Statistics

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BSTM 2026

ISS

Stat ophul

Sequence of Random variables

Session → II

Chebyshev's Inequality

ISL

MB vs MC

1 - 3

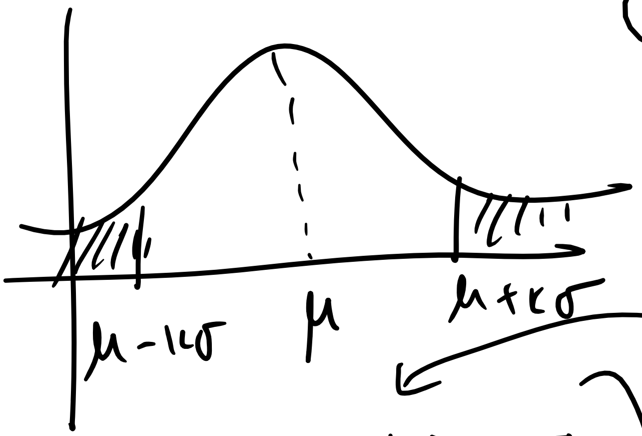
2 1-0

$X \quad \mu, \sigma^2$

$$P\left[|X-\mu| \geq k\right] \leq \frac{\sigma^2}{k^2} \quad \forall k \in \mathbb{R}^+$$

2 tailed

(i) Distribution
(ii) Inequality
(iii) CI



UB

LB

$$1 - P\left[|X-\mu| \geq k\sigma\right] \leq \frac{1}{k^2}$$

$$- \frac{1}{k^2}$$

$$P\left[|X-\mu| < k\sigma\right] \geq 1 \quad k$$

All dist ISS

ISI MStat

JAM Stat

Gate Stat

PUMDET

PUMDEI
Shp

$k > 0$. $P[|x-\mu| \geq k\sigma] \leq \frac{1}{k^2}$

$k \to \frac{k}{\sigma} \to P[|x-\mu| \geq k] \leq \frac{\sigma^2}{k^2}$

alternating forms

$P[|x-\mu| \geq r\cdot\sigma] \leq \frac{1}{r^2}$

$>$ $\geq 1 - 1/r^2$

$<$ $\geq 1 - 1/r^2$

$\leq$

$P[\mu - r\sigma < x < \mu + r\cdot\sigma] \geq 1 - \frac{1}{r^2}$

$\leq \quad \leq \quad \geq 1 - \frac{1}{r^2}$

$\langle x_n \rangle$ iid

for $\bar{X}_n \to P[|\bar{x} - \mu| < \epsilon] \geq 1 - \frac{\sigma^2}{\epsilon^2 n}$

Qm

$f(x) = 630x^4(1-x)^4 \quad 0 < x < 1$

o/w

θ $f(x) = 630\, x^4 (1-x)^4$ $\quad 0 < x < 1$

$= 0$ $\quad$ o/w

Find the exact value of $\left[|X-\mu| \leq 2\sigma\right]$

Approx value using CB Inequality

$E(x) = \int_0^1 x f(x) dx$

$\int_0^1 630\, x^5 (1-x)^4 dx$ → Beta Gamma fn

$= 630 \cdot \frac{5!\,4!}{(5+4+1)!}$

$= 630 \times \frac{2880}{3628800} = 0.5$

$V(X) = \int_0^1 x^2 f(x) dx - \mu_x^2$

$= \int_0^1 630\, x^6 (1-x)^4 dx - 1/4$

$= 630 \cdot \frac{6!\,4!}{11!} - \frac{1}{4}$

$= 1/44$

$\sigma = 0.15$

$P\left[|X-\mu| \leq 2\sigma\right]$

$\Rightarrow P\left[|X-0.5| \leq 0.3\right]$

$\Rightarrow P\left[0.2 \leq X \leq 0.8\right]$

$\int^{0.8} x^4 (1-x)^4 dx$

$$\Rightarrow \int_{0.2}^{0.8} 630x^4(1-x)^4\,dx$$

$$\Rightarrow 0.96$$

If we have CB inequality

$$P(|x-\mu| \le 2\sigma) \ge 1-1/4 = 0.75$$

the P is at least 0.75

CB inequality

0.96

0.90

ISI 2014

Prob that X lies b/w $\mu - k\sigma$ & $\mu + k\sigma$ is ≥ 0.95

for k ?

$f_m \cdot K$!

$$P\left[|X-\mu| < K\sigma\right] \geq 1-\frac{1}{K^2}$$

$$P\left[\mu-K\sigma < X < \mu+K\sigma\right] \geq 1-\frac{1}{K^2}$$

$$1-\frac{1}{K^2} \geq 0.75$$

$$\frac{1}{K^2} \leq 0.05$$

$$K^2 \geq 20 \qquad K \geq \sqrt{20}$$

(K)

$$\mu=0 \quad \sigma^2=4$$

$$P\left[|x| \geq 8\right]$$

$$P\left[|x-\mu| > K\right] \leq \frac{V(x)}{K^2}$$

$$E(x)=\mu=0$$

$$P\left[|x| \geq 8\right] = P\left[|x-0| \geq 8\right]$$

$$P\left[\right. \qquad \leq V(x)$$

$$\le \frac{V(X)}{8^2}$$

$$= \frac{4}{64} = \frac{1}{16}$$

(*) 2D RV

$E(X) = E(Y) = 3$

$V(X) = V(Y) = 1$

$Cov(X,Y) = 1/2$

$P(|X-Y| \ge 6) = ?$

An

$E(X-Y) = E(X) - E(Y) = 0$

$V(X-Y) = V(X) + V(Y) - 2Cov(X,Y)$

$= 1 + 1 - \frac{2}{2} = 1$

$$P\left[\left|(X-Y) - E(X-Y)\right| \ge 6\right] \le \frac{V(X-Y)}{6^2}$$

$$= \frac{1}{6^2} = \frac{1}{36} < \frac{1}{6}$$

Markov

~~Only negative Values~~

nt

$$P[X \geq a] \leq \frac{E(X)}{a} \quad \forall\ a \in R^+$$

(*) $X \to$ Exponentially distribution with parameter λ

Find upper Bound for $P(X \geq a)$

$E(X) = \frac{1}{\lambda} \qquad V(X) = \frac{1}{\lambda^2}$

using Markov inequality $\quad P(X \geq a) \leq \frac{E(X)}{a} = \frac{1}{\lambda a}$

$$P(X \geq a) = e^{-\lambda a} \Rightarrow \frac{1}{\lambda a}$$

$$\frac{1}{\lambda a} \geq e^{-\lambda a}$$

using cdf of Exponential distribution

$X \to$ B R V.

Q $X \to$ B R V

$$\alpha = P(|X - np| \geq \sqrt{n})$$

which holds for:

$0 < \alpha \leq 1/4$, $1/4 \leq \alpha \leq \frac{1}{2}$, $\frac{1}{2} \leq \alpha \leq \frac{3}{4}$, $3/4 \leq \alpha \leq 1$

$$P(|X - \mu| \geq k) \leq \frac{V(X)}{k^2}$$

$$\alpha = P(|X - np| \geq \sqrt{n}) \leq \frac{V(X)}{n} \Rightarrow p(1-p)$$

$\alpha = \phi(p)$

$\phi(p) = p - p^2$

$$\frac{d\,\phi(p)}{dp} = 1 - 2p = 0 \quad \boxed{p = \frac{1}{2}}$$

$1 - 2p \quad 2p > 1$

$p \geq 1/2$

p attains max when $0 \leq \alpha \leq \phi(\frac{1}{2}) = 1/4$

WLLN $\leftarrow$ Chebyshev The

Theorem

Them

X_i iid → Sample mean behav

$E[X_i]$

↓

Seqnce of R.V. iid

n ↑ Sample mean → Population mean

1000 5 → 10 → 20 → 820

1sc

Strong law of large number

Adv

w.p.

$p = 1$

Sample mean of iid

↓

Converges almost surely to the population mean

↓

$\boxed{n \to \infty}$

σ |

→

$n \to$ finite

Population $\to \infty$

$n \to \infty$

1000 10^{1000} $10^{10^{1000}}$

n n n

Drinking habit

drinking water

water + ice + lemon + ...

90ml   120ml

$$\boxed{Y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n}$$