Sequences & Series Question

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Progression

90623-95123

AP GP HP

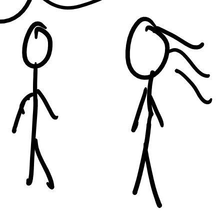
AM GM HM

$\frac{a+b+c}{3}$ $\sqrt[3]{abc}$ $\frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

HM is used for Ratio Based things

$\frac{a}{b}+\frac{c}{d}+\frac{e}{f}=\frac{a+c+e}{b+d+f}$ X

5 km/hr



8 km/hr

$\frac{K}{a}+\frac{K}{b}$

$\frac{2K}{2h}$

Average Speed $\frac{5+8}{2}=6.5$ X

HM $\frac{2}{\frac{1}{a}+\frac{1}{b}}=\frac{2ab}{a+b}$

Why 5 $\frac{K}{4}$

$=\frac{2\cdot5\cdot8}{5+8}=\frac{80}{13}$

$\Rightarrow 6.1$

Like AM HM also lies between the 2 numbers but not exactly the same point

Whats App..

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3 friends are walking

4, 5, 6 → find average speed?

AM ≠ 5

3abc

in $F(5)$

$$\frac{3abc}{ab+bc+ca} \rightarrow \frac{3.5.4.6}{20+30+24} = ?$$

4 friends ??

$$\frac{4abcd}{(abc+abd+cda+cdb)}$$

$$\frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

AHP ..

$$1+2+\cdots n = \frac{n(n+1)}{2} = \frac{n^2}{2}+\frac{n}{2}$$

$$1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{(n^2+n)(2n+1)}{6} = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}$$

$$1^3+2^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2(n^2+2n+1)}{4} = \frac{n^4}{4}+\frac{n^3}{2}+\frac{n^2}{4}$$

$$1^4+2^4+\cdots+n^4 \Rightarrow ! \quad n^5/5+n^4/2+$$

$$1^5+2^5+\cdots+n^5 \Rightarrow ?$$

$$1+2+\cdots+n = \frac{n(n+1)}{2} \qquad \frac{n}{2}[2.1+(n-1).1] = \frac{n}{2}[2+n-1]$$

n odd → $1+3+5+\cdots \Rightarrow \frac{n}{2}[2.1+(n-1).2] \Rightarrow \frac{n(n+1)}{2}$

$$\Rightarrow \frac{n}{2}[2+2n-2]$$

$$\Rightarrow \frac{n}{2}\left[2 + 2n - 2\right]$$
$$= n^2$$

$$1^2 + 2^2 + \cdots + n^2 \Rightarrow \sum n^2 = \frac{n(n+1)(2n+1)}{6}$$

$r \Rightarrow 1$ to 1

$\sum r = 1 + 2 + \cdots + n = \sum n$

We know, $r^3 - (r-1)^3 = 3r^2 - 3r + 1$

$r \neq 0$

$$\sum_{r=1}^{n}\left[r^3 - (r-1)^3\right] = 3\sum r^2 - 3\sum r + \sum 1$$

$$n^3 - 0^3 = 3\sum n^2 - 3\sum n + n$$

$$n^3 = 3\sum n^2 - 3\cdot\frac{n(n+1)}{2} + n$$

$$\sum n^2 = \frac{1}{3}\left[n^3 + \frac{3n(n+1)}{2} - n\right]$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$1^4 + 2^4 + \cdots + n^4 \Rightarrow ? \quad \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$1 + 16 + 81 + 256 \Rightarrow 354 \qquad \frac{2\cdot 4\cdot 5\cdot 9\,(48 + 12 - 1)}{30}$$

$$\Rightarrow 6(59) \rightarrow 354$$

$$r^5 - (r-1)^5 = r^5 - (r^5 - 5C_1 r^4 + 5C_2 \cdots)$$

$$r^5 - (r^5 - 5C_1 r^4 + 5C_2 r^3 - 5C_3 r^2 + 5C_4 r - 5C_5)$$

$$\Rightarrow 5\sum n^4 - 10\sum n^3 + 10\sum n^2 - 5\sum n + \sum 1$$

$5\sum n^4$ ...

$\Rightarrow 5\sum n^4 - 10\sum n^3 + 10\sum n^2 - 5\sum n + \sum 1$

$n^5 = 5\sum n^4 - 10\sum n^3 + 10\sum n^2 - 5\sum n + n$

$n^5 = 5\sum n^4 - 10\frac{n^2(n+1)^2}{4} + 10\cdot\frac{n(n+1)(2n+1)}{6} - \frac{5n(n+1)}{2} + n$

$5\sum n^4 = n^5 + \frac{10}{4}n^2(n+1)^2 + \frac{10}{6}n(n+1)(2n+1) - \frac{5}{2}n(n+1) + n$

Subjective

$T_n = an^3 + bn^2 + cn + d$

$S_n = a\sum n^3 + b\sum n^2 + c\sum n + \sum d$

$1^2 + 3^2 + 5^2 + \dots$

$1 + 9 + 25 + 49 + 81$

8, 16, 24, 32

8, 8

2, 8, 34

1, 3, 5 ...

$(2n-1)$

$n=1$, $n=2$

$2n^2$: $2\cdot1^2 = 2$, $2\cdot2^2 = 8$, $2\cdot3^2 = 18$, $2\cdot4^2 = 32$; 6, 10, 14; 4, 4

$T_n = [1 + (n-1)2]^2 = (2n-1)^2 = (4n^2 - 4n + 1)$

@ the end difference of the differences will be zero.

$\sum T_n = \sum(4n^2 - 4n + 1)$

$S_n = \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n$

$$S_n = \quad \frac{\quad}{6} \qquad 2$$

$$= \frac{n(4n^2-1)}{3}$$

What if there is NO Relation???

$(1+5+12+22+35+\cdots\cdots)$

$\quad 4 \quad 7 \quad 10 \quad 13 \quad \cdots$

$$S_n = 1+5+12+22+35+\cdots + T_{n-1}+T_n \quad (1)$$

$$S_n = 0+1+5+12+22+\cdots\cdots + (T_{n-2}+T_{n-1}) \cdots T_{n-1}+T_n$$

Subtracting (ii) from (i)

$$0 = 1+4+7+10+13+\cdots+(T_n - T_{n-1}) - T_n$$

$$T_n = 1+4+7+10+13 \cdots n \text{ term}$$

$$\sum T_n = \frac{n}{2}\{2 \cdot 1+(n-1)3\} = \sum \frac{1}{2}(3n^2-n)$$

$$= \frac{3\sum n^2}{2} - \frac{\sum n}{2}$$

$$= \frac{3}{2}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{1}{2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{n(n+1)}{2}(2n+1-1)$$

$$= \frac{1}{2}n^2(n+1) = \frac{1}{2}(n^3+n^2)$$

HW $1+3+7+15+31+\ldots$

find max value x^3y^4.

If $2x+3y=7$ $x\geq 0 \,||\, y\geq 0$

Lagrangian Multiplier Method..

$(x)(x)(x)(y)(y)(y)(y)$

x repeats 3 times   y repeats 4 times

$3\left(\frac{2x}{3}\right)+4\left(\frac{3y}{4}\right)=7$

or, $\frac{2x}{3}+2x/3+2x/3+3y/4+3y/4+3y/4+3y/4=7$

1 2 3 4 5 6 7

$k=7$ $n=7$

Greater value of $\left(\frac{2x}{3}\right)^3\left(\frac{3y}{4}\right)^4 \rightarrow \left(\frac{7}{7}\right)^7$

max value $\rightarrow \frac{2^3}{3^3}\cdot\frac{3^4}{4^4}\; x^3y^4 \rightarrow (1)$

AM $\geq$ GM

$\frac{2x}{3}\;\frac{2x}{3}\;\frac{3y}{4}$

$$\frac{2x}{3} \quad \frac{2x}{3} \quad \frac{3y}{4}$$

$$\frac{2x}{3}+\frac{2x}{3}+\frac{3y}{4} \geq \left[\left(\frac{2x}{3}\right)\left(\frac{2x}{3}\right)\left(\frac{3y}{4}\right)\right]^{\frac{1}{3}}$$

Partial derivative

$$L = x^3y^4 + \lambda\left[7-2x-3y\right]$$

$$L_x = 3x^2y^4 - 2\lambda = 0$$

$$L_y = 4y^3x^3 - 3\lambda = 0$$

$$L_\lambda = 7-2x-3y = 0$$

$$\frac{3x^2y^4}{2} = \frac{4y^3x^3}{3}$$

$$\frac{3y}{2} = \frac{x}{3}$$

$$\frac{9y}{2} = x$$

$$7 - 2\cdot\frac{9y}{2} - 3y = 0$$

$$\frac{7}{10} = y$$

$$x = \frac{9}{2}\left(\frac{7}{10}\right) = \frac{63}{20}$$

note max value $\Rightarrow \leq$

min value $\geq$

Whenever max/min subject to a constraint

$$(f(x,y)) + \lambda\left[\text{Some Constraint in (Sided form)}\right]$$

max $(x)(y)$ subject to $x+y=10$

$1,9 \to 9$

$2,8 \to 16$

$3,7 \to 21$

25

$z = xy + \lambda[10 - x - y]$

$z_x = y - \lambda = 0$ $x = y$

$z_y = x - \lambda = 0$ $x = y = 5$

$z_\lambda = 10 - x - y = 0$

3, 7 → 21
4, 6 → 24
5, 5 → 25

$x^2 y^2$

Sequences & Series ...

$3333 = 3(1111)$

4. Let S_n $(1 \le n \le 9)$ denotes the sum of n terms of the series $1 + 22 + 333 + \ldots + \underbrace{999\ldots9}_{9 \text{ times}}$, then for $2 \le n \le 9$

(a) $S_n - S_{n-1} = \frac{1}{9}(10^n - n^2 + n)$

(b) $S_n = \frac{1}{9}(10^n - n^2 + 2n - 2)$

(c) $9(S_n - S_{n-1}) = n(10^n - 1)$

(d) None of the above

$1 + 22 + 333$

$S_3 - S_2$
$= 1 + 22 + 333 - 1 - 22$
$= 333$

$S_n - S_{n-1} = nnnn\ldots n$
$= n(111\ldots1)$ (n times)
$= n(10^{n-1} + 10^{n-2} + \ldots + 10 + 1)$

$S_n - S_{n-1} = n\left(\frac{10^n - 1}{10 - 1}\right)$

$9(S_n - S_{n-1}) = n(10^n - 1)$

6. Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots$ is equal to

(a) $2^n - n - 1$ (b) $1 - 2^{-n}$

(c) $n + 2^{-n} - 1$ (d) $2^n - 1$

$= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+\ldots$

$= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{2^2}\right)+\left(1-\frac{1}{2^3}\right)+\left(1-\frac{1}{2^4}\right)+\ldots+\left(1-\frac{1}{2^n}\right)$

$= n-\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2^{n-1}}\right)$

$= n-\frac{1}{2}+\frac{1\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\frac{1}{2}\right)}$

$= n-1+\frac{1}{2^n} = n-1+2^{-n}$

10. If $a(b-c)x^2 + b(c-a)xy + c(a-b)y^2$ is a perfect square, the quantities a, b, c are in

(a) AP (b) GP

(c) HP (d) None of these

$b^2(c-a)^2 - 4a(b-c)\,c(a-b)$

$b^2(c-a)^2 = 4a(b-c)\,c(a-b)$

$= 4ac(a-b)(b-c)$

$[a(b-c) + c(a-b)]^2 = 4ac(a-b)(b-c)$

$[\because a(b-c) + b(c-a) + c(a-b) = 0]$

$[a(b-c) - c(a-b)]^2 = 0$

$ab - ac - ac + bc = 0$

$b(a+c) = 2ac$

$b = \frac{2ac}{a+c}$

HP

11. The sum to infinity of the series,

$S = 1 + 2\left(1 - \frac{1}{n}\right) + 3\left(1 - \frac{1}{n}\right)^2 + \ldots$ is — (1)

(a) n^2 (b) $n(n+1)$

(c) $n\left(1 + \frac{1}{n}\right)^2$ (d) None of these

$\left(1-\frac{1}{n}\right)S = \left(1-\frac{1}{n}\right) + 2\left(1-\frac{1}{n}\right) + \ldots + \infty$ — (2)

$S\left(1-1+\frac{1}{n}\right) = 1 + (1-1/n) + (1-1/n)^2 + \ldots \infty$

$\frac{S}{n} = \frac{1}{1-(1-1/n)} = \frac{1}{1/n} = n$

$S = n^2$

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20 marks

4 — ...

Solve 12, 15, 17, 19, 21

4 ✓
+1 leave
0 X

12. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in AP, x is equal to

(a) 2 (b) 3

(c) 4 (d) 2, 3

15. If the sides of a right angled triangle form an AP, the sines of the acute angles are

(a) $\frac{3}{5}, \frac{4}{5}$

(b) $\sqrt{3}, \frac{1}{3}$

(c) $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$

(d) $\frac{\sqrt{3}}{2}, \frac{1}{2}$

17. If the arithmetic progression whose common difference is non-zero, the sum of first $3n$ terms is equal to the sum of the next n terms. The ratio of the sum of the first $2n$ terms to the next $2n$ terms is

(a) $\frac{1}{5}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) None of these

19. Consider the pattern shown below:

Row 1 1

Row 2 3 5

Row 3 7 9 11

Row 4 13 15 17, 19, etc.

The number at the end of row 60 is

(a) 3659 (b) 3519

(c) 3681 (d) 3731

21. If a_1, a_2, a_3, a_4, a_5 are in HP, then $a_1a_2 + a_2\,a_3 + a_3a_4 + a_4a_5$ is equal to

(a) $2\,a_1a_5$

(b) $3a_1a_5$

(c) $4a_1a_5$

(d) $6a_1a_5$

22. If a, b, c and d are four positive real numbers such that $abcd = 1$, the minimum value of $(1 + a)(1 + b)(1 + c)(1 + d)$ is

(a) 1　　(b) 4

(c) 16　　(d) 64

23. If a, b, c are in AP and $(a+2b-c)(2b+c-a)(c+a-b)$ $= \lambda\, abc$, then λ is

(a) 1 (b) 2

(c) 4 (d) None of these

25. The sum of the first ten terms of an AP is four times the sum of the first five terms, the ratio of the first term to the common difference is

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) 4

27. If 11 AM's are inserted between 28 and 10, the number of integral AM's is

(a) 5 (b) 6

(c) 7 (d) 8

28. If x, y, z are in GP $(x, y, z > 1)$, then $\frac{1}{2x + \ln x}, \frac{1}{4x + \ln y}, \frac{1}{6x + \ln z}$ are in

(a) AP (b) GP

(c) HP (d) None of these

29. The minimum value of the quantity

$$\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc},$$

where $a, b, c \in R^+$, is

(a) $\frac{11^3}{2^3}$ (b) 125

(c) 25 (d) 27

31. If $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{2^n - 1}$, then

(a) $a(100) < 100$

(b) $a(100) > 100$

(c) $a(200) > 100$

(d) $a(200) < 100$

32. If the first and $(2n-1)$ th term of an AP, GP and HP are equal and their nth terms are a, b and c respectively, then

(a) $a = b = c$ (b) $a \geq b \geq c$

(c) $a + c = b$ (d) $ac - b^2 = 0$

36. If the sum of n consecutive odd numbers is $25^2 - 11^2$, then

(a) $n = 14$ (b) $n = 16$

(c) first odd number is 23 (d) last odd number is 49

37. The GM of two positive numbers is 6. Their AM is A and HM is H satisfy the equation $90A + 5H = 918$, then A may be equal to

(a) $\frac{1}{5}$ (b) 5 (c) $\frac{5}{2}$ (d) 10

40. Let $E = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$, then

(a) $E < 3$ (b) $E > \frac{3}{2}$ (c) $E < 2$ (d) $E > 2$

71. If the coefficient of x in the expansion of $\prod_{r=1}^{110}(1+rx)$ is $\lambda(1+10)(1+10+10^2)$, then the value of λ is

73. If n is a positive integer satisfying the equation
$2+(6\cdot 2^2-4\cdot 2)+(6\cdot 3^2-4\cdot 3)+\ldots+(6\cdot n^2 - 4\cdot n)=140,$
then the value of n is

76. Let (a_1, b_1) and (a_2, b_2) are the pair of real numbers such that 10, a, b, ab constitute an arithmetic progression. Then, the value of $\left(\frac{2a_1a_2 + b_1b_2}{10}\right)$ is

84. **Statement 1** 4, 8, 16 are in GP and 12, 16, 24 are in HP.

Statement 2 If middle term is added in three consecutive terms of a GP, resultant will be in HP.

93. The sequence of odd natural numbers is divided into groups 1; 3, 5; 7, 9, 11; ... and so on. Show that the sum of the numbers in *n*th group is n^3.

95. If the first four terms of an arithmetic sequence are a, $2a$, b and $(a - 6 - b)$ for some numbers a and b, find the sum of the first 100 terms of the sequence.

101. Consider the sequence $S = 7 + 13 + 21 + 31 + \ldots + T_n$, find the value of T_{70}.

106. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row of two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of the sides, then contains 8 balls less than each side of the triangle. Determine the initial number of balls.