14 July 2023 20:11

Q. Consider a consumer living for 2 Periods - Pd 1, Pd 2. $u(c_1, c_2) = \log c_1 + \beta \log c_2$, $\beta \in (0,1)$. Suppose that the consumer earns a wage 'W' only in Pd 1 & nothing in Pd 2. Suppose the govt implements a scheme where an amt $\boxed{T \geq 0}$ is collected from agents in Pd 1 and given back in Pd 2. Find the utility maximizing level of T for the consumer. [savings earn interest @ r]

$u(c_1, c_2) = \log c_1 + \beta \log c_2, \beta \in (0,1)$

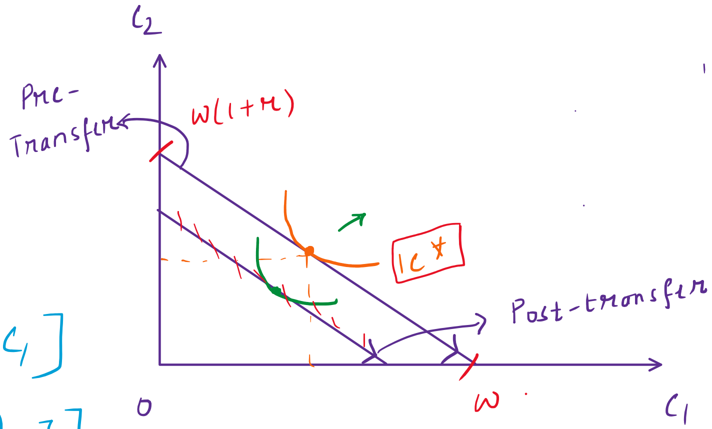
	Income	Consump	Savings
Pd 1	W (-T)	c_1	s_1
Pd 2	— (+T)	c_2	—

Pre-transfer:

B.L : $c_2 = 0 + (1+r) s_1$

$$c_2 = (1+r)[W - c_1]$$

$$c_1 + \frac{c_2}{1+r} = W \quad \text{--- (i)}$$



Post-Transfer:

B.L = $c_2 = T + (1+r) s_1$

$$c_2 = T + (1+r)[(W-T) - c_1]$$

$$c_2 = T + (1+r)[(W - c_1) - T]$$

$$c_2 = T + (1+r)[W - c_1] - (1+r)T$$

$$c_2 = \underbrace{(1+r)[W - c_1]} - \boxed{rT}$$

$r \geq 0, T \geq 0,$

Post transfer $c_2 \leq$ Pre-transfer c_2

Opt $T = 0$.

Q. Consider a community having a fixed amt of exhaustible resource X. The utility fn. of ...

... community having a fixed amt of exhaustible resource X. The utility fn of the community is $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$, δ = rate of discount, C_t = level of consumption by the community in pd 't'. Find the optimal level of consumption of the resource C_t^* for each period.

$$U = \delta^0 \ln C_0 + \delta^1 \ln C_1 + \delta^2 \ln C_2 + \cdots\cdots$$

$$= \ln C_0 + \delta \ln C_1 + \delta^2 \ln C_2 + \cdots\cdots \rightarrow \text{utility fn}$$

C_t : consumption level in pd 't'.

Constraint: $C_0 + C_1 + C_2 + \cdots\cdots = X \Rightarrow \sum_{t=0}^{\infty} C_t = X$. → BL

Max: $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$ s.t. $X = \sum_{t=0}^{\infty} C_t$.

$C_0, C_1, C_2, \ldots$

The Lagrangian:

$$\mathcal{L} = (\ln C_0 + \delta \ln C_1 + \delta^2 \ln C_2 + \cdots\cdots) + \lambda \left[X - \sum_{t=0}^{\infty} C_t \right]$$

$$\frac{\partial \mathcal{L}}{\partial C_0} = 0 \Rightarrow \frac{1}{C_0} + \lambda(-1) = 0 \Rightarrow \frac{1}{C_0} = \lambda.$$

$$\frac{\partial \mathcal{L}}{\partial C_1} = 0 \Rightarrow \delta \cdot \frac{1}{C_1} + \lambda(-1) = 0 \Rightarrow \frac{\delta}{C_1} = \lambda \Rightarrow \frac{\delta}{C_1} = \frac{1}{C_0} \Rightarrow C_1 = \delta C_0$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = 0 \Rightarrow \delta^2 \cdot \frac{1}{C_2} + \lambda(-1) = 0 \Rightarrow \frac{\delta^2}{C_2} = \lambda \Rightarrow \frac{\delta^2}{C_2} = \frac{1}{C_0} \Rightarrow C_2 = \delta^2 C_0$$

$\therefore$ In general:

$$\frac{\partial \mathcal{L}}{\partial C_K} = 0 \Rightarrow C_K = \delta^K C_0$$

$$B.L \Rightarrow X = \sum_{t=0}^{\infty} C_t = C_0 + C_1 + C_2 + \cdots\cdots$$

$$X = C_0 + \delta C_0 + \delta^2 C_0 + \cdots\cdots$$

$$X = C_0 [1 + \delta + \delta^2 + \cdots\cdots \infty]$$

Infinite GP

$a, ar, ar^2, \cdots\cdots \infty, |r| < 1$

$S = a + ar + ar^2 + \cdots \infty$

$= \frac{a}{\ldots}$ $|r| < 1$

$$X = c_0 \left[\underbrace{1 + \delta + \delta^2 + \cdots \infty}_{} \right] \qquad \begin{aligned} S &= a + ar + ar^2 + \cdots \infty \\ &= \frac{a}{1-r}, \ |r| < 1. \end{aligned}$$

$\times\delta \quad \times\delta$

$\hookrightarrow [a = 1, r = \delta]$

$$X = c_0 \cdot \frac{1}{1-\delta}$$

$$c_0^* = X(1-\delta)$$

$$\left.\begin{aligned} c_1 = \delta c_0 \Rightarrow c_1^* &= \delta c_0^* = \delta(1-\delta)X \\ c_2^* &= \delta^2 c_0^* = \delta^2(1-\delta)X \end{aligned}\right\}$$

$$\therefore c_K^* = \delta^K c_0^* = \delta^K(1-\delta)X$$

Soln: $c_0^* = (1-\delta)X, \quad c_K^* = \delta^K(1-\delta)X, \quad K = 1, 2, \ldots\ldots.$