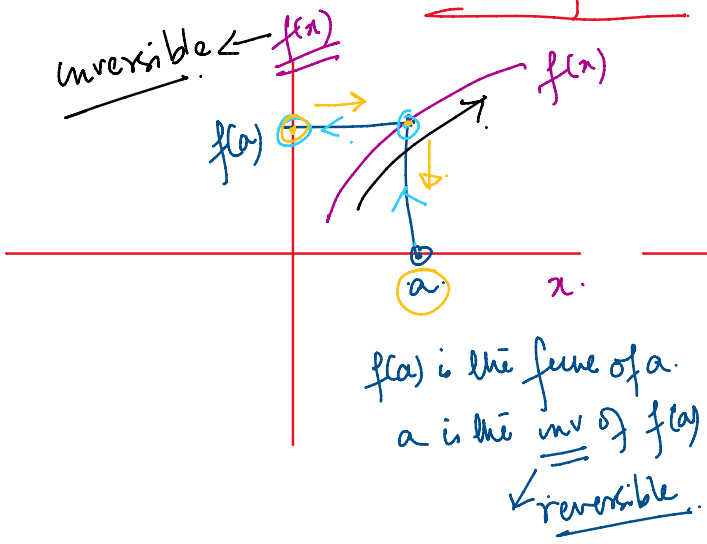
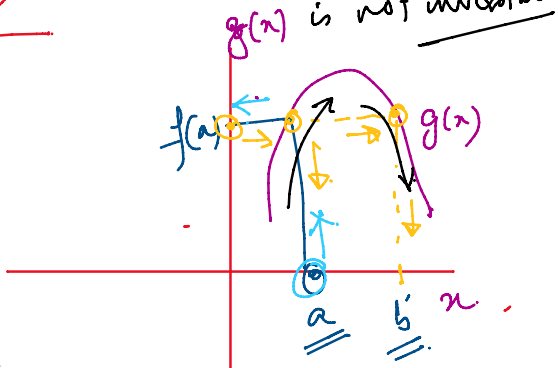
09 December 2023 17:01

Inverse functions



f(a) is the func of a.
a is the inv of f(a)
↙ reversible.



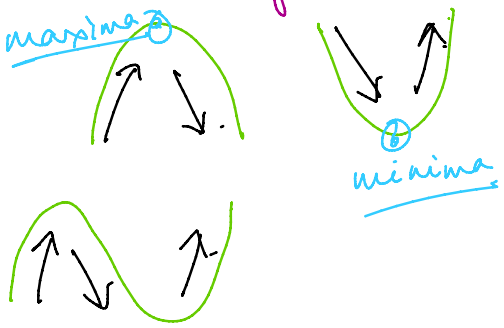
f(a) is the func of a & b
a, b are the inverses of f(a)

Invertible functions → strictly increasing or strictly decreasing ⇒ one-one & onto (bijective)

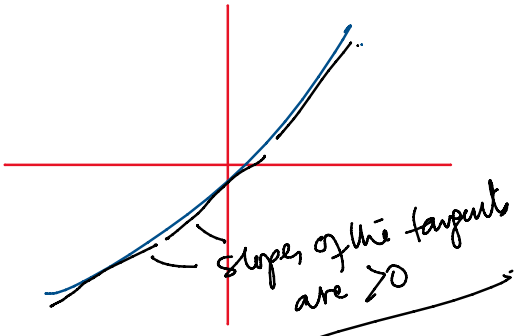
Non-invertible functions → both increasing & decreasing.

↙ not one-one

↓ maxima | minima.

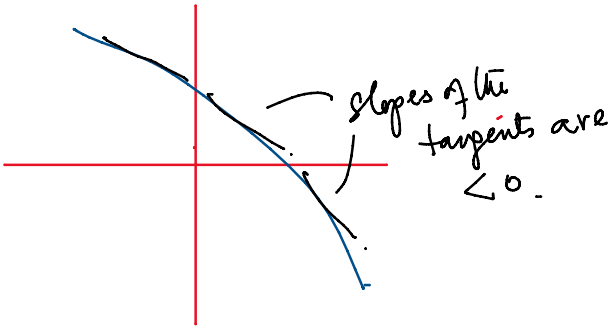


Invertible functions



$f'(x) > 0$.

OR

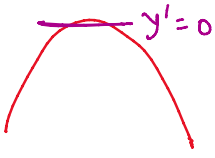


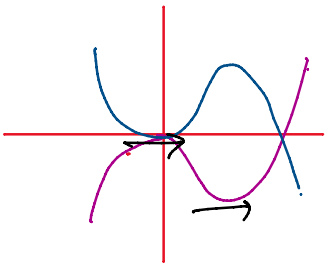
$f'(x) < 0$

$y = x^{99} - 3x^{47} + 4$ ×

$y' = 99x^{98} - 141x^{46} = 0$.

$x^{46}(99x^{52} - 141) = 0$.





$x^{46}(99x^{52} - 141) = 0$

$x = 0, \left(\frac{141}{99}\right)^{1/52}$

$= 0, >0$

$y = x^5$ ✓ $y' = 5x^4 > 0$

$f(x) = ax^{(2n+1)}$ → invertible.

$f'(x) = a(2n+1)x^{2n}$ ≥ 0

$f'(x) > 0$; $a > 0$

$f'(x) < 0$, $a < 0$.

$f(x) = ax^{2n}$ $f'(x) = a \cdot 2n x^{2n-1}$

↓

Not invertible.

$x^3 > 0, x > 0$

$< 0, x < 0$

① Calculate $f'(x)$

② check if $f'(x) = 0$.

↓

either a maxima / minima.

↓

Non-invertible.

if not

③ $f'(x) > 0$ OR $f'(x) < 0$.

$\Rightarrow$ Invertible

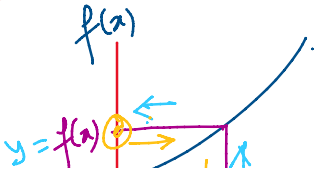
$y = \frac{x-2}{x+2}$ $y' = \frac{x+2-(x-2)}{(x+2)^2}$ $\frac{u'v - v'u}{v^2}$

$y' = \frac{4}{(x+2)^2} > 0$.

≥ 0

To find the inverse

$y = x^3 = f(x)$.



$y = f(x)$

↓

$x \to y$

$x \leftarrow y$

swap my pain

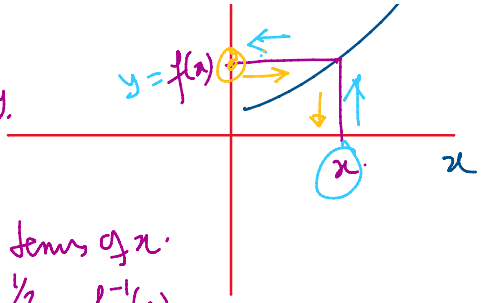
$y = x^3 = f(x)$.

① swap x and y.

$x = y^3$.

② Express y in terms of x.

$y = x^{1/3} = f^{-1}(x)$



swap my y...

↓

$x = f(y)$

$f^{-1}(x) = f^{-1}f(y) = y$.

$f(x) = y = x^2$

$x = y^2$

$y = \pm\sqrt{x}$. ✗

$y = \frac{x-2}{x+2}$. ✓ invertible.

① $x = \frac{y-2}{y+2}$.

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$.

② express y in terms of x.

$\frac{x}{1} = \frac{y-2}{y+2} \Rightarrow \frac{x+1}{x-1} = \frac{2y}{-4} = -\frac{y}{2}$.

$xy + 2x = y - 2$.

$xy - y = -2x - 2$.

$y(x-1) = -2(x+1)$

$f^{-1}(x) = y = -2\frac{(x+1)}{(x-1)}$

$y = \frac{x^2+3}{x+3}$. ✗

$y' = \frac{2x(x+3) - (x^2+3)}{(x+3)^2}$

$= \frac{x^2+6x-3}{(x+3)^2}$

$x^2 + 6x - 3 = 0$?

$\Delta = 6^2 - 4\times(-3)$

$= 48 > 0$

> 0.

$y = \log_e(\sin x)$ ✗

$y' = \frac{1}{\sin x} \cdot \cos x = \cot x$

$y = \log_e(\sin x)$ $0 \le x \le \pi/2$.

$x = \log_e(\sin y)$

$\sin y = e^x$.

$y = \sin^{-1}(e^x)$ ✗

$y = \log_e(\sin x)$ $\quad 0 \le x \le \pi/2$

$y' = \cot x > 0$

$y = x + \frac{1}{x}$. $\quad y' = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$.

$y = x - \frac{1}{x}$. ✓ $\quad y' = 1 + \frac{1}{x^2} > 0$.

>0

$x = y - \frac{1}{y}$.

HW

$y^2 - 1 = xy$. $\quad y^2 - xy - 1 = 0$.

$y = \frac{x \pm \sqrt{x^2+4}}{2}$.