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Measures of Dispersion

- **Absolute Measure**
  1. Range
  2. Quartile Deviation
  3. Mean Deviation
  4. Standard Deviation
- **Relative Measure**
  1. Coefficient of variation (C.V.)
  2. Coefficient of Quartile Deviation
  3. Coeff of Mean Dev.

Formulas of Absolute Measures of Dispersion:

(1). Range = Maximum value − Minimum value

(2). Quartile Deviation (Q.D) = $\dfrac{Q_3 - Q_1}{2}$

(3) Mean Deviation (M.D.) = $\dfrac{1}{n} \Sigma |x_i - \bar{x}|$

(4) Stander Deviation (S.D) or $\sigma = \sqrt{\dfrac{1}{n} \Sigma (x_i - \bar{x})^2}$

and variance $= \sigma^2 = \dfrac{1}{n} \Sigma (x_i - \bar{x})^2$

Note: (3) and (4) ⇒ formula for data without frequency.

→ if $x_1, x_2, \dots, x_n$ have corresponding frequencies so if $x_1, x_2, \ldots, x_n$ have corresponding frequencies $f_1, f_2, \ldots f_n$ such that $N = \sum_{i=1}^{n} f_i$

then $M.D(\bar{x}) = \frac{1}{N} \sum_{i=1}^{n} |x_i - \bar{x}| f_i$

or, $M.D \text{ about Median} = \frac{1}{N} \sum_{i=1}^{n} |x_i - M_e| f_i$

and $S.D, \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{n} (x_i - \bar{x})^2 f_i}$

Alternatively: Variance, $\sigma^2 = \frac{1}{N} \Sigma (x_i - \bar{x})^2 f_i$

or, $\sigma^2 = \frac{1}{N} \Sigma x_i^2 f_i - (\bar{x})^2$

(i) without frequency:

variance, $\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$= \frac{1}{n} \sum_{i=1}^{n} (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$

$\boxed{\bar{x} = \frac{1}{n} \Sigma x_i \quad \therefore \Sigma x_i = n\bar{x}}$

$= \frac{1}{n} \Sigma x_i^2 - \frac{2\bar{x}}{n} \Sigma x_i + \frac{1}{n} \sum_{i=1}^{n} \bar{x}^2$

$= \frac{1}{n} \Sigma x_i^2 - 2\bar{x} \cdot \bar{x} + \frac{1}{\cancel{n}} \cancel{n} \bar{x}^2$

$\sigma^2 = \frac{1}{n} \Sigma x_i^2 - 2\bar{x}^2 + \bar{x}^2$

$\therefore \boxed{\sigma^2 = \frac{1}{n} \Sigma x_i^2 - \bar{x}^2}$

and $S.D, \sigma = \sqrt{\frac{1}{n} \Sigma x_i^2 - \bar{x}^2}$

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In case of with frequency, $SD(\sigma) = \sqrt{\frac{1}{N}\Sigma x_i^2 f_i - \bar{x}^2}$

Question: Calculate Quartile Deviation from the following:

Class intervals:	10-15	15-20	20-25	25-30	30-40	40-50	50-60	60-70	Total
Frequency:	4	12	16	22	10	8	6	4	82

Soln:

Class Boundary	Commulative Frequency (Less than)
10	0
15	4
20	16 ← 20.5
Q_1 → 25	32
30	54 ← 61.5
Q_3 → 40	64
50	72
60	78
70	$\boxed{82 = N}$

Q_1 corresponds to $\frac{N}{4} = \frac{82}{4} = 20.5$

Q_3 corresponds to $\frac{3N}{4} = \frac{3 \times 82}{4} = 61.5$

using interpolation method:

$$\frac{Q_1 - 20}{25 - 30} = \frac{20.5 - 16}{32 - 16}$$

or, $\frac{Q_1 - 20}{5} = \frac{4.5}{16}$

or, $Q_1 = \frac{4.5}{16} \times 5 + 20$

or, $Q_1 = 21.407$

Again, $\frac{Q_3 - 30}{40 - 30} = \frac{61.5 - 54}{64 - 54}$

$\frac{Q_3 - 30}{\cancel{10}} = \frac{7.5}{\cancel{10}}$

$\boxed{Q_3 = 37.5}$

$\therefore$ Quartile Deviation,

$QD = \frac{Q_3 - Q_1}{2}$

$Q.D = \frac{37.5 - 21.4}{2}$

$= \frac{16.1}{2}$

$\boxed{Q.D = 8.05}$

Q2 Find the mean deviation of the following:

x	10	11	12	13	14	Total
f	3	12	18	12	3	48

N

$$MD = \frac{1}{N}\sum |x_i - \bar{x}| f_i$$

x	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$ $(\bar{x}=12)$	$\lvert x_i - \bar{x}\rvert f_i$
10	3	30	$\lvert -2\rvert = 2$	6
11	12	132	$\lvert -1\rvert = 1$	12
12	18	216	0	0
13	12	156	1	12
14	3	42	2	6
	$N = \sum f = 48$	$\sum x_i f_i = 576$		$\sum \lvert x_i - \bar{x}\rvert f_i = 36$

A.M, $\bar{x} = \frac{1}{N}\sum x_i f_i = \frac{576}{48} = 12$

$\therefore$ $M.D(\bar{x}) = \frac{1}{N}\sum |x_i - \bar{x}| f_i = \frac{36}{48} = 0.75$ (ans)

Properties of Standard Deviation (S.D.)

1. If all values of observations are same, then standard deviation of the set of data will be equal to 0.

Proof: Let $x_1, x_2 \ldots x_n$ (set of n observations) be equal to k, then,

mean of obs $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$= \frac{1}{n}\sum_{i=1}^{n} k$

$\bar{x} = \frac{1}{n} \cdot nk$

12

$\bar{x} = 1/n \cdot nk$

$\therefore SD = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$ $\quad \bar{x} = k$

$SD = \sqrt{\frac{1}{n}\Sigma(k-k)} = 0$ (Proved).

2. SD is independent of the change in origin but depends on the change in scale.

Proof: Let $y_i = a + bx_i$ (origin: a, scale: b), then from property of AM, $\bar{y} = a + b\bar{x}$

$y_i - \bar{y} = (a+bx_i) - (a+b\bar{x})$

$\frac{1}{n}\Sigma(y_i - \bar{y})^2 = \frac{1}{n}\Sigma\left\{(a+bx_i) - (a+b\bar{x})\right\}^2$

$\frac{1}{n}\Sigma(y_i - \bar{y})^2 = \frac{1}{n}\Sigma(\cancel{a}+bx_i - \cancel{a} - b\bar{x})^2$

$\frac{1}{n}\Sigma(y_i - \bar{y})^2 = \frac{1}{n}\Sigma(b(x_i - \bar{x}))^2$

$\frac{1}{n}\Sigma(y_i - \bar{y})^2 = \frac{1}{n}\cdot b^2\,\Sigma(x_i - \bar{x})^2$

$\sigma_y^2 = b^2\sigma_x^2$

$\boxed{\sigma_y = |b|\,\sigma_x}$ $\quad$ ($\because$ SD cannot be -ve)

3. SD calculated from two values x_1 and x_2 of a variable x is equal to half their difference.

(ie prove that $\sigma = \frac{1}{2}|x_1 - x_2|$)

Proof: We know by definition, $\sigma^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$

$$\therefore \sigma^2 = \frac{1}{2}\left[(x_1-\bar{x})^2 + (x_2-\bar{x})^2\right]$$

~~Since there~~ are two values x_1 and x_2

$$\therefore \bar{x} = \frac{x_1+x_2}{2}$$

then $x_1 - \bar{x} = x_1 - \frac{1}{2}(x_1+x_2) = \frac{2x_1 - x_1 - x_2}{2}$

$$\text{ie, } \boxed{x_1 - \bar{x} = \frac{x_1 - x_2}{2}}$$

and $x_2 - \bar{x} = x_2 - \frac{1}{2}(x_1+x_2) = \frac{2x_2 - x_1 - x_2}{2}$

$$\text{ie, } \boxed{x_2 - \bar{x} = \frac{x_2 - x_1}{2}}$$

$$\therefore \sigma^2 = \frac{1}{2}\left\{\left(\frac{x_1-x_2}{2}\right)^2 + \left(\frac{x_2-x_1}{2}\right)^2\right\}$$

$$\sigma^2 = \frac{1}{2}\left\{\frac{1}{4}\left[(x_1-x_2)^2 + (x_2-x_1)\right]^2\right\}$$

$$\delta^2 = \frac{1}{\cancel{2}} \times \frac{1}{4} \times \cancel{2}(x_1 - x_2)^2$$

$$\delta^2 = \frac{1}{4}(x_1 - x_2)^2$$

$$\therefore SD, \delta = \sqrt{\frac{1}{4}(x_1 - x_2)^2}$$

$$\boxed{\delta = \frac{1}{2}|x_1 - x_2|}$$ ($\because$ δ cannot be -ve).

(Proved).

4. What is the mean and S.D of the first 'n' natural numbers.

first 'n' natural numbers are,

$$x = 1, 2, 3, 4, \ldots, n$$

sum of 'n' natural number, $\sum_{x=1}^{n} x = \frac{n(n+1)}{2}$

$$\therefore \text{Mean, } \bar{x} = \frac{\cancel{n}(n+1)}{2 \cdot \cancel{n}}$$

$$\boxed{\bar{x} = \frac{n+1}{2}}$$

$\boxed{\bar{x} = \frac{n+1}{2}}$

$\sigma^2 = \frac{1}{n}\sum x^2 - \bar{x}^2$

$x^2 : 1^2 \quad 2^2 \quad 3^2 \dots n^2$

sum of first n (sq) number $\sum x^2 = \frac{n(2n+1)(n+1)}{6}$

$$\therefore \sigma^2 = \frac{1}{n}\sum x^2 - \bar{x}^2$$

$$\sigma^2 = \cancel{\frac{1}{n}} \times \frac{\cancel{n}(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$\sigma^2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$\sigma^2 = \frac{2(2n^2+n+2n+1) - 3(n^2+2n+1)}{12}$$

$$\sigma^2 = \frac{4n^2 + \cancel{2n} + \cancel{4n} + 2 - 3n^2 - \cancel{6n} - 3}{12}$$

Variance, $\sigma^2 = \frac{n^2-1}{12}$

$\therefore$ SD, $\sigma = \sqrt{\frac{n^2-1}{12}}$ (ans)