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Relative measures of Dispersion

① Coefficient of variation (CV) = $\frac{S.d}{\bar{x}} \times 100$

② Coefficient of M.D = $\frac{M.D(\bar{x})}{\bar{x}} \times 100$

or $\frac{MD(Median)}{Median} \times 100$

③ Coefficient of Q.D = $\frac{Q.D}{Median} \times 100$

Q1 For some financial statistics, it is found that the monthly average Electricity Charges was Rs 2460 and S.D Rs 120.
The monthly average Direct wages was Rs 42000 and SD was Rs 1200.
State which is the more variable?

Soln

	Electricity Charges	Direct wages
Mean	$\bar{x}_A = 2460$	$\bar{x}_B = 42000$
S.d	$\delta_A = 120$	$\delta_B = 1200$

Coeff of variation of Electricity Charges $(CV_A) = \frac{\delta_A}{\bar{x}} \times 100$

$= \frac{120}{246\cancel{0}} \times 10\cancel{0}$

$CV_A = \frac{1200}{246} = 4.9$

Coefficient of variation of Direct monthly wage $(CV_B) = \frac{\sigma_B}{\bar{x}_B} \times 100$

$= \frac{120\cancel{0}}{42\cancel{000}} \times 100$

$= \frac{120}{42} = 2.9$

$\therefore CV_A > CV_B$

$\Rightarrow$ electricity charges are more variable than monthly wages.

Q2 : You are given two variables A and B. Using Quartile Deviation, state which is more variable.

A

$72.25 \times 3 = 216.75$

Midpoints	Frequency	cf
15	15	15
20	33	48
25	56	104
30	103	207
35	40	247
40	32	279
45	10	289 = N
	289 = N	

$Q_1 \rightarrow 25$

$Q_2 \rightarrow$ median 30

$Q_3 \rightarrow 35$

$N/4 = 72.25$

$N/2 = 144.5$

$3N/4 = 216.75$

B

Mid points	Frequency	cf
100	340	
150	492	
200	890	
250	1420	
300	620	
350	360	
400	187	
450	140	

$QD = \frac{Q_3 - Q_1}{...}$ and Coeff of $QD = \frac{QD}{...} \times 100$

$$Q.D = \frac{Q_3 - Q_1}{2} \quad \text{and coeff of } QD = \frac{QD}{\text{Median}} \times 100$$

$$\text{Median} = Q_2$$

$Q_3 \longrightarrow$ corresponds to $\frac{3N}{4}$

$Q_2 \longrightarrow \ldots\ldots \frac{2N}{4} = N/2$

$Q_1 \longrightarrow \ldots\ldots N/4$

$\hookrightarrow \frac{289}{4} = 72.25$

Q2 In a batch of 10 children, the I.Q of a dull boy is 36 below the average I.Q. of the other children. Show that the standard deviation of I.Q. for all the children cannot be less than 10.8. If this standard deviation is actually 11.4, determine what the s.d will be when the dull boy is left out?

$\boxed{\sigma \geq 10.8}$

Soln: $N = 10$

Group I $\rightarrow$ 9 children (not dull)

$\hookrightarrow n_1$

$\bar{x}_1 = a$ (let)

$\sigma_1 =$ s.d of grp 1.

Group II $\rightarrow$ 1 dull boy

$\bar{x}_2 = a - 36$

$\sigma = 0$

combined mean,

$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{\ldots} = \frac{9a + (a-36)}{\ldots}$

$x_2 = a - 36$

Combined mean,

$\bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{N} = \frac{9a + (a-36)}{10} = a - 3.6$ $\quad \sigma_2 = 0$

Using the formula for composite / combined variance

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{(n_1+n_2)} + \frac{n_1 d_1^2 + n_2 d_2^2}{(n_1+n_2)}$$

Here, $d_1 = \bar{x}_1 - \bar{x} = a - (a - 3.6)$

$= 3.6$

$d_2 = \bar{x}_2 - \bar{x} = (a-36) - (a - 3.6)$

$= -32.4$

$$\therefore \sigma^2 = \frac{9\sigma_1^2 + 0}{10} + \frac{9(3.6)^2 + (-32.4)^2}{10}$$

$10\sigma^2 = 9\sigma_1^2 + 1166.4$

$\sigma^2 = 0.9\sigma_1^2 + 116.64$

comparing both sides $\quad \sigma^2 \geq 116.64$

or $\quad \sigma \geq \sqrt{116.64}$

or $\quad \sigma \geq 10.8$

(Proved).

$\therefore$ if $\sigma = 11.4$

then $10 \times (11.4)^2 = 9\sigma_1^2 + 1166.4$

or, $\quad \sigma_1^2 = \frac{10 \times (11.4)^2 - 1166.4}{9}$

$$\overline{\quad} \qquad 9$$

or, $\sigma_1^2 = 14.8$

$\therefore \sigma_1 = \sqrt{14.8} = 3.85$ (ans)

Q What is the mean and s.d. of the first 'n' natural numbers?

$1, 2, \ldots, n$

Sum of first 'n' natural numbers $\Sigma x = 1+2+\cdots+n$

$$= \frac{n(n+1)}{2}$$

$$\therefore \bar{x} = \frac{1}{n}\Sigma x = \frac{1}{\cancel{n}} \times \frac{\cancel{n}(n+1)}{2}$$

$$= \frac{n+1}{2}$$

$$\Sigma x^2 = 1^2+2^2+\cdots+n^2 = \frac{n(2n+1)(n+1)}{6}$$ ✓

$$\therefore \text{variance } (x) = \frac{1}{n}\Sigma x^2 - \bar{x}^2$$

$$= \frac{\cancel{n}(2n+1)(n+1)}{\cancel{n}6} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{2n^2+2n+n+1}{6} - \frac{(n+1)^2}{4}$$

$$= 2n^2+3n+1 - \frac{(n^2+1+2n)}{}$$

$$= \frac{2n^2+3n+1}{6} - \frac{(n^2+1+2n)}{4}$$

$$= \frac{8n^2+\cancel{12n}+4-6n^2-6-\cancel{12n}}{24}$$

$$= \frac{2n^2-2}{24} = \frac{n^2-1}{12}$$

$$\therefore s.d = \sqrt{v(x)} = \sqrt{\frac{n^2-1}{12}} \text{ (ans).}$$

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Moments, skewness and Kurtosis

Moments

- Raw moments $(\mu_r' \text{ or } m_r')$ (deviation about any arbitrary constant A)
- Central moments $(\mu_r \text{ or } m_r)$ (deviation about mean, $\bar{x}$)

μ_r' or m_r' is the rth order raw moment. $= \frac{1}{n}\sum(x_i-A)^r$

when $r=1$

$$m_1' = \frac{1}{n}\sum(x_i-A)$$

$$m_2' = \frac{1}{n}\sum(x_i-A)^2$$

if $A=0$, then

$$m_0' = \frac{1}{n}\sum(x_i-0)^0 = 1$$

$$m_1' = \frac{1}{n}\sum x_i = \bar{x}$$

$$m_2' = \frac{1}{n}\sum x_i^2$$

$m_2' = \frac{1}{n} \Sigma (x_i - A)^2$

$m_3' = \frac{1}{n} \Sigma (x_i - A)^3$

$m_2' = \frac{1}{n} \Sigma x_i^2$

$\vdots$

and μ_r or m_r is the rth order central moment

$= \frac{1}{n} \Sigma (x_i - \bar{x})^r$

$m_1 = \frac{1}{n} \Sigma (x_i - \bar{x})^1 = 0$

$m_2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2 = \sigma^2 =$ variance

$m_3 = \frac{1}{n} \Sigma (x_i - \bar{x})^3$

$m_4 = \frac{1}{n} \Sigma (x_i - \bar{x})^4$

$\vdots$