

Integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$Q1. \int x^5 dx = \frac{x^6}{6} + c$$

$$\sqrt{\quad} \quad \frac{1}{2}$$

$$\sqrt[3]{\quad} \quad \frac{1}{3}$$

$$\sqrt[5]{\quad} \quad \frac{1}{5}$$

$$Q2. \int x^{-2} dx = \frac{x^{-1}}{-1} + c$$

$$= -\frac{1}{x} + c$$

$$Q3. \int \frac{1}{x^5 \sqrt{x}} dx = \int \frac{1}{x \cdot x^{5/2}} dx$$

$$= \int \frac{1}{x^{9/2}} dx$$

$$= \int x^{-6/5} dx$$

$$= \frac{x^{-6/5+1}}{-6/5+1} + c$$

$$= \frac{x^{-1/5}}{1/5} + c$$

$$x^m \cdot x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$a^m \cdot b^m = (ab)^m$$

$$= \frac{5}{x^{1/5}} + c$$

$$= \frac{5}{\sqrt[5]{x}} + c \quad (\text{ans})$$

$$Q4. \int \frac{x+4}{3x\sqrt{x}} dx = \int \frac{x+4}{3x^{3/2}} dx$$

$$= \int \frac{x}{3x^{3/2}} dx + \int \frac{4}{3x^{3/2}} dx$$

$$= \frac{1}{3} \int x^{-1/2} dx + \frac{4}{3} \int x^{-3/2} dx$$

$$\begin{aligned}
 &= \frac{1}{3} \int x^{-1/2} dx + \frac{4}{3} \int x^{-3/2} dx \\
 &= \frac{1}{3} \left[\frac{x^{1/2}}{1/2} \right] + \frac{4}{3} \left[\frac{x^{-1/2}}{-1/2} \right] + C \\
 &= \frac{2}{3} \sqrt{x} - \frac{8}{3\sqrt{x}} + C
 \end{aligned}$$

(ii) $\int \frac{1}{x} dx = \log |x| + C$ (iii) $\int e^{mx} dx = \frac{e^{mx}}{m} + C$

(iv) $\int a^{mx} dx = \frac{a^{mx}}{m \log_e a} + C$

Q5. evaluate: $\int \frac{3 - 2x^{5/2} + \sqrt{x}}{x \sqrt{x^3}} dx$

$$= \int \frac{3 - 2x^{5/2} + \sqrt{x}}{x^1 x^{3/4}} dx$$

$$\left(\frac{\sqrt{x^3}}{x^{3/4}} \right)$$

$$= \int \frac{3 - 2x^{5/2} + x^{1/2}}{x^{7/4}} dx$$

$$= 3 \int \frac{1}{x^{7/4}} dx - 2 \int \frac{x^{5/2}}{x^{7/4}} dx + \int \frac{x^{1/2}}{x^{7/4}} dx$$

$$\begin{aligned}
 &\int x^{5/2 - 7/4} \\
 &\int x^{\frac{10-7}{4}} = \int x^{3/4} \\
 &- 7/4
 \end{aligned}$$

$$\begin{aligned}
 &= 3 \frac{x^{-7/4+1}}{\left(\frac{-7}{4}+1\right)} - 2 \frac{x^{3/4+1}}{\left(\frac{3}{4}+1\right)} + \frac{x^{-3/2+1}}{\left(\frac{-3}{2}+1\right)} + C
 \end{aligned}$$

$$\begin{aligned}
 & \int x^{11/5} \cdot x^{-7/4} dx \\
 & = \int x^{4-35/20} dx \quad \left(x^{-31/20} \right) \\
 & = \int x^{(7/4)+1} dx \quad \left(\frac{7}{4}+1 \right) \\
 & = \int x^{(3/4)+1} dx \quad \left(\frac{3}{4}+1 \right) \\
 & = \int x^{(-31/20)+1} dx \quad \left(-\frac{31}{20}+1 \right) \\
 & = \frac{1}{3} x^{-4/4} - \frac{4}{7} x^{-3/4} - 2 \times \frac{4}{7} x^{3/4} - \frac{20}{11} x^{-11/20} + c \\
 & = \frac{1}{3} x^{-1} - \frac{4}{7} x^{-3/4} - \frac{8}{7} x^{3/4} - \frac{20}{11} \frac{1}{x^{11/20}} + c \\
 & \quad \text{(ans)}
 \end{aligned}$$

$$\textcircled{6} \int (e^{5x} + 2x^{3/2} - 5x^{-1}) dx$$

$$= \int e^{5x} dx + 2 \int x^{3/2} dx - 5 \int \frac{1}{x} dx$$

$$= \frac{e^{5x}}{5} + 2 \left(\frac{x^{5/2}}{5/2} \right) - 5 \log|x| + c$$

$$= \frac{1}{5} e^{5x} + \frac{4}{5} x^{5/2} - 5 \log|x| + c$$

$$\begin{aligned}
 & \frac{x^{3/2+1}}{3/2+1} \\
 & = \frac{x^{5/2}}{5/2} \\
 & = 2 \cdot \frac{x^{5/2}}{5} \\
 & = \frac{2 \times 2}{5} x^{5/2} \\
 & = \frac{4}{5} x^{5/2}
 \end{aligned}$$

$$\textcircled{7} \int (3^{5x} + 2^{3x} - 10^{-x}) dx$$

$$= \frac{3^{5x}}{5 \log_e 3} + \frac{2^{3x}}{3 \log_e 2} + \frac{10^{-x}}{1 \log_e 10} + c$$

$$\textcircled{8} \int \frac{2x^3 - x^2 + 5x + 3}{(2x-3)} dx$$

$$= \int \frac{2x^3 - 3x^2 + 2x^2 - 3x - 2x + 3}{(2x-3)} dx$$

$$= \int \frac{x^2(2x-3) + x(2x-3) - 1(2x-3)}{(2x-3)} dx$$

$$= \int \frac{(x^2 + x - 1)(2x-3)}{(2x-3)} dx = \frac{x^3}{3} + \frac{x^2}{2} - x + c \quad (\text{ans})$$

TRY:

$$\int \frac{1}{\sqrt{x}} \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^3 dx$$

$$\int \frac{2x^{4/5} - 3x \sqrt[4]{x^3} + 5x^2 \sqrt{x^2}}{7x \sqrt{x^3}} dx$$