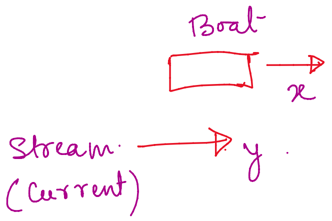
24 January 2024 17:01

Speed, Time and distance

Relative speed / velocity

Case 1. a boat moving upstream/downstream.

Case 2. 2 different bodies are moving in the same direction or in the opposite direction.

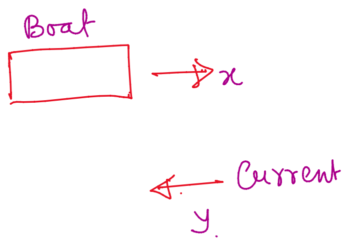


Boat and current are in the same direction

$\Rightarrow$ DOWNSTREAM

(Boat is helped by the current)

Net speed of the boat = $x+y$

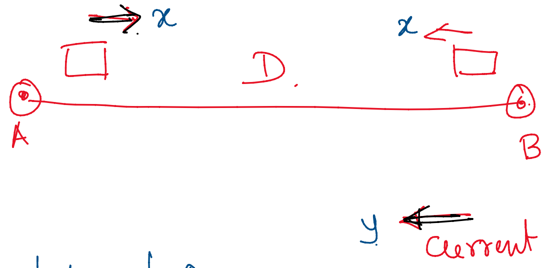


Boat and current are in opposite direction

$\Rightarrow$ UPSTREAM

(Boat is opposed by the current)

Net speed of the boat = $x-y$



A → B upstream.

B → A downstream.

$T_{upstream}$ = Time taken to go from A to B = $\frac{D}{\text{Net Speed}} = \frac{D}{x-y}$

Time taken to go from B to A $= \frac{D}{x+y} = T_{downstream}$

$(x+y) > (x-y)$

$\frac{D}{x+y} < \frac{D}{x-y}$

$T_{downstream} < T_{upstream}$

Total time taken by the boat to go from A to B and return to A is

$T_{Total} = T_{up} + T_{down}$

$T = \frac{D}{x-y} + \frac{D}{x+y}$ → $\frac{T}{D} = \frac{2x}{x^2-y^2}$

Total time = 4 hrs
Distance = 10 km.

$x = 30$ km/hr.

find $y = ?$

$\frac{T}{D} = \frac{1}{x-y} + \frac{1}{x+y} = \frac{2x}{x^2-y^2}$

$\frac{4}{10} = \frac{2\times 30}{30^2 - y^2}$

$900 - y^2 = \frac{2 \times 30 \times 10}{4} = 150$

$900 - y^2 = 150$ $y^2 = 900 - 150 = 750$

$y = \sqrt{750}$

$y = 10$ km/hr

find x.

$\frac{4}{10} = \frac{2x}{x^2 - 10^2}$

$x^2 - 10^2 = \frac{2x \times 10}{4}$ $x^2 - 100 = 5x$.

$x^2 - 5x - 100 = 0$.

$ax^2 + bx + c = 0$.

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-100)}}{2\times 1}$

$x = \frac{5 \pm \sqrt{25 + 400}}{2}$

$\sqrt{425} = 20 + \frac{25}{2\times 20} = 20 + \frac{5}{8}$

$\sqrt{400} = 20$

$= 20.6$

$x = \frac{5 \pm \sqrt{425}}{2} = \frac{5 \pm 20.6}{2}$

$x = \frac{5 + 20.6}{2} = \frac{25.6}{2} = 12.8$

$T = 40$ min $D = 10$ km. $x = 10$ m/s. find y in km/hr.

$\frac{T}{D} = \frac{2x}{x^2 - y^2}$

$1 \text{ km/hr} = \frac{1 \text{ km}}{1 \text{ hr}} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{5}{18}$ m/s

1 km/hr $= \frac{5}{18}$ m/s.

$T = \frac{40}{60}$ hr $= \frac{2}{3}$ hr. $10 \text{ m/s} = 10 \times \frac{18}{5}$ km/hr

1 m/s $= \frac{18}{5}$ km/hr.

$x = 36$ km/hr

$$\frac{\frac{2}{3}}{10} = \frac{2\times 36}{36^2 - y^2}$$

$$36^2 - y^2 = \cancel{2}\times 36 \times \frac{10}{\cancel{2}} \times 3 = 1080$$

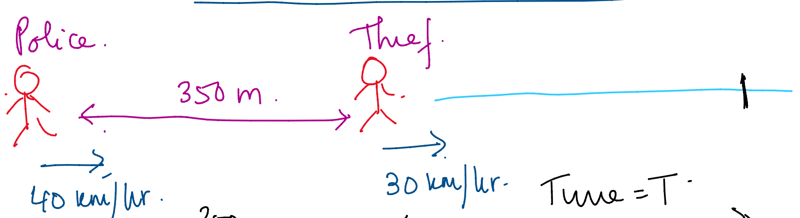
$$1296 - y^2 = 1080$$

$$y^2 = 216 \qquad y = \sqrt{216} = 14.7$$

$$\sqrt{216} = 14 + \frac{\cancel{20}\,10\,5}{\cancel{2}\times \cancel{14}\,7} = 14.7$$

$$\sqrt{196} = 14$$

2 bodies are moving in the same direction



Time = T

350 m

x = distance run by the thief

$x + 350$ = distance by police

Time = T

$$T = \frac{\text{Distance}}{\text{speed}}$$

Thief $T = \dfrac{x}{30}$

Police $T = \dfrac{x + 0.35}{40}$

350 m = 0.35 km

$$\frac{x}{30} = \frac{x + 0.35}{40}$$

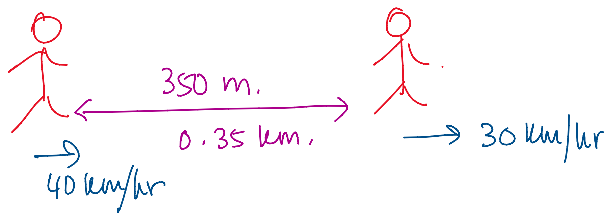
$$40x = 30x + 10.5$$

$$10x = 10.5$$

$$x = 1.05 \text{ km}$$

$$T = \frac{x}{30} = \frac{1.05}{30} \text{ hrs} = \frac{0.105}{3} \text{ hrs} = 0.035 \text{ hrs}$$

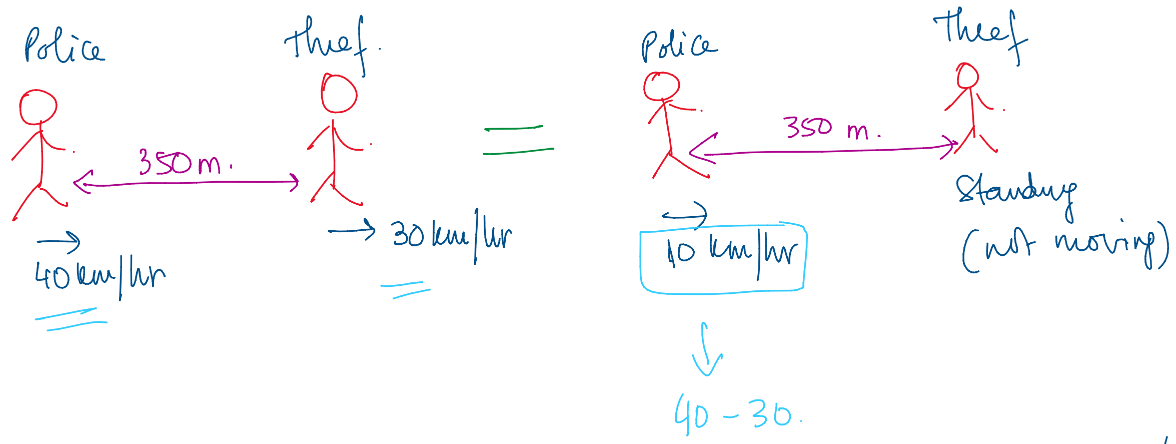
$$= \frac{35}{1000} \text{ hrs} = \frac{\cancel{35}\,7}{\cancel{1000}\,\cancel{20}\,10} \times \cancel{60}\,3 \text{ min} = 2.1 \text{ min}$$



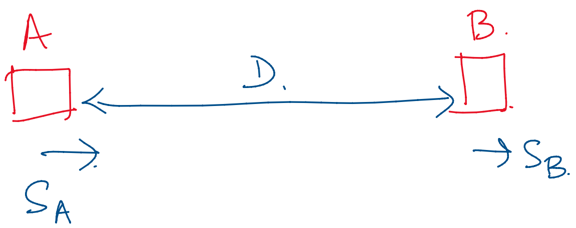
⇒ If the thief is stationary then the Relative Speed of the police w.r.t the thief = 40 − 30 = 10 km/hr.

If the thief is stationary then the Relative speed of the police w.r.t the thief $= 40 - 30 = 10$ km/hr.

$$\text{Time} = \frac{0.35}{10} = 0.035 \text{ hr} = 2.1 \text{ min}$$



Relative speed of the Police w.r.t the thief = Speed of the police − speed of the thief.



Then the time required by A to catch B

$$= \frac{D}{\text{Rel speed of A w.r.t B}} = \frac{D}{S_A - S_B}$$