27 December 2023 16:52

FUNCTIONS

$f(x) = x$

4 types

1. Algebraic (Polynomials)
2. Trigonometric.
3. Exponential
4. Logarithmic

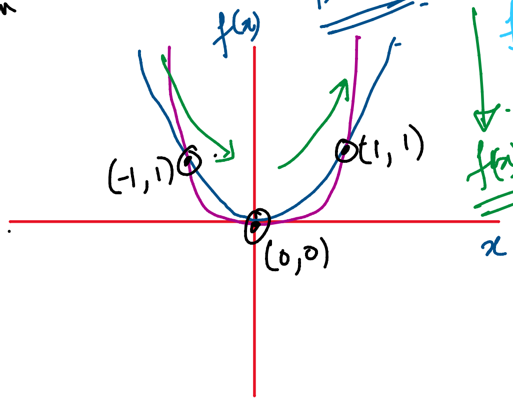
$f(x) = x^n$ where n is an integer.

n is even, $n = 2m$. $f(x) = x^2, x^4, \ldots$

n is odd. $n = 2m+1$. $f(x) = x, x^3, \ldots$

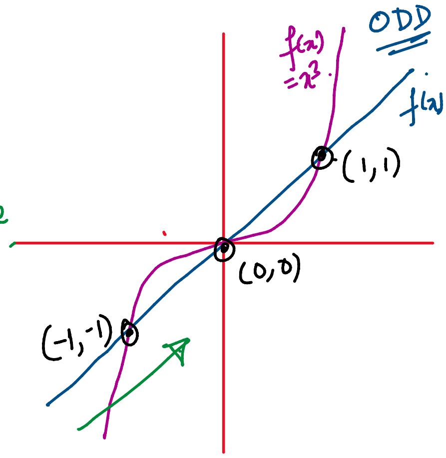
where the power is very large.

EVEN $f(x) = x^2, x^4$ -

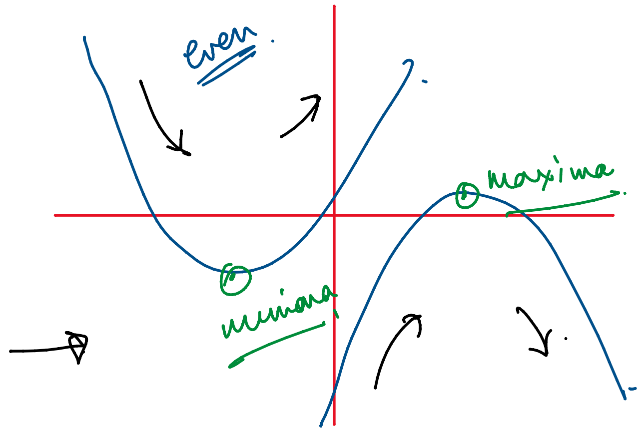


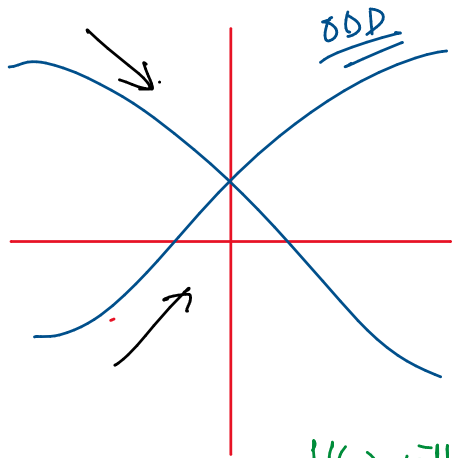
$f(x) \geq 0$ only +ve

ODD. $f(x) = x, x^3, \ldots$



$f(x)$ can be both +ve and -ve.





$f'(x) \rightarrow$ +ve as well as -ve

$\Downarrow$

$f(x)$ will have at least 1 maxima/minima.

$f(x) = x + \frac{1}{x}$.

$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2} = \frac{(x+1)(x-1)}{x^2}$, $x^2 \geq 0$

<0, $-1 < x < 1$.

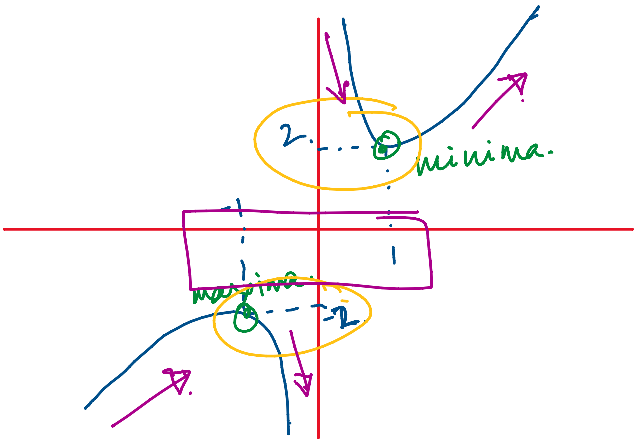
$=0$, $x = 1, -1$

>0. $x < -1$, $x > 1$

$f'(x)$ will be either >0 or <0.

$= 0 \quad , \quad x = 1, -1$

$> 0 \quad , \quad x < -1, \; x > 1$



MINIMA > MAXIMA

$f(x) = x + \frac{1}{x}.$

$f(x) = x - \frac{1}{x}.$

$f'(x) = 1 + \frac{1}{x^2} > 0$ except for $x = 0$.

$f(x) = x + \frac{1}{x}; \quad [f(x)]^3 = f(x^3) + \lambda f\left(\frac{1}{x}\right)$ find λ.

$[f(x)]^3 = \left[x + \frac{1}{x}\right]^3$ —(1)

$f\left(\frac{1}{x}\right) = x + \frac{1}{x} = f(x)$ —(3)

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$f(x^3) = x^3 + \frac{1}{x^3}$ —(2)

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + \lambda\left(x + \frac{1}{x}\right)$$

$\downarrow$ 3

even $f(-x) = f(x)$

odd $f(-x) = -f(x)$

$f(x) = x + \frac{1}{x}.$

$f(-x) = -x - \frac{1}{x} = -f(x)$ ✓

$f(x) = x + 2.$

$f(-x) = -x + 2 \neq -f(x)$

$f(x) = \frac{x+2}{x-1}$

$$f'(x) = \frac{x-1-(x+2)}{(x-1)^2} = \frac{-3}{(x-1)^2} < 0 \text{ except at } x=1$$

$(x-1)^2 \geq 0$

$$a\underbrace{f(x)}_{p} + b\underbrace{f\left(\frac{1}{x}\right)}_{q} = \frac{1}{x} - 5 \qquad \text{find } f(2).$$

replace x with $\frac{1}{x}$.

$$a\underbrace{f\left(\frac{1}{x}\right)}_{q} + b\underbrace{f(x)}_{p} = x - 5.$$

$(ap + bq = \frac{1}{x} - 5) \times b.$

$(aq + bp = x - 5) \times a.$

$\cancel{abp} + b^2 q = \frac{b}{x} - 5b.$

$-(\cancel{abp} + a^2 q = ax - 5a.)$

$(b^2 - a^2)q = \frac{b}{x} - ax + 5(a-b)$

$q = \left[\frac{b}{x} - ax + 5(a-b)\right] \times \frac{1}{(b^2-a^2)}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{b}{x} - ax + 5(a-b)}{b^2 - a^2}.$$

put $x = \frac{1}{2}$

$$f(2) = \frac{2b - a/2 + 5(a-b)}{b^2 - a^2}$$

$$= \frac{\frac{9}{2}a - 3b.}{b^2 - a^2}.$$

$$\underbrace{f(x)}_{p} + 2\underbrace{f(x-4)}_{q} = x. \qquad \text{find } f(x)$$

$p + 2q = x$ $\qquad$ $p = ?$

replace x with $x-4$.

$f(x-4) + 2f(x-8) = x - 4.$

$q + 2f(x-8) = x - 4.$

replace $x-8$ with x.

$f(x-8+4) + 2f(x-8) = x - 8 + 4.$

$f(x+4) + 2f(x) = x + 4.$

$f(x+4) + 2p = x + 4. \text{ — } (1)$

replace $x-4$ with $x+4$.

$f(x+4) + 2f(x) = x+4. \quad (2)$

$f(x) + 2f(x-4) = x.$

$x, \; x-4$

$x - 4 = (x-2) - 2$

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$f(x) + 2f(x-4) = x.$

$x,\ x-4$

$x-2$

$x-4 = (x-2)-2$

x

replace x with $x-2$.

$f(x-2) + 2f(x-2) = x-2.$

$3f(x-2) = x-2.$