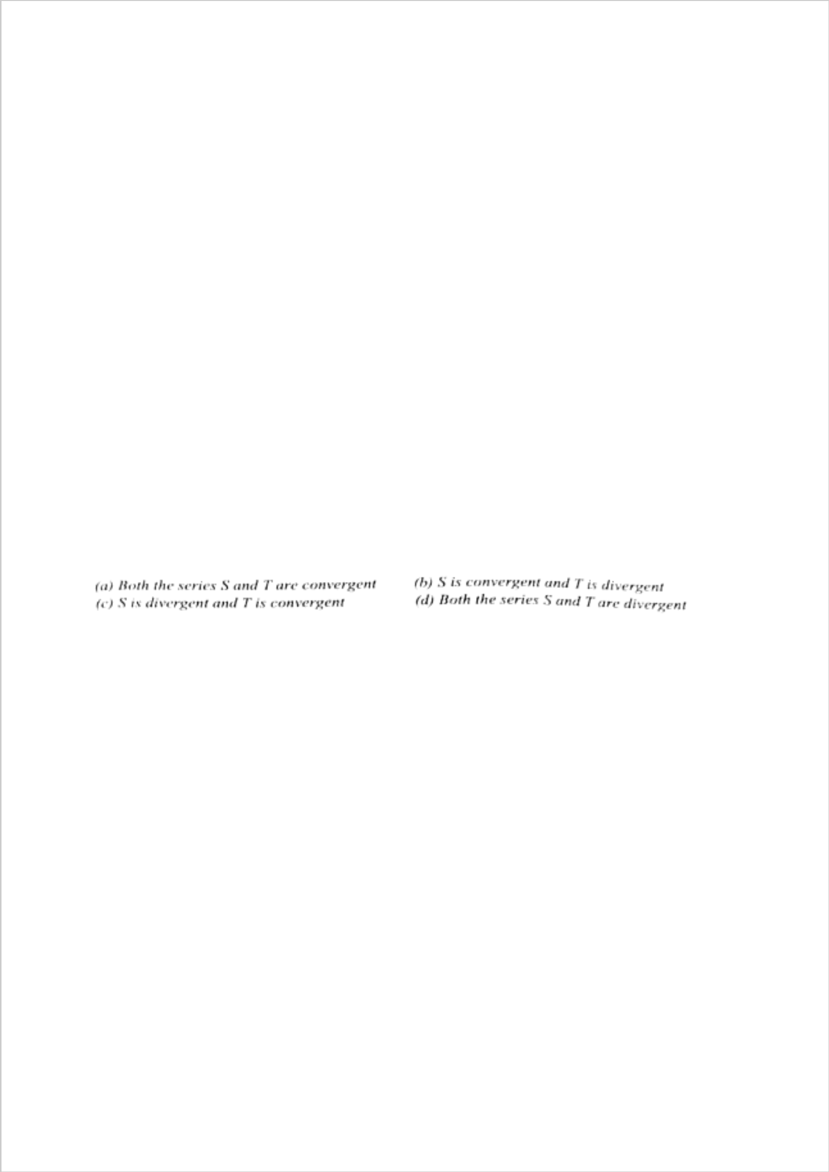
61. Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then, $S \cap T$ is **(MCQ)**

(a) closed but not compact (b) not closed

(c) compact (d) neither closed nor compact



62. *Let S be the series* $\sum_{k=1}^{\infty} \frac{1}{(2k-1)2^{(2k-1)}}$ *and T be the series* $\sum_{k=2}^{\infty} \left(\frac{3k-4}{3k+2}\right)^{\frac{(k+1)}{3}}$ *of real numbers. Then, which one of the following is TRUE?* ***(MCQ)***



(a) Both the series S and T are convergent

(b) S is convergent and T is divergent

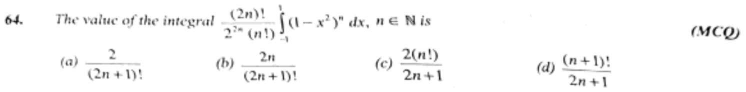
(c) S is divergent and T is convergent

(d) Both the series S and T are divergent

63. *Let* $\{a_n\}$ *be a sequence of positive real numbers satisfying* $\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}$, $n \geq 1$, $a_1 = 1$. *Then, all the terms of the sequence lie in*

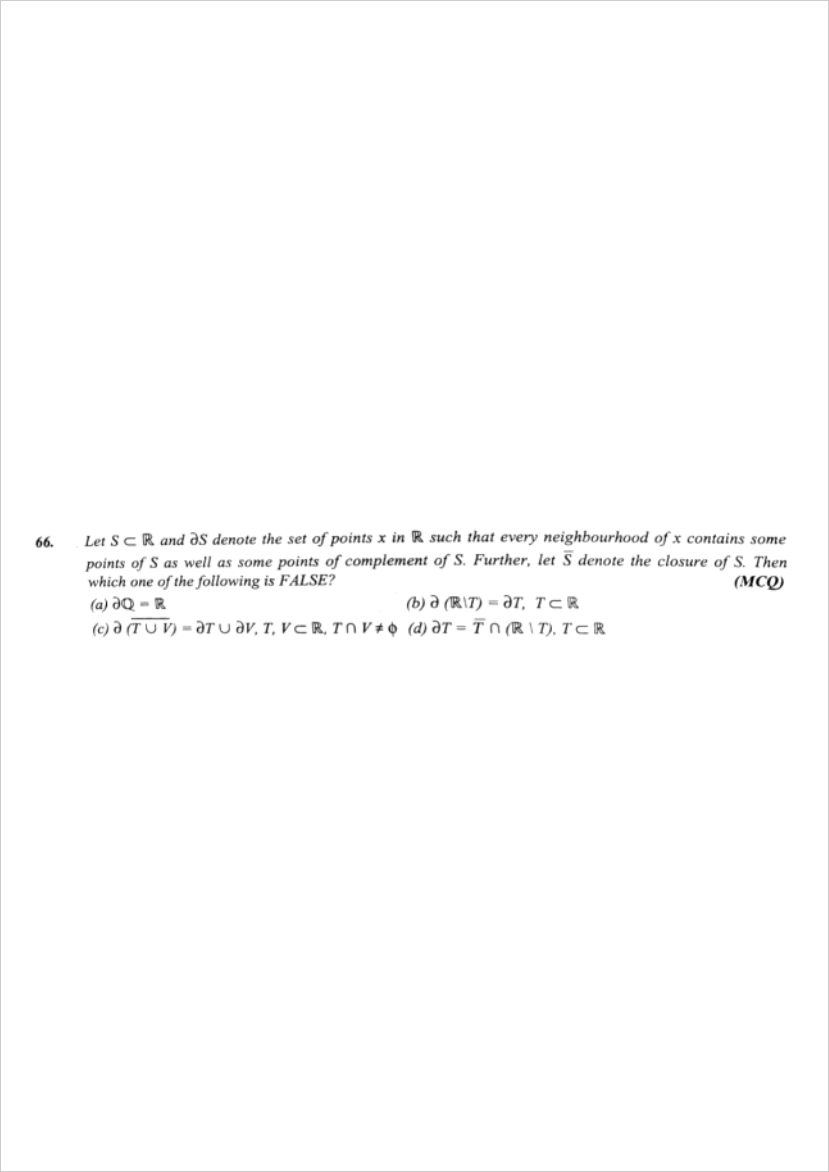
(MCQ)

(a) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (b) $[0, 1]$ (c) $[1, 2]$ (d) $[1, 3]$



64. *The value of the integral* $\frac{(2n)!}{2^{2n}\,(n!)}\int_{-1}^{1}(1-x^2)^n\,dx,\ n\in\mathbb{N}$ *is* ***(MCQ)***

(a) $\frac{2}{(2n+1)!}$ (b) $\frac{2n}{(2n+1)!}$ (c) $\frac{2(n!)}{2n+1}$ (d) $\frac{(n+1)!}{2n+1}$



66. *Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S. Further, let $\overline{S}$ denote the closure of S. Then which one of the following is FALSE?* ***(MCQ)***

(a) $\partial \mathbb{Q} = \mathbb{R}$

(b) $\partial (\mathbb{R} \setminus T) = \partial T, \ T \subset \mathbb{R}$

(c) $\partial \, (\overline{T \cup V}) = \partial T \cup \partial V, \ T, V \subset \mathbb{R}, \ T \cap V \neq \phi$

(d) $\partial T = \overline{T} \cap (\mathbb{R} \setminus T), \ T \subset \mathbb{R}$

67. *The sum of the series* $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+n-2}$ *is* **(MCQ)**

(a) $\frac{1}{3}\ell n 2 - \frac{5}{18}$ (b) $\frac{1}{3}\ell n 2 - \frac{5}{6}$ (c) $\frac{2}{3}\ell n 2 - \frac{5}{18}$ (d) $\frac{2}{3}\ell n 2 - \frac{5}{6}$

68. *Let* $f:\mathbb{R}\to\mathbb{R}$ *be defined as* $f(x)=\begin{cases} x(1+x^{\alpha}\sin(\ell n x^2)) & if\ \ x\neq 0 \\ 0 & if\ \ x=0 \end{cases}$

(MCQ)

Then, at x = 0, the function f is

(a) continuous and differentiable when $\alpha = 0$

(b) continuous and differentiable when $\alpha > 0$

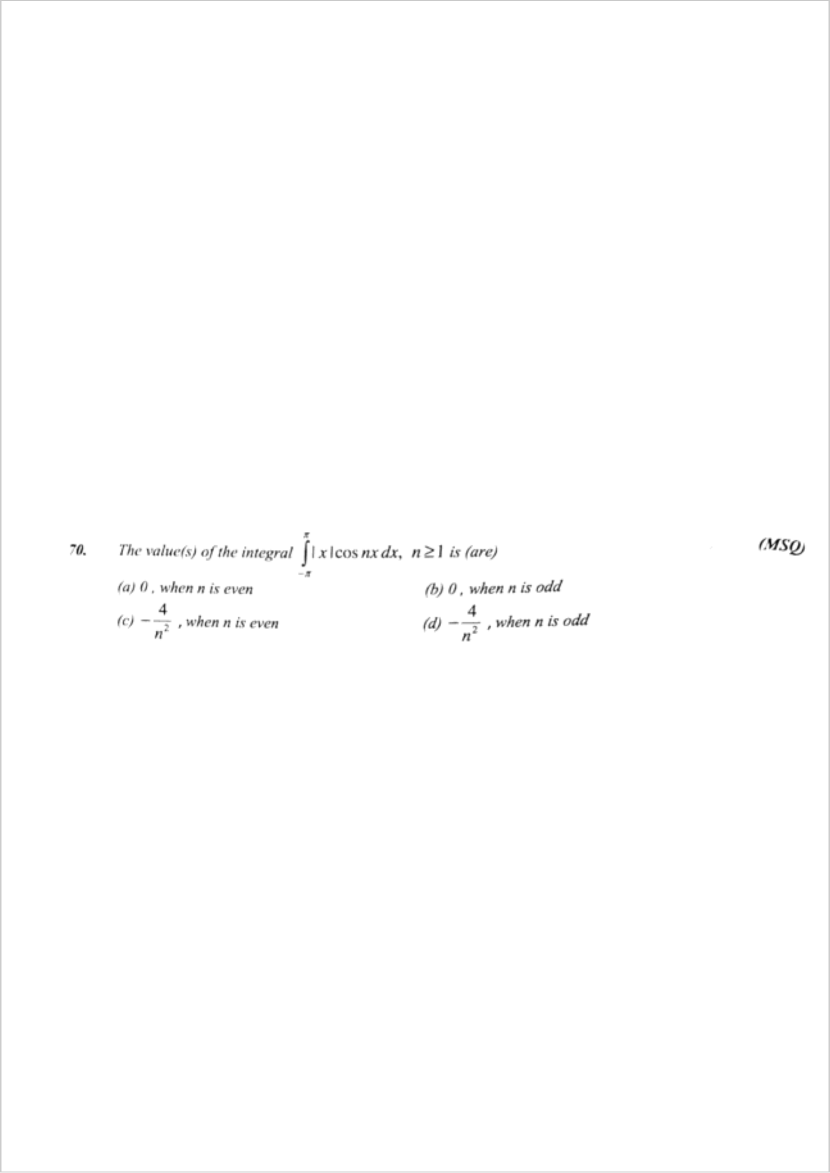
(c) continuous and differentiable when $-1<\alpha<0$

(d) continuous and differentiable when $\alpha< -1$

69. *Let* $\{s_n\}$ *be a sequence of positive real numbers satisfying* $2s_{n+1} = s_n^2 + \frac{3}{4}, n \geq 1$. *If* α *and* β *are the roots of the equation* $x^2 - 2x + \frac{3}{4} = 0$ *and* $\alpha < s_1 < \beta$*, then which of the following statement(s) is(are) TRUE?* ***(MSQ)***

(a) $\{s_n\}$ *is monotonically decreasing* (b) $\{s_n\}$ *is monotonically increasing*

(c) $\lim_{n \to \infty} s_n = \alpha$ (d) $\lim_{n \to \infty} s_n = \beta$



70. *The value(s) of the integral* $\int_{-\pi}^{\pi} |x| \cos nx \, dx,\ n \geq 1$ *is (are)* ***(MSQ)***

(a) 0 *, when* n *is even*

(b) 0 *, when* n *is odd*

(c) $-\frac{4}{n^2}$ *, when* n *is even*

(d) $-\frac{4}{n^2}$ *, when* n *is odd*

71. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \begin{cases} \dfrac{xy}{|x|} & , \text{ if } x \neq 0 \\ 0 & , \text{ elsewhere} \end{cases}$.

Then, at the point (0,0), which of the following statement(s) is (are) TRUE? **(MSQ)**

(a) f is not continuous

(b) f is continuous

(c) f is differentiable

(d) Both first order partial derivatives of f exist

72. *Which of the following statement(s) is (are) TRUE?* **(MSQ)**

(a) *There exists a connected set in* $\mathbb{R}$ *which is not compact.*

(b) *Arbitrary union of closed intervals in* $\mathbb{R}$ *need not be compact*

(c) *Arbitrary union of closed intervals in* $\mathbb{R}$ *is always closed*

(d) *Every bounded infinite subset* V *of* $\mathbb{R}$ *has a limit point in* V *itself*

73. Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is (are) TRUE?

(MSQ)

(a) The equation $P(x) = 0$ has exactly one solution in $\mathbb{R}$

(b) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$

(c) The equation $P(x) = 0$ has exactly two solutions in $\mathbb{R}$

(d) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$

74. Let $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \,\middle|\, n, m \in \mathbb{N} \right\}$. *Then, which of the following statement(s) is(are) TRUE?* ***(MSQ)***

(a) S is closed

(b) S is not open

(c) S is connected

(d) 0 is a limit point of S

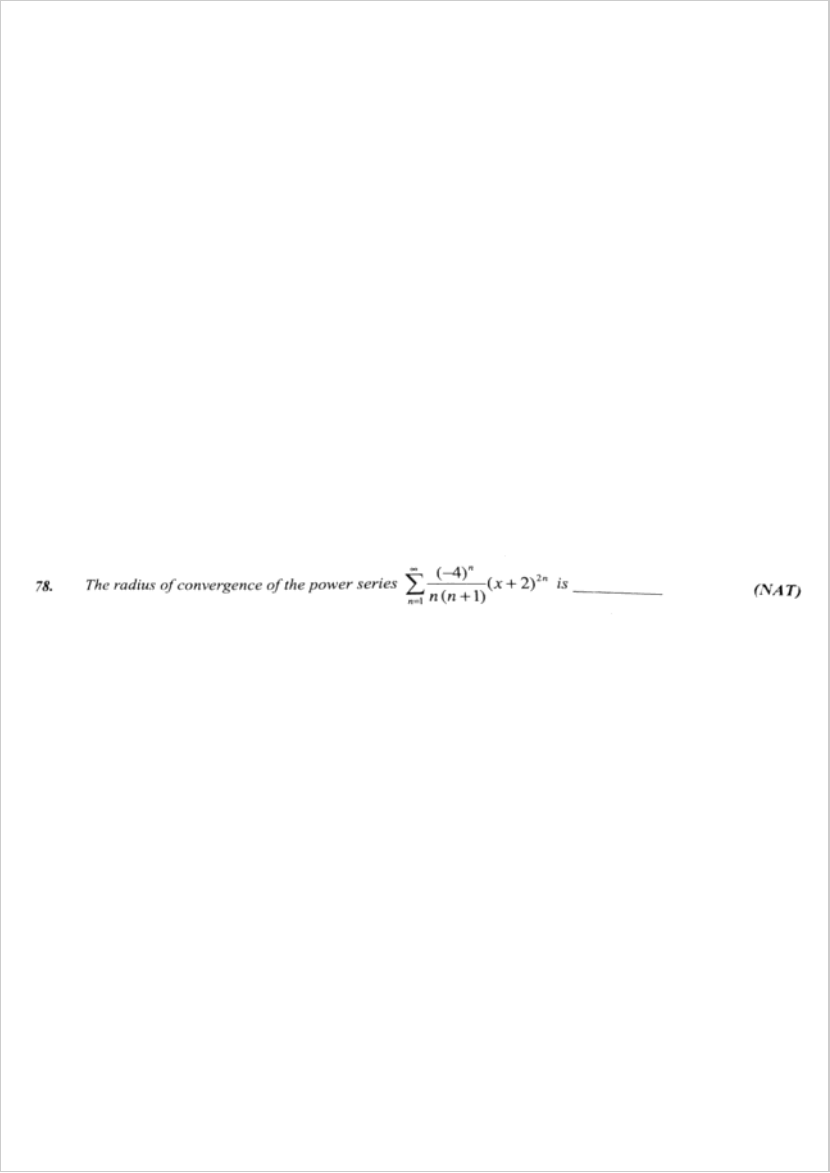
75. Let $\{s_n\}$ be a sequence of real numbers given by $s_n = 2^{(-1)^n}\left(1-\frac{1}{n}\right)\sin\frac{n\pi}{2}$, $n \in \mathbb{N}$.

Then the least upper bound of the sequence $\{s_n\}$ is ______________ **(NAT)**

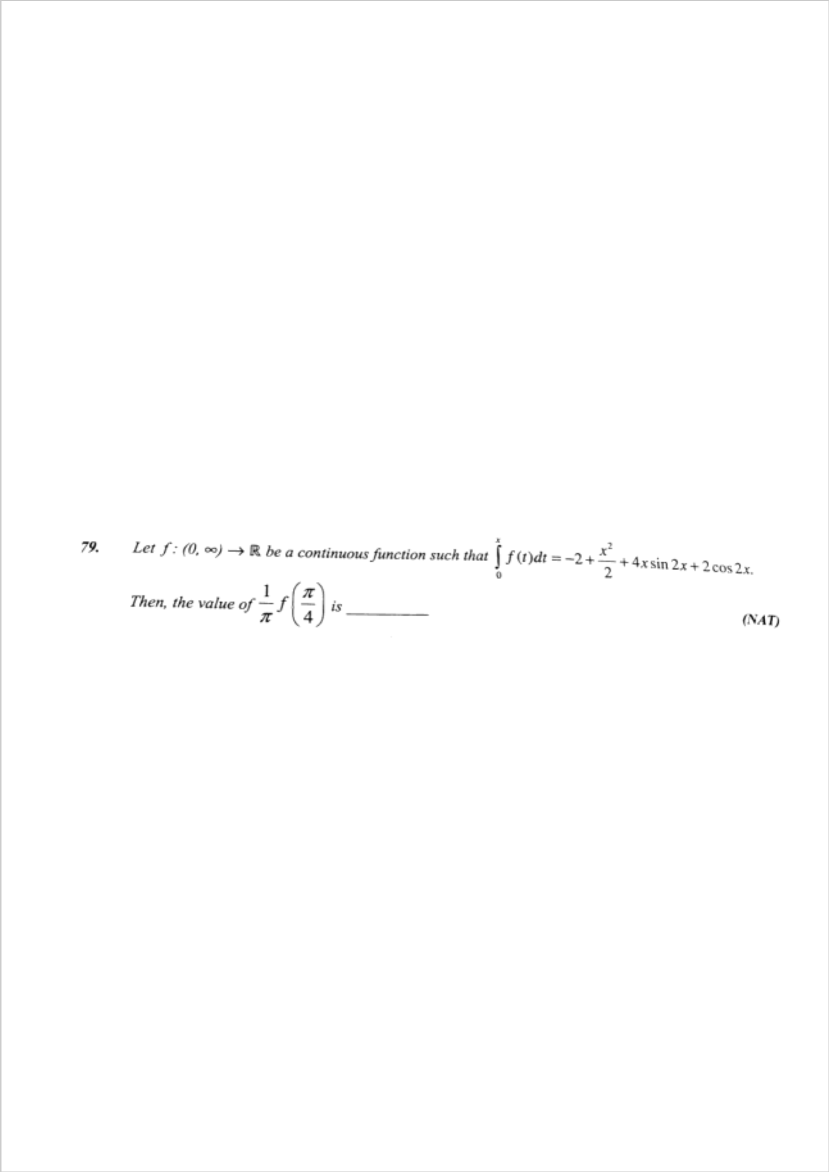
76. *Let $\{s_k\}$ be a sequence of real numbers, where $s_k = k^{\alpha k}$, $k \geq 1$, $\alpha > 0$.*
Then, $\lim\limits_{n \to \infty}(s_1 s_2 \ldots s_n)^{1/n}$ is ______________

(NAT)

77. *If* $f : (-1, \infty) \to \mathbb{R}$ *defined by* $f(x) = \dfrac{x}{1+x}$ *is expressed as* $f(x) = \dfrac{2}{3} + \dfrac{1}{9}(x-2) + \dfrac{c(x-2)^2}{(1+\xi)^3}$,
where ξ *lies between 2 and x, then, the value of c is* ________________ ***(NAT)***



78. *The radius of convergence of the power series* $\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)}(x+2)^{2n}$ *is* __________ **(NAT)**



79. *Let* $f : (0, \infty) \to \mathbb{R}$ *be a continuous function such that* $\int_0^x f(t)dt = -2 + \frac{x^2}{2} + 4x\sin 2x + 2\cos 2x.$

Then, the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ *is* __________

(NAT)

80. *The value of* $\lim_{n\to\infty}\left(8n-\frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$ *is equal to* ________ ***(NAT)***

81. *Let* $f_1(x), f_2(x), g_1(x), g_2(x)$ *be differentiable functions on* $\mathbb{R}$. *Let* $F(x)=\begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ *be the determinant of the matrix* $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$. *Then* $F'(x)$ *is equal to* ***(MCQ)***

(a) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$

(b) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(c) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(d) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

82. *Let* $f(x)=\dfrac{x+|x|(1+x)}{x}\sin\left(\dfrac{1}{x}\right)$, $x \neq 0$. *Write* $L=\lim_{x\to 0^-} f(x)$ *and* $R=\lim_{x\to 0^+} f(x)$. *Then which of the following is TRUE?* ***(MCQ)***

(a) L exists but R does not exist

(b) L does not exist but R exists

(c) Both L and R exist

(d) Neither L nor R exists

83. $\lim_{n \to \infty} \frac{\pi}{n} \sum_{k=1}^{n} \sin\left(\frac{\pi}{2} + \frac{5\pi}{2} \cdot \frac{k}{n}\right) =$ *(MCQ)*

(a) $\frac{2\pi}{5}$ (b) $\frac{5}{2}$ (c) $\frac{2}{5}$ (d) $\frac{5\pi}{2}$

84. If $\lim\limits_{T\to+\infty}\int_0^T e^{-x^2}dx = \dfrac{\sqrt{\pi}}{2}$, then $\lim\limits_{T\to+\infty}\int_0^T x^2 e^{-x^2}dx =$ (MCQ)

(a) $\dfrac{\sqrt{\pi}}{4}$ (b) $\dfrac{\sqrt{\pi}}{2}$ (c) $\sqrt{2\pi}$ (d) $2\sqrt{\pi}$

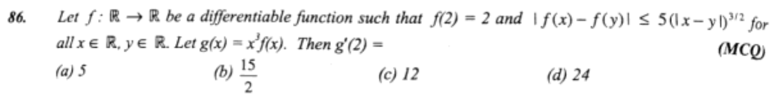
85. *Let S be an infinite subset of $\mathbb{R}$ such that $S \setminus \{\alpha\}$ is compact for some $\alpha \in S$. Then which one of the following is TRUE?* **(MCQ)**

(a) S is a connected set

(b) S contains no limit points

(c) S is a union of open intervals

(d) Every sequence in S has a subsequence converging to an element in S

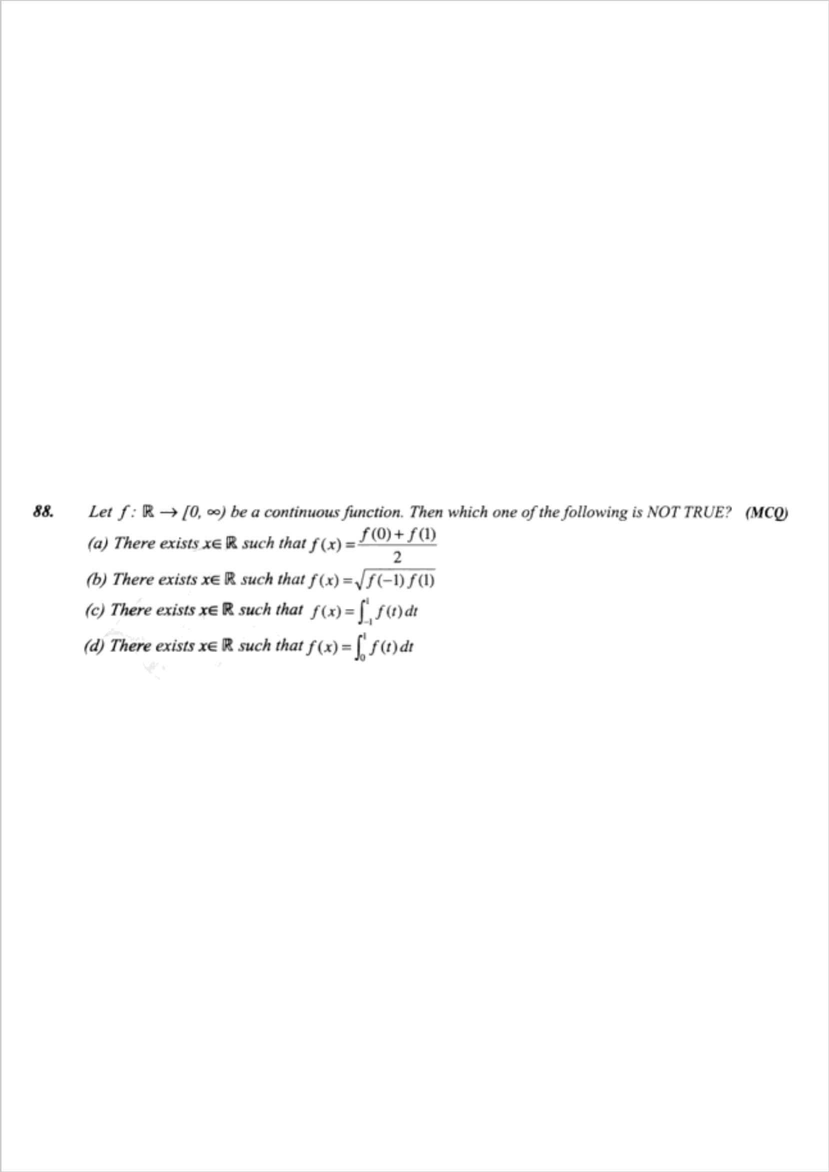


86. *Let* $f:\mathbb{R}\to\mathbb{R}$ *be a differentiable function such that* $f(2)=2$ *and* $|f(x)-f(y)| \le 5(|x-y|)^{3/2}$ *for all* $x\in\mathbb{R}$, $y\in\mathbb{R}$. *Let* $g(x)=x^3f(x)$. *Then* $g'(2)=$ ***(MCQ)***

(a) 5 *(b)* $\frac{15}{2}$ *(c)* 12 *(d)* 24

87. $\sum_{n=1}^{\infty} \tan^{-1} \frac{2}{n^2} =$ ***(MCQ)***

(a) $\frac{\pi}{4}$ *(b)* $\frac{\pi}{2}$ *(c)* $\frac{3\pi}{4}$ *(d)* π



88. *Let* $f : \mathbb{R} \rightarrow [0, \infty)$ *be a continuous function. Then which one of the following is NOT TRUE?* **(MCQ)**

(a) There exists $x \in \mathbb{R}$ *such that* $f(x) = \frac{f(0) + f(1)}{2}$

(b) There exists $x \in \mathbb{R}$ *such that* $f(x) = \sqrt{f(-1)\, f(1)}$

(c) There exists $x \in \mathbb{R}$ *such that* $f(x) = \int_{-1}^{1} f(t)\, dt$

(d) There exists $x \in \mathbb{R}$ *such that* $f(x) = \int_{0}^{1} f(t)\, dt$

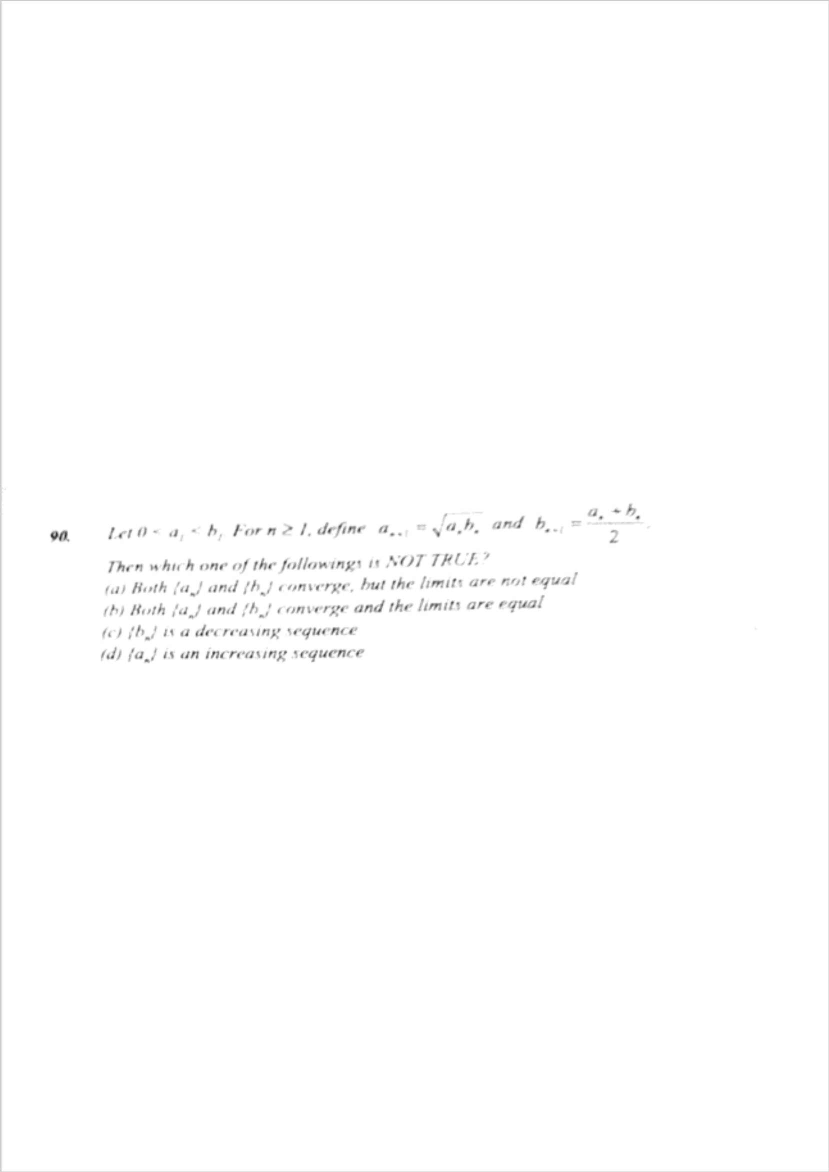
89. Let $f(x, y) = \dfrac{x^2 y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Then

(a) $\dfrac{\partial f}{\partial x}$ and f are bounded

(b) $\dfrac{\partial f}{\partial x}$ is bounded and f is unbounded

(c) $\dfrac{\partial f}{\partial x}$ is unbounded and f is bounded

(d) $\dfrac{\partial f}{\partial x}$ and f are unbounded



90. Let $0 < a_1 < b_1$. For $n \geq 1$, define $a_{n+1} = \sqrt{a_n b_n}$ and $b_{n+1} = \frac{a_n + b_n}{2}$.

Then which one of the followings is NOT TRUE?

(a) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal

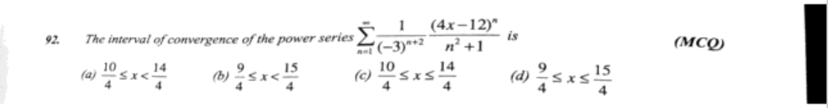
(b) Both $\{a_n\}$ and $\{b_n\}$ converge and the limits are equal

(c) $\{b_n\}$ is a decreasing sequence

(d) $\{a_n\}$ is an increasing sequence

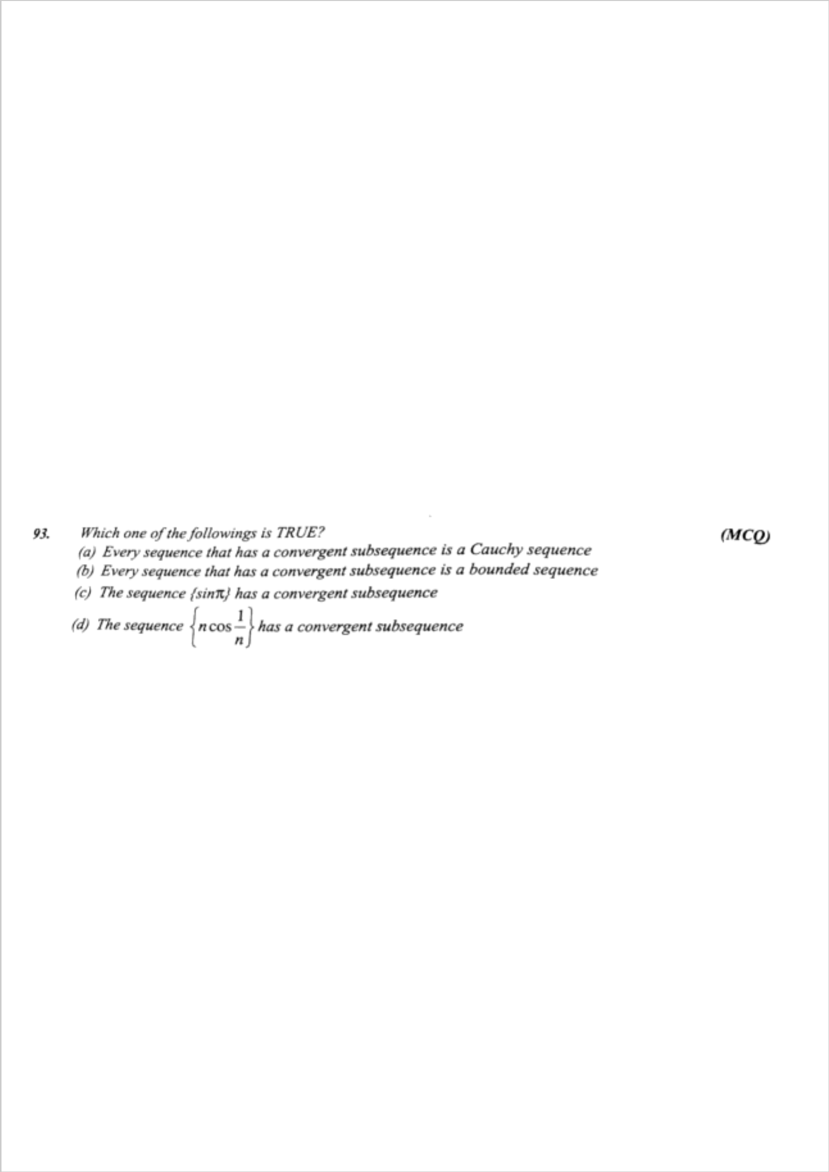
91. $\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{9}}+\ldots+\frac{1}{\sqrt{3n}+\sqrt{3n+3}}\right)=$ ***(MCQ)***

(a) $1+\sqrt{3}$ *(b)* $\sqrt{3}$ *(c)* $\frac{1}{\sqrt{3}}$ *(d)* $\frac{1}{1+\sqrt{3}}$



92. *The interval of convergence of the power series* $\sum_{n=1}^{\infty}\frac{1}{(-3)^{n+2}}\frac{(4x-12)^n}{n^2+1}$ *is* **(MCQ)**

(a) $\frac{10}{4}\le x<\frac{14}{4}$ (b) $\frac{9}{4}\le x<\frac{15}{4}$ (c) $\frac{10}{4}\le x\le\frac{14}{4}$ (d) $\frac{9}{4}\le x\le\frac{15}{4}$



93. *Which one of the followings is TRUE?* ***(MCQ)***

(a) Every sequence that has a convergent subsequence is a Cauchy sequence
(b) Every sequence that has a convergent subsequence is a bounded sequence
(c) The sequence $\{\sin\pi\}$ *has a convergent subsequence*
(d) The sequence $\left\{n\cos\frac{1}{n}\right\}$ *has a convergent subsequence*

94. Let $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Then $\lim_{n \to \infty} M^n x$ **(MCQ)**

(a) does not exist (b) is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (c) is $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (d) is $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

95. Let S be the set of all rational numbers in $(0, 1)$. Then which of the following statements is/are TRUE?

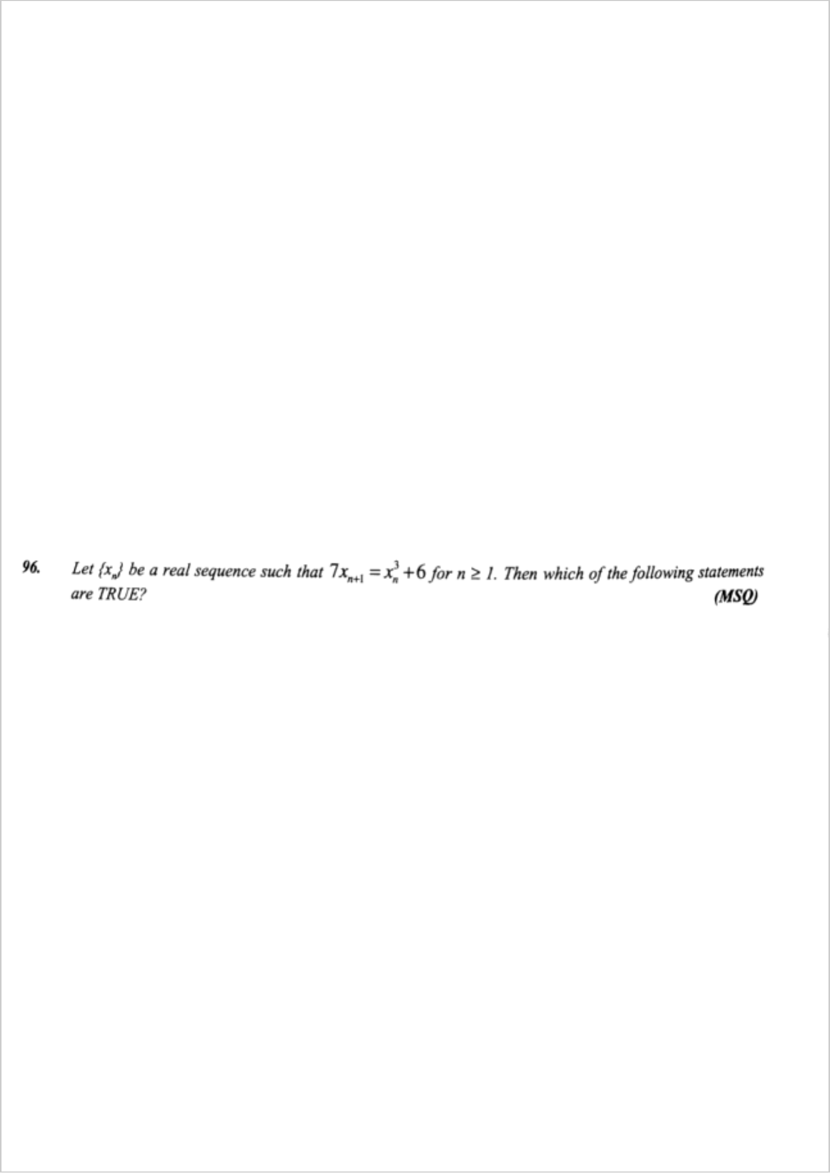
(MSQ)

(a) S is a closed subset of $\mathbb{R}$

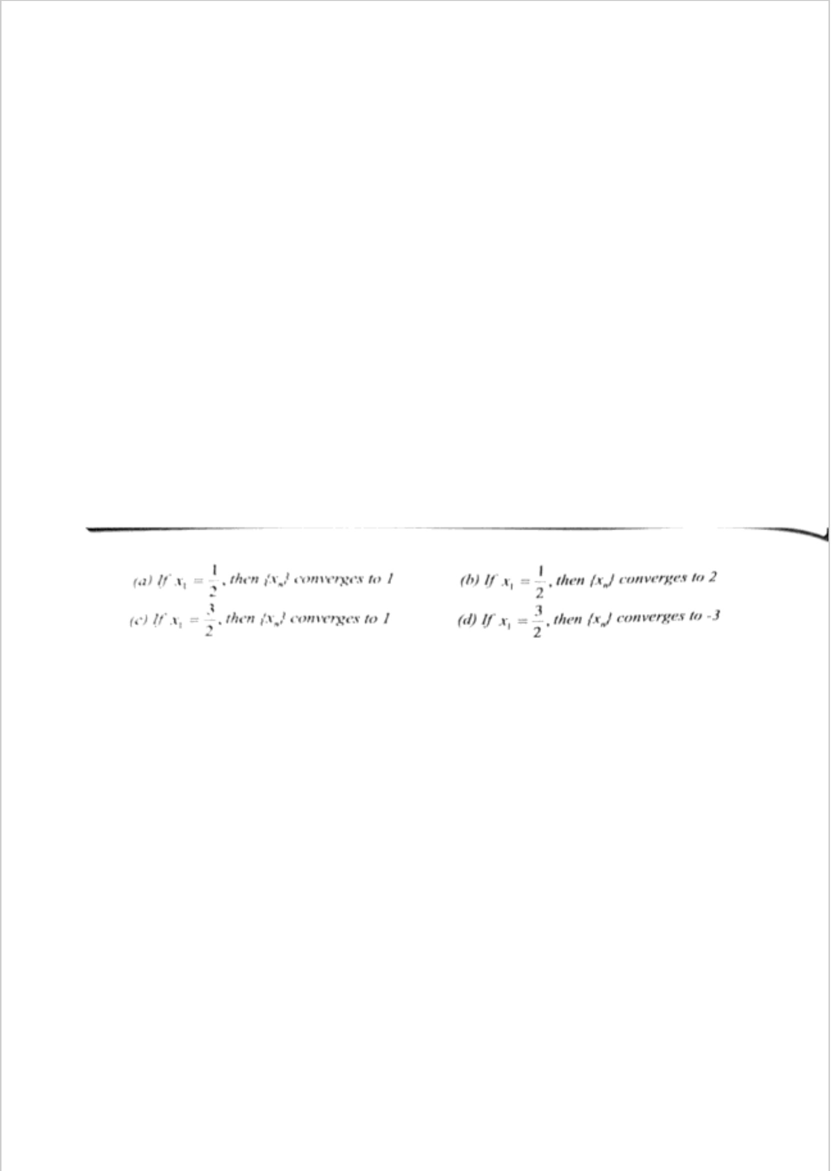
(b) S is not a closed subset of $\mathbb{R}$

(c) S is an open subset of $\mathbb{R}$

(d) Every $x \in (0, 1) \setminus S$ is a limit point of S



96. *Let $\{x_n\}$ be a real sequence such that $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$. Then which of the following statements are TRUE?* ***(MSQ)***



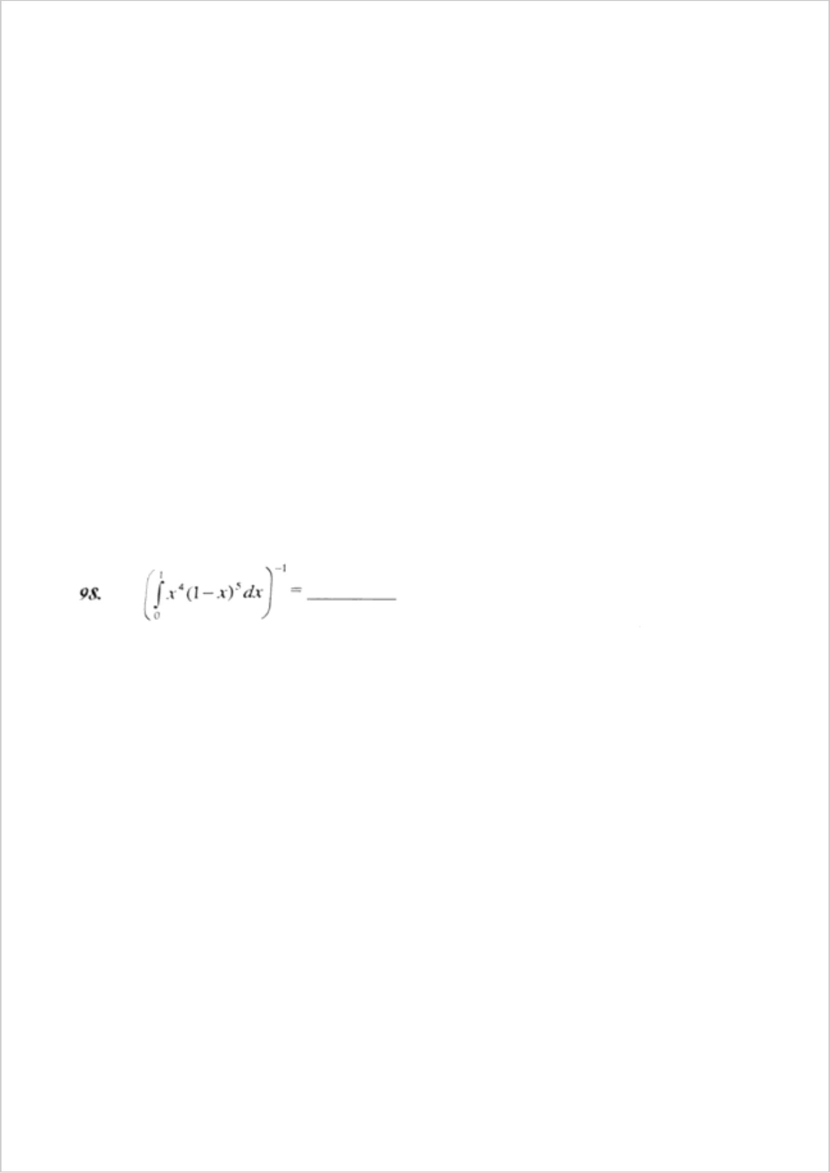
(a) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 1

(b) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 2

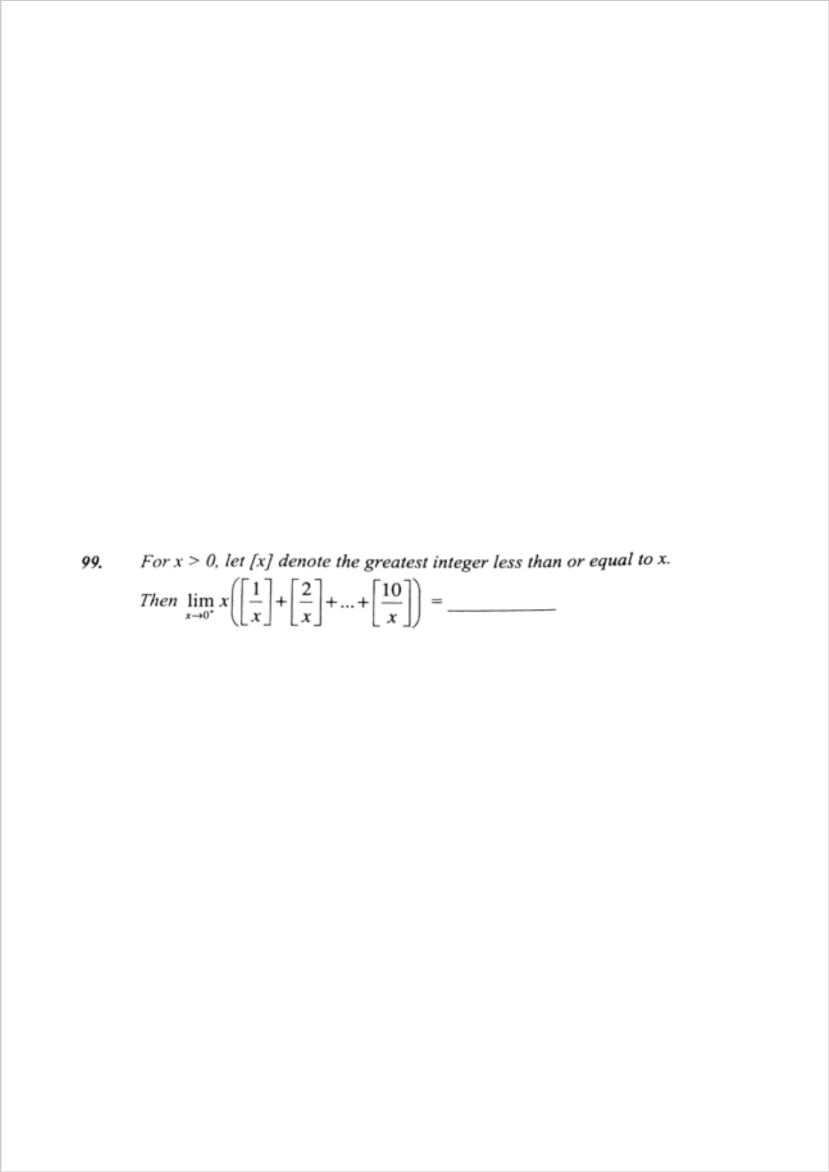
(c) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to 1

(d) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to -3

97. $\frac{1}{2\pi}\left(\frac{\pi^3}{1!\ 3}-\frac{\pi^5}{3!\ 5}+\frac{\pi^7}{5!\ 7}-\ldots+\frac{(-1)^{n-1}\pi^{2n+1}}{(2n-1)!\ (2n+1)}+\ldots\right)=$ __________



98. $\left(\int_0^1 x^4(1-x)^5\,dx\right)^{-1} =$ __________



99. *For $x > 0$, let $[x]$ denote the greatest integer less than or equal to x.*

Then $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{10}{x}\right]\right) =$ ____________

100. *If* $y(x)=\int_{\sqrt{x}}^{x}\frac{e^{t}}{t}\,dt, x>0$, *then* $y'(1)=$ __________