Integration by parts

1. $\int \underbrace{4x}_{v} \underbrace{(x+1)^3}_{u} dx =$

$\int \overset{u}{f(x)}\, \overset{v}{g(x)}\, dx$

$$\boxed{\int u v\, dx = u\int v \cdot dx - \int \left[u' \int v\, dx\right] dx}$$

OR $\boxed{\int u \cdot dv = uv - \int v\, du}$

$\int 4x\, (x+1)^3\, dx$

$= 4\int x \cdot (x+1)^3 dx = 4\left[x\int (x+1)^3 dx - \int\left[\frac{dx}{dx}\int (x+1)^3 \cdot dx\right]dx\right]$

$= 4\left[x\,\frac{(x+1)^4}{4} - \int \frac{(x+1)^4}{4}\, dx\right]$

$= 4\left[\frac{x(x+1)^4}{4} - \frac{(x+1)^5}{20}\right] + c$

$= x(x+1)^4 - \frac{(x+1)^5}{5} + c$

(ans)

$= 2x$ 3 c

2:

$$\int 2x \cdot e^x \, dx = 2\int x \cdot e^x \, dx$$

$$= 2\left[x \cdot \int e^x \, dx - \int \left\{ \frac{dx}{dx} \int e^x \cdot dx \right\} dx \right]$$

$$= 2\left[x \cdot e^x - \int 1 \cdot e^x \cdot dx \right]$$

$$= 2\left[x \cdot e^x - e^x \right] + c$$

$$= 2x \cdot e^x - 2e^x + c$$

$$= 2e^x (x-1) + c \quad \text{(ans)}$$

3

$$15\int x \, (x+4)^{3/2} \, dx .$$

$$= 15\left[x \int (x+4)^{3/2} dx - \int \left\{ \frac{dx}{dx} \int (x+4)^{3/2} dx \right\} dx \right]$$

$$= 15\left[x \, \frac{(x+4)^{3/2+1}}{\frac{3}{2}+1} - \int \left\{ 1 \cdot \frac{(x+4)^{3/2+1}}{3/2+1} \right\} dx \right]$$

$$= 15\left[x \cdot \frac{(x+4)^{5/2}}{5/2} - \frac{2}{5} \int (x+4)^{5/2} dx \right]$$

$$= 15\left[\frac{2x}{5} (x+4)^{5/2} - \frac{2}{5} \, \frac{(x+4)^{5/2+1}}{5/2+1} \right] + C$$

$$= \overset{3}{\cancel{15}} \times 2x \; (x+4)^{5/2} - 15 \times 2 \times 2 \times (x+4)^{7/2} + c$$

$$= \frac{\cancel{15}^{\,3} \times 2x}{\cancel{8}_{\,4}} (x+4)^{5/2} - \frac{15\times 2}{5} \times \frac{2}{7} \times (x+4)^{7/2} + c$$

$$= 6x \, (x+4)^{5/2} - \frac{\cancel{15}^{\,3}\times 4}{7\,\cancel{35}} (x+4)^{7/2} + c \quad \text{(ans)}$$

$$= 6x \, (x+4)^{5/2} - \frac{12}{7} (x+4)^{7/2} + c.$$

(4) $$\int \frac{2x}{(x-8)^3}\, dx = 2\int x\,(x-8)^{-3}\, dx$$

$$= 2\left[x \int (x-8)^{-3} dx - \int \left\{ \frac{dx}{dx} \int (x-8)^{-3} dx \right\} dx \right]$$

$$= 2\left[x \, \frac{(x-8)^{-2}}{-2} - \int 1 \cdot \frac{(x-8)^{-2}}{-2}\, dx \right]$$

$$= 2\left[\frac{x(x-8)^{-2}}{-2} + \frac{1}{2} \cdot \frac{(x-8)^{-1}}{(-1)} \right] + c$$

$$= -x(x-8)^{-2} - (x-8)^{-1} + c$$

(ans)

Definite Integrals:

(1) $$\int_1^3 2x^3\, dx = 2\int_1^3 x^3\, dx$$

$$= 2\, \frac{\left[x^{3+1} \right]_1^3}{3+1}$$

$$= \frac{\cancel{2}}{\cancel{4}_{\,2}} \left[x^4 \right]_1^3 = \frac{1}{2}\left[3^4 - 1^4 \right]$$

$= \frac{\cancel{2}}{\cancel{4}12} [x^{?}]_1 \ldots$

$= \frac{1}{2}[71 - 1]$

$= \frac{1}{2} \times 70 = 35$ (ans)

② $\int_1^3 (x^3 + x + 6)\,dx = \int_1^3 x^3\,dx + \int_1^3 x\,dx + 6\int_1^3 dx$

$= \left[\frac{x^4}{4}\right]_1^3 + \left[\frac{x^2}{2}\right]_1^3 + 6[x]_1^3$

$= \frac{3^4 - 1^4}{4} + \frac{3^2 - 1^2}{2} + 6(3-1)$

$= \frac{81-1}{4} + \frac{8}{2} + 6\times 2$

$= 20\ \cancel{\frac{80}{4}} + 4 + 12 = 20 + 16$

$= 36$ (ans)

④ $\int_0^3 8x(2x^2+3)\,dx$

let $u = 2x^2 + 3$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$= \int \overset{2}{\cancel{8x}}\ u\ \frac{du}{\cancel{4x}} = \int 2u\,du$

$= \cancel{2}\left(\frac{u^2}{\cancel{2}}\right)$

$= \left[(2x^2+3)^2\right]_0^3$

$= (2\times(3)^2+3)^2 - (2\times 0^2+3)^2$

$$\begin{array}{r} 21 \\ \times\ 21 \\ \hline 21 \\ 42\times \\ \hline 441 \end{array}$$

$$= \left(2 \times (3)^2 + 3\right)^2 - \left(2 \times 0^2 + 3\right)^2$$

$$= (18+3)^2 - 3^2$$

$$= 21^2 - 9 = 441 - 9$$

$$= 432 \text{ (ans)}$$