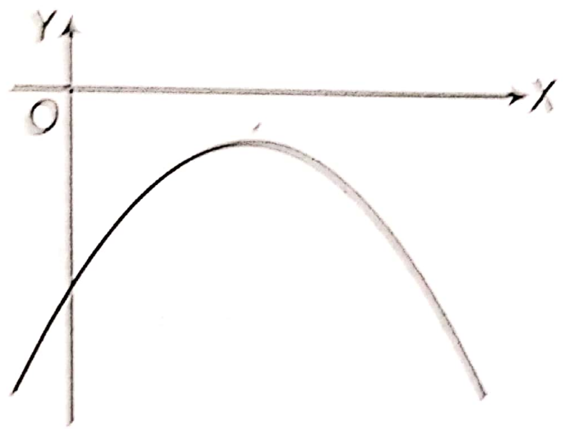
1. If a, b, c are real and $a \neq b$, the roots of the equation $2(a-b)x^2 - 11(a+b+c)x - 3(a-b) = 0$ are

(a) real and equal (b) real and unequal

(c) purely imaginary (d) None of these

2. The graph of a quadratic polynomial $y = ax^2 + bx + c$; $a, b, c \in R$ is as shown.



Which one of the following is not correct?

(a) $b^2 - 4ac < 0$ (b) $\frac{c}{a} < 0$

(c) c is negative

(d) Abscissa corresponding to the vertex is $\left(-\frac{b}{2a}\right)$

3. There is only one real value of 'a' for which the quadratic equation $ax^2 + (a+3)x + a - 3 = 0$ has two positive integral solutions. The product of these two solutions is

(a) 9 (b) 8 (c) 6 (d) 12

4. If for all real values of a one root of the equation $x^2 - 3ax + f(a) = 0$ is double of the other, $f(x)$ is equal to

(a) $2x$ (b) x^2 (c) $2x^2$ (d) $2\sqrt{x}$

5. A quadratic equation the product of whose roots x_1 and x_2 is equal to 4 and satisfying the relation

$$\frac{x_1}{x_1-1}+\frac{x_2}{x_2-1}=2$$ is

(a) $x^2-2x+4=0$ (b) $x^2-4x+4=0$

(c) $x^2+2x+4=0$ (d) $x^2+4x+4=0$

6. If both roots of the quadratic equation $x^2-2ax+a^2-1=0$ lie in $(-2,2)$, which one of the following can be $[a]$? (where $[\cdot]$ denotes the greatest integer function)

(a) -1 (b) 1 (c) 2 (d) 3

7. If $(-2,7)$ is the highest point on the graph of $y=-2x^2-4ax+\lambda$, then λ equals

(a) 31 (b) 11 (c) -1 (d) $-\frac{1}{3}$

8. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unity, the number of integral values of p is

(a) 1 (b) 2 (c) 3 (d) 4

9. Solution set of the equation $3^{2x^2} - 2 \cdot 3^{x^2 + x + 6} + 3^{2(x+6)} = 0$ is

(a) $\{-3, 2\}$ (b) $\{6, -1\}$ (c) $\{-2, 3\}$ (d) $\{1, -6\}$

10. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$ $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$, then

(a) α will be AM of the roots of $f(x) = 0$ and $g(x) = 0$
(b) α will be AM of the roots of $f(x) = 0$
(c) α will be AM of the roots of $f(x) = 0$ or $g(x) = 0$
(d) α will be AM of the roots of $g(x) = 0$

11. If x_1 and x_2 are the arithmetic and harmonic means of the roots of the equation $ax^2 + bx + c = 0$, the quadratic equation whose roots are x_1 and x_2, is

(a) $abx^2 + (b^2 + ac)x + bc = 0$

(b) $2abx^2 + (b^2 + 4ac)x + 2bc = 0$

(c) $2abx^2 + (b^2 + ac)x + bc = 0$

(d) None of the above

12. $f(x)$ is a cubic polynomial $x^3 + ax^2 + bx + c$ such that $f(x) = 0$ has three distinct integral roots and $f(g(x)) = 0$ does not have real roots, where $g(x) = x^2 + 2x - 5$, the minimum value of $a + b + c$ is

(a) 504 (b) 532 (c) 719 (d) 764

13. The value of the positive integer n for which the quadratic equation $\sum_{k=1}^{n}(x + k - 1)(x + k) = 10n$ has solutions α and $\alpha + 1$ for some α, is

(a) 7 (b) 11 (c) 17 (d) 25

14. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime, while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is

(a) 8 (b) 16 (c) 24 (d) 32

15. Number of real roots of the equation $\sqrt{x} + \sqrt{x - \sqrt{(1-x)}} = 1$ is

(a) 0 (b) 1 (c) 2 (d) 3

16. The value of $\sqrt{7 + \sqrt{7 - \sqrt{7 + \sqrt{7 - \dots}}}}$ upto ∞ is

(a) 5 (b) 4

(c) 3 (d) 2

17. For any real x, the expression $2(k - x)[x + \sqrt{x^2 + k^2}]$ cannot exceed

(a) k^2
(b) $2k^2$
(c) $3k^2$
(d) None of these

18. Given that, for all $x \in R$, the expression $\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ lies between $\frac{1}{3}$ and 3, the values between which the expression $\frac{9 \cdot 3^{2x} + 6 \cdot 3^x + 4}{9 \cdot 3^{2x} - 6 \cdot 3^x + 4}$ lies, are

(a) -3 and 1
(b) $\frac{3}{2}$ and 2
(c) -1 and 1
(d) 0 and 2

19. Let α, β, γ be the roots of the equation $(x - a)(x - b)(x - c) = d$, $d \neq 0$, the roots of the equation $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ are

(a) a, b, d
(b) b, c, d
(c) a, b, c
(d) $a + d, b + d, c + d$

20. If one root of the equation $ix^2 - 2(1+i)x + 2 - i = 0$ is $(3 - i)$, where $i = \sqrt{-1}$, the other root is

(a) $3 + i$
(b) $3 + \sqrt{-1}$
(c) $-1 + i$
(d) $-1 - i$

21. The number of solutions of $|[x] - 2x| = 4$, where $[x]$ denotes the greatest integer $\leq x$ is

(a) infinite (b) 4 (c) 3 (d) 2

22. If $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d$, the real root of $ax^3 + bx^2 + cx + d = 0$ is

(a) $-\frac{d}{a}$ (b) $\frac{d}{a}$ (c) $\frac{a}{d}$ (d) None of these

23. The value of x which satisfy the equation

$$\sqrt{(5x^2 - 8x + 3)} - \sqrt{(5x^2 - 9x + 4)} = \sqrt{(2x^2 - 2x)} - \sqrt{(2x^2 - 3x + 1)},$$ is

(a) 3
(b) 2
(c) 1
(d) 0

24. The roots of the equation

$$(a+\sqrt{b})^{x^2-15}+(a-\sqrt{b})^{x^2-15}=2a$$

where $a^2-b=1$, are

(a) $\pm 2, \pm\sqrt{3}$ (b) $\pm 4, \pm\sqrt{14}$

(c) $\pm 3, \pm\sqrt{5}$ (d) $\pm 6, \pm\sqrt{20}$

25. The number of pairs (x, y) which will satisfy the equation

$x^2-xy+y^2=4(x+y-4)$, is

(a) 1 (b) 2

(c) 4 (d) None of these

26. The number of positive integral solutions of $x^4-y^4=3789108$ is

(a) 0 (b) 1 (c) 2 (d) 4

27. The value of 'a' for which the equation $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$, have a common root, is

(a) $a = 2$ (b) $a = -2$

(c) $a = 0$ (d) None of these

28. The necessary and sufficient condition for the equation $(1 - a^2)x^2 + 2ax - 1 = 0$ to have roots lying in the interval $(0, 1)$, is

(a) $a > 0$ (b) $a < 0$

(c) $a > 2$ (d) None of these

29. Solution set of $x - \sqrt{1 - |x|} < 0$, is

(a) $\left[-1, \frac{-1 + \sqrt{5}}{2}\right)$ (b) $[-1, 1]$

(c) $\left[-1, \frac{-1 + \sqrt{5}}{2}\right]$ (d) $\left(-1, \frac{-1 + \sqrt{5}}{2}\right)$

30. If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0 (b \neq c)$ have a common root, $a + 4b + 4c$, is equal to

(a) -2 (b) -1

(c) 0 (d) 1

31. If $0 < a < b < c$ and the roots α, β of the equation $ax^2 + bx + c = 0$ are non-real complex numbers, then

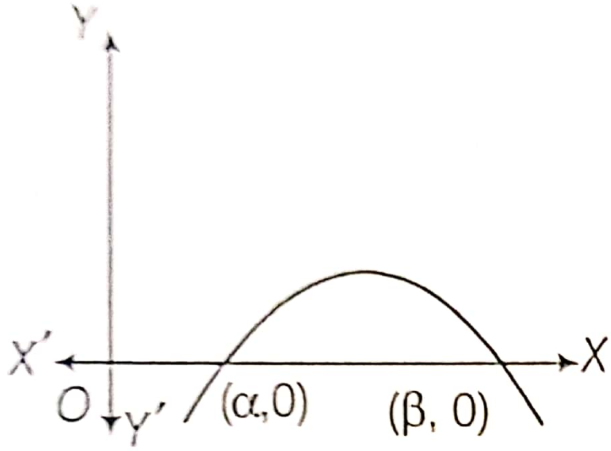
(a) $|\alpha| = |\beta|$ (b) $|\alpha| > 1$

(c) $|\beta| < 1$ (d) None of these

32. If A, G and H are the arithmetic mean, geometric mean and harmonic mean between unequal positive integers. Then, the equation $Ax^2 - |G|x - H = 0$ has

(a) both roots are fractions
(b) atleast one root which is negative fraction
(c) exactly one positive root
(d) atleast one root which is an integer

33. The adjoining graph of $y = ax^2 + bx + c$ shows that



(a) $a < 0$
(b) $b^2 < 4ac$
(c) $c > 0$
(d) a and b are of opposite signs

34. If the equation $ax^2 + bx + c = 0\,(a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then

(a) $b^2 - 4ac > 0$ (b) $c < 0$
(c) $a + |b| + c < 0$ (d) $4a + 2|b| + c < 0$

35. If $b^2 \geq 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all the roots of the equation will be real, if

(a) $b > 0, a < 0, c > 0$ (b) $b < 0, a > 0, c > 0$
(c) $b > 0, a > 0, c > 0$ (d) $b > 0, a < 0, c < 0$

36. If roots of the equation $x^3 + bx^2 + cx - 1 = 0$ from an increasing GP, then

(a) $b + c = 0$

(b) $b \in (-\infty, -3)$

(c) one of the roots is 1

(d) one root is smaller than one and one root is more than one

37. Let $f(x) = ax^2 + bx + c$, where $a, b, c \in R, a \neq 0$. Suppose $|f(x)| \leq 1, \forall x \in [0, 1]$, then

(a) $|a| \leq 8$ (b) $|b| \leq 8$

(c) $|c| \leq 1$ (d) $|a| + |b| + |c| \leq 17$

38. $\cos\alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, the value of $\sin 2\alpha$ is

(a) $\frac{24}{25}$ (b) $-\frac{12}{25}$

(c) $-\frac{24}{25}$ (d) $\frac{20}{25}$

39. If $a, b, c \in R(a \neq 0)$ and $a + 2b + 4c = 0$, then equation $ax^2 + bx + c = 0$ has

(a) atleast one positive root

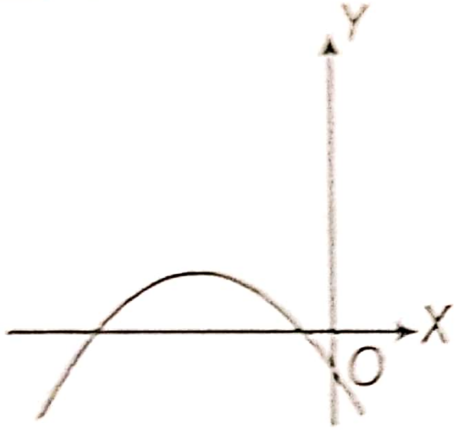
(b) atleast one non-integral root

(c) both integral roots

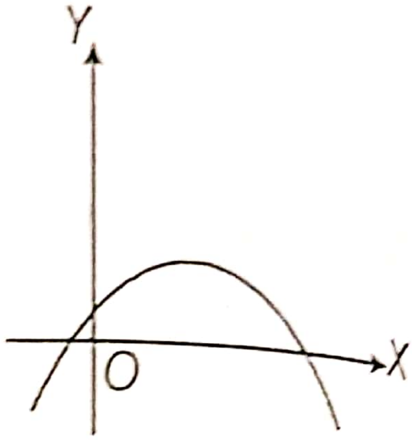
(d) no irrational root

40. For which of the following graphs of the quadratic expression $f(x) = ax^2 + bx + c$, the product of abc is negative

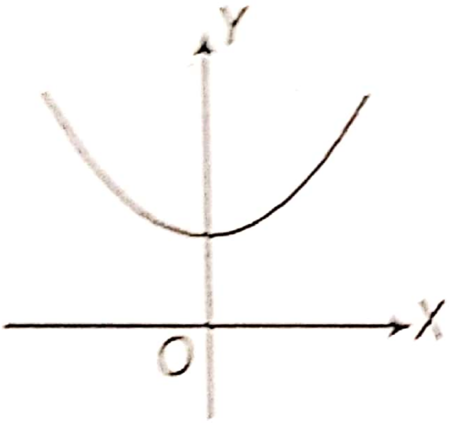
(a)



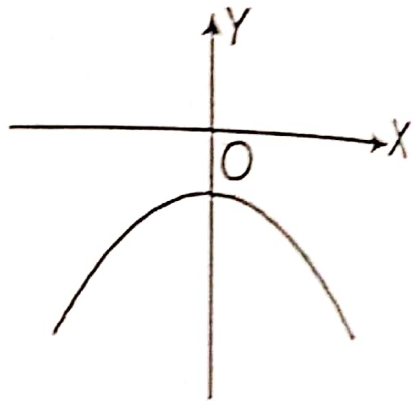
(b)



(c)



(d)



41. If $a, b \in R$ and $ax^2 + bx + 6 = 0, a \neq 0$ does not have two distinct real roots, the

(a) minimum possible value of $3a + b$ is -2
(b) minimum possible value of $3a + b$ is 2
(c) minimum possible value of $6a + b$ is -1
(d) minimum possible value of $6a + b$ is 1

42. If $x^3 + 3x^2 - 9x + \lambda$ is of the form $(x - \alpha)^2(x - \beta)$, then λ is equal to

(a) 27
(b) -27
(c) 5
(d) -5

43. If $ax^2 + (b - c)x + a - b - c = 0$ has unequal real roots for all $c \in R$, then

(a) $b < 0 < a$ (b) $a < 0 < b$

(c) $b < a < 0$ (d) $b > a > 0$

44. If the equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with the given cubic equation, then

(a) $a = b = 0$

(b) $a = 0, b = 3$

(c) $a = b = 3$

(d) a, b are roots of $x^2 + x + 2 = 0$

45. If the equation $ax^2 + bx + c = 0\ (a > 0)$ has two real roots α and β such that $\alpha < -2$ and $\beta > 2$, which of the following statements is/are true?

(a) $4a - 2|b| + c < 0$

(b) $9a - 3|b| + c < 0$

(c) $a - |b| + c < 0$

(d) $c < 0, b^2 - 4ac > 0$

67. The sum of all the real roots of the equation $|x-2|^2+|x-2|-2=0$ is

68. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2-(4+\sqrt{5})x+8+2\sqrt{5}=0$ is

69. If product of the real roots of the equation,

$$x^2-ax+30=2\sqrt{(x^2-ax+45)},\ a>0,$$

is λ and minimum value of sum of roots of the equation is μ. The value of (μ) (where $(\cdot)$ denotes the least integer function) is

70. The minimum value of $\dfrac{\left(x+\frac{1}{x}\right)^6-\left(x^6+\frac{1}{x^6}\right)-2}{\left(x+\frac{1}{x}\right)^3+x^3+\frac{1}{x^3}}$ is (for $x>0$)

71. Let a, b, c, d are distinct real numbers and a, b are the roots of the quadratic equation $x^2-2cx-5d=0$. If c and d are the roots of the quadratic equation $x^2-2ax-5b=0$, the sum of the digits of numerical values of $a+b+c+d$ is

72. If the maximum and minimum values of $y = \frac{x^2 - 3x + c}{x^2 + 3x + c}$ are 7 and $\frac{1}{7}$ respectively, the value of c is

73. Number of solutions of the equation

$\sqrt{x^2} - \sqrt{(x-1)^2} + \sqrt{(x-2)^2} = \sqrt{5}$ is

74. If α and β are the complex roots of the equation $(1+i)x^2 + (1-i)x - 2i = 0$, where $i = \sqrt{-1}$, the value of $|\alpha - \beta|^2$ is

75. If α, β be the roots of the equation $4x^2 - 16x + c = 0$, $c \in R$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral values of c, are

76. Let r, s and t be the roots of the equation $8x^3 + 1001x + 2008 = 0$ and if $99\lambda = (r+s)^3 + (s+t)^3 + (t+r)^3$, the value of $[\lambda]$ is (where $[\cdot]$ denotes the greatest integer function)

88. For what values of m, the equation

$(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$ has $(m \in R)$

(i) both roots are imaginary?

(ii) both roots are equal?

(iii) both roots are real and distinct?

(iv) both roots are positive?

(v) both roots are negative?

(vi) roots are opposite in sign?

(vii) roots are equal in magnitude but opposite in sign?

(viii) atleast one root is positive?

(ix) atleast one root is negative?

(x) roots are in the ratio $2:3$?

89. For what values of m, then equation $2x^2 - 2(2m+1)x + m(m+1) = 0$ has $(m \in R)$

(i) both roots are smaller tha 2?

(ii) both roots are greater than 2?

(iii) both roots lie in the interval (2, 3)?

(iv) exactly one root lie in the interval (2, 3)?

(v) one root is smaller than 1 and the other root is greater than 1?

(vi) one root is greater than 3 and the other root is smaller than 2?

(vii) atleast one root lies in the interval (2, 3)?

(viii) atleast one root is greater than 2?

(ix) atleast one root is smaller than 2?

(x) roots α and β, such that both 2 and 3 lie between α and β?

90. If r is the ratio of the roots of the equation $ax^2 + bx + c = 0$, show that $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$.

91. If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, show that $p + q = 2r$ and that the product of the roots is equal to $\left(-\frac{p^2+q^2}{2}\right)$.

92. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the nth power of the other, then show that

$$(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0.$$

93. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and γ, δ those of equation $lx^2 + mx + n = 0$, then find the equation whose roots are $\alpha\gamma + \beta\delta$ and $\alpha\delta + \beta\gamma$.

94. Show that the roots of the equation

$$(a^2 - bc)x^2 + 2(b^2 - ac)x + c^2 - ab = 0$$

are equal, if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$.

95. If the equation $x^2 - px + q = 0$ and $x^2 - ax + b = 0$ have a common root and the other root of the second equation is the reciprocal of the other root of the first, then prove that $(q - b)^2 = bq(p - a)^2$.

96. If the equation $x^2 - 2px + q = 0$ has two equal roots, then the equation $(1 + y)x^2 - 2(p + y)x + (q + y) = 0$ will have its roots real and distinct only, when y is negative and p is not unity.

97. Solve the equation $x^{\log_x (x+3)^2} = 16$.

98. Solve the equation

$$(2 + \sqrt{3})^{x^2 - 2x + 1} + (2 - \sqrt{3})^{x^2 - 2x - 1} = \frac{101}{10(2 - \sqrt{3})}.$$

99. Solve the equation $x^2 + \left(\dfrac{x}{x-1}\right)^2 = 8$.

100. Solve the equation

$$\sqrt{(x+8) + 2\sqrt{(x+7)}} + \sqrt{(x+1) - \sqrt{(x+7)}} = 4.$$

101. Find all values of a for which the inequation

$4^{x^2} + 2(2a+1)2^{x^2} + 4a^2 - 3 > 0$ is satisfied for any x.

102. Solve the inequation $\log_{x^2 + 2x - 3}\left(\frac{|x+4| - |x|}{x-1}\right) > 0$.

103. Solve the system $|x^2 - 2x| + y = 1, x^2 + |y| = 1$.

104. If α, β, γ are the roots of the cubic $x^3 - px^2 + qx - r = 0$. Find the equations whose roots are

(i) $\beta\gamma + \frac{1}{\alpha}, \gamma\alpha + \frac{1}{\beta}, \alpha\beta + \frac{1}{\gamma}$

(ii) $(\beta + \gamma - \alpha), (\gamma + \alpha - \beta), (\alpha + \beta - \gamma)$

Also, find the value of $(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma)$.

105. If $A_1, A_2, A_3, \ldots, A_n, a_1, a_2, a_3, \ldots, a_n, a, b, c \in R$, show that the roots of the equation

$$\frac{A_1^2}{x-a_1}+\frac{A_2^2}{x-a_2}+\frac{A_3^2}{x-a_3}+\ldots+\frac{A_n^2}{x-a_n}$$

$$= ab^2 + c^2 x + ac \text{ are real.}$$

106. For what values of the parameter a the equation $x^4 + 2ax^3 + x^2 + 2ax + 1 = 0$ has atleast two distinct negative roots?

107. If $[x]$ is the integral part of a real number x. Then solve $[2x] - [x+1] = 2x$.

108. Prove that for any value of a, the inequation $(a^2 + 3)x^2 + (a+2)x - 6 < 0$ is true for atleast one negative x.

109. How many real solutions of the equation $6x^2 - 77[x] + 147 = 0$, where $[x]$ is the integral part of x ?

110. If α, β are the roots of the equation $x^2 - 2x - a^2 + 1 = 0$ and γ, δ are the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$, such that $\alpha, \beta \in (\gamma, \delta)$, find the value of '$a$'.

111. If the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive real roots, find the minimum value of pr.