$$Y = (\alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3)$$

$$\boxed{dy = d\alpha + \beta_1 dx_1 + \beta_2 dx_2 + \beta_3 dx_3}$$

$$dy = \beta_1 dx_1$$

$$\boxed{\frac{dy}{dx_1} = \beta_1}$$

Segregation of effect..

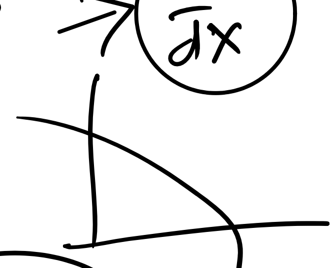
Global max, Global min, Local max, Local min → Interval

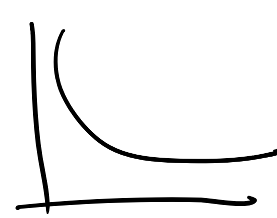
Neighbourhood

f(x) →

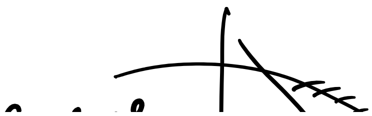
↑ ↓ → $\frac{d}{dx}$

Curvature

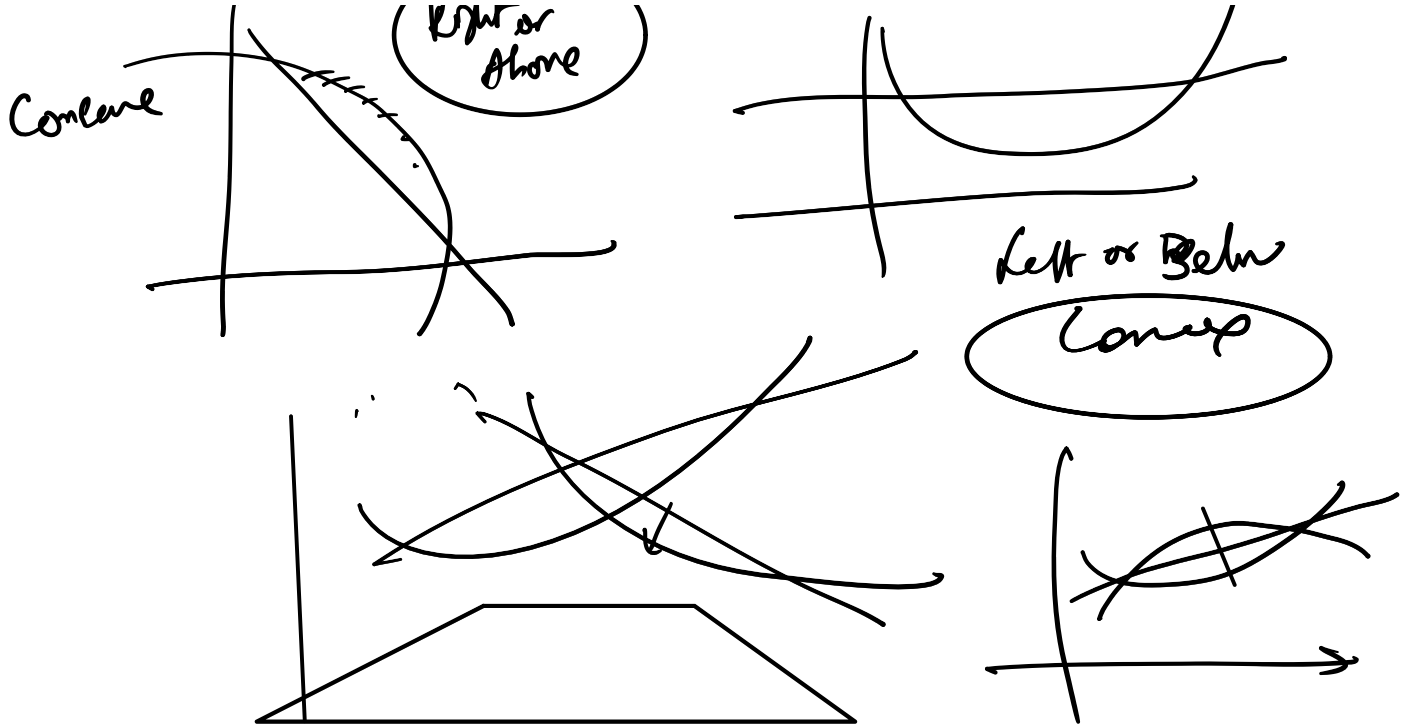




$\frac{d^2y}{dx^2}$



Right or Above



Derivative function On the basis of definition..

[redacted]
MATH DIF...

1 If $f:\mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x, y) = \begin{cases} \dfrac{x^3}{x^2 + y^4}, & \text{if } (x, y) \neq (0,0) \\ 0, & \text{if } (x, y) = (0,0) \end{cases}$ → non-homogeneous function..

$\cancel{\dfrac{x^3}{x^2}}$

Then

(a) $f_x(0, 0) = 0$ and $f_y(0, 0) = 0$
(b) $f_x(0, 0) = 1$ and $f_y(0, 0) = 0$
(c) $f_x(0, 0) = 0$ and $f_y(0, 0) = 1$
(d) $f_x(0, 0) = 1$ and $f_y(0, 0) = 1$

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$y = x_1^2 + x_2^2$

$y_{x_1} = 2x_1$

$\{0, \quad \text{if } (x,y) = (0,0)$

$y = x_1 + x_2$

$y_{x_1} = 2x_1^2$

Then

(a) $f_x(0, 0) = 0$ and $f_y(0, 0) = 0$ (b) $f_x(0, 0) = 1$ and $f_y(0, 0) = 0$

(c) $f_x(0, 0) = 0$ and $f_y(0, 0) = 1$ (d) $f_x(0, 0) = 1$ and $f_y(0, 0) = 1$

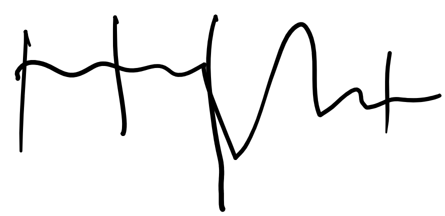
$$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{h - 0}{h} = 1$$

$$f_y(0,0) = \frac{f(0,k) - f(0,0)}{k}$$

$$= \frac{0 - 0}{k} = 0$$

$\boxed{f_x \neq f_y}$



2 The set of points at which the function $f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$, $(x,y) \in \mathbb{R}^2$ **attains local maximum** is ______ (NAT)

$(0,0)$ $\to$ 1

$$\frac{\partial f}{\partial x} = 4x^3 - 2x = 0 \qquad \frac{\partial^2 f}{\partial x^2} = 12x^2 - 2 \quad r$$

$$\frac{\partial f}{\partial y} = 4y^3 - 2y = 0 \qquad \frac{\partial^2 f}{\partial y^2} = 12y^2 - 2 = t$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0 = s$$

for max/min value of $f(x,y)$

$f_x = 0 = f_y$

$x = 0, \pm \frac{1}{\sqrt{2}}$

$y = 0, \pm \frac{1}{\sqrt{2}}$

The stationary points are $(0,0)$, $\left(0, \pm\frac{1}{\sqrt{2}}\right)$, $\left(\pm\frac{1}{\sqrt{2}}, 0\right)$, $\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$

The stationary points are

$$(0,0),\ \left(0, \pm\frac{1}{\sqrt2}\right),\ \left(\pm\frac{1}{\sqrt2}, 0\right)\ \left(\pm\frac{1}{\sqrt2}, \pm\frac{1}{\sqrt2}\right)$$

	$\frac{\partial^2 f}{\partial x^2}$	t	$rt - s^2$	
$(0,0)$	<0	<0	>0	max
$\left(0, \pm\frac{1}{\sqrt2}\right)$	<0	>0	<0	X
$\left(\pm\frac{1}{\sqrt2}, 0\right)$	>0	<0	<0	X
$\left(\pm\frac{1}{\sqrt2}, \pm\frac{1}{\sqrt2}\right)$	>0	>0	>0	mini

$f(\max) = 1$

3. Let $f : [a, b] \to \mathbb{R}$ be a continuous function, such that f is differentiable in (a,c) and (c, b), $a<c<b$. If $\lim_{x\to c} f'(x)$ exists, then prove that f is differentiable at 'c' and $f'(c)=\lim_{x\to c} f'(x)$.

$a < c < b$

$\lim_{x\to c} f'(x)$ exist $\Rightarrow$ $\lim_{x\to c^-} f'(x)$ & $\lim_{x\to c^+} f'(x)$ exist & equal

$\boxed{a < c}$ $\quad \lim_{x\to c^-} \frac{f(x)-f(c)}{x-c} = f'(c-)$ exist

$$\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c} = \lim_{x\to c^+} \frac{f(x)-f(c)}{x-c}$$

$$f'(c^-) = f'(c^+)$$

Then, $f(x)$ is diff $x=c$ a $f'(c) = \lim_{x\to c} f'(x)$

Proved..

$f(x) = |x^2-25|\ \forall x \in R$

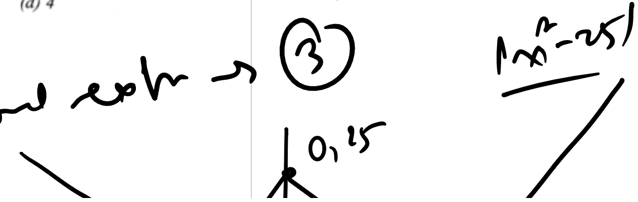
$f'(x) = 0 \Rightarrow \frac{(x^2-25)2x}{|x^2-25|} = 0$

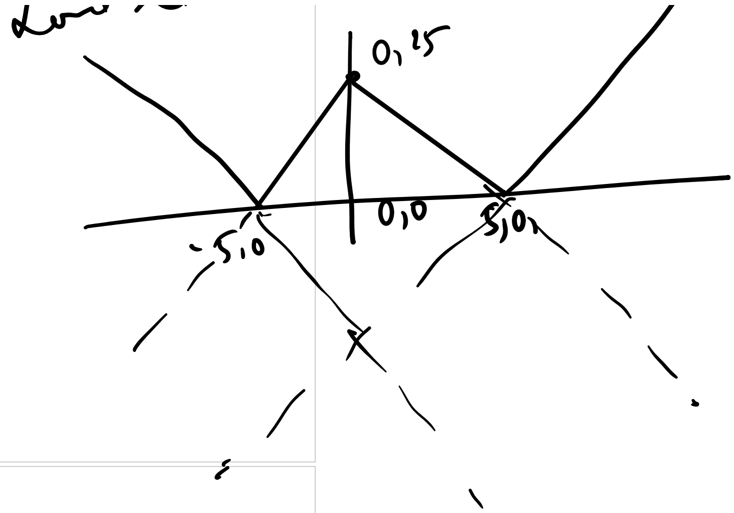
4. Let $f(x) = |x^2-25|$ for all $x \in \mathbb{R}$. The total number of points of $\mathbb{R}$ at which f attains a local extremum (minimum or maximum) is (MCQ)

(a) 1 (b) 2 (c) 3 (d) 4

$\Rightarrow x = 0, 5, -5$

Local extrm $\to$ 3





5 Let $f(x,y)=\sum_{k=1}^{10}(x^2-y^2)^k$ for all $(x,y)\in\mathbb{R}^2$. Then for all $(x,y)\in\mathbb{R}^2$,

(a) $x\frac{\partial f}{\partial x}(x,y)-y\frac{\partial f}{\partial y}(x,y)=0$

(b) $x\frac{\partial f}{\partial x}(x,y)+y\frac{\partial f}{\partial y}(x,y)=0$

(c) $y\frac{\partial f}{\partial x}(x,y)-x\frac{\partial f}{\partial y}(x,y)=0$

(d) $y\frac{\partial f}{\partial x}(x,y)+x\frac{\partial f}{\partial y}(x,y)=0$ ✓

$$\frac{\partial f}{\partial x}=\sum K(x^2-y^2)^{K-1}(2x)$$

$$\frac{\partial f}{\partial y}=\sum K(x^2-y^2)^{K-1}(-2y)$$

$x\neq 0$
$y\neq 0$

$$\sum K(x^2-y^2)^{K-1}=\frac{1}{2x}\frac{\partial f}{\partial x}$$

$$\sum K(x^2-y^2)^{K-1}=-\frac{1}{2y}\frac{\partial f}{\partial y}$$

$$\frac{1}{2x}\frac{\partial f}{\partial x}=-\frac{1}{2y}\frac{\partial f}{\partial y}$$

So, $y\frac{\partial f}{\partial x}+x\frac{\partial f}{\partial y}=0$ ✓

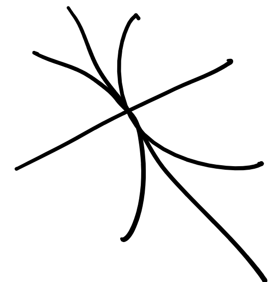
If $x=0=y$ then $=0$ ✓

If $x=0=y$ then $=0$ ✓

6 The value of $\alpha \in \mathbb{R}$ for which the curves $x^2+\alpha y^2=1$ and $y=x^2$ intersect orthogonally is

(a) -2 (b) $\frac{-1}{2}$ (c) $\frac{1}{2}$ (d) 2 ✓

They will become Orthogonal S



α_1, α_2 $\boxed{\alpha_1\alpha_2=-1}$

$x^2+\alpha y^2=1 \Rightarrow 2x+2\alpha y\frac{dy}{dx}=0$

$\frac{dy}{dx}=$ $\boxed{\alpha_1=-\frac{x}{\alpha y}}$

Also, $y=x^2 \Rightarrow \frac{dy}{dx}=2x$ $\boxed{\alpha_2=2x}$

So, $\left(-\frac{x}{\alpha y}\right)(2x)=-1$

$\frac{2x^2}{\alpha y}=1$ As $y=x^2$

$\frac{2x^2}{\alpha x^2}=1$

$\frac{2}{\alpha}=1$

$\boxed{\alpha=2}$

7 For all $(x, y)\in\mathbb{R}^2$, let $f(x,y)=\begin{cases}\frac{x}{|x|}\sqrt{x^2+y^2} & \text{if } x\neq 0,\\ 0 & \text{if } x=0\end{cases}$. Then $\frac{\partial f}{\partial x}(0,0)+\frac{\partial f}{\partial y}(0,0)$ equals **(MCQ)**

(a) -1 (b) 0 (c) 1 ✓ (d) 2

$f_x(0,0)=\lim \frac{f(h,0)-f(0,0)}{h}=\lim \frac{\frac{h}{|h|}\sqrt{h^2}-0}{h}$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{|h|\sqrt{\cdots}-0}{h}$$

$$= \lim_{h\to 0} \frac{|h|}{|h|} = 1$$

fm, $f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k}$

$$= \lim_{k\to 0} \frac{0-0}{k} = 0$$

$$1+0 = 1$$

$$f_z g_w - f_w g_z = 1$$

$$\frac{\partial f}{\partial z}\frac{\partial g}{\partial w} - \frac{\partial f}{\partial w}\cdot\frac{\partial g}{\partial z} = 1$$

$$\frac{\partial f}{\partial z}\frac{\partial g}{\partial w}\cdot\frac{\partial z}{\partial x} - \frac{\partial f}{\partial w}\frac{\partial g}{\partial z}\frac{\partial z}{\partial x} = \frac{\partial z}{\partial x}$$

8. Suppose that the dependent variables z and w are functions of the independent variables x and y, defined by the equations $f(x, y, z, w) = 0$ and $g(x, y, z, w) = 0$, where $f_z g_w - f_w g_z = 1$. Which one of the following is correct? **(MCQ)**

(a) $z_x = f_w g_x - f_x g_w$ (b) $z_x = f_x g_w - f_w g_x$ (c) $z_x = f_z g_x - f_x g_z$ (d) $z_x = f_z g_w - f_z g_x$

$$\Rightarrow \left(\frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial x}\right)\frac{\partial g}{\partial w} - \frac{\partial f}{\partial w}\left(\frac{\partial g}{\partial z}\frac{\partial z}{\partial x}\right) = \frac{\partial z}{\partial x}$$

$$\Rightarrow \frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial w} - \frac{\partial f}{\partial w}\frac{\partial g}{\partial x} = \frac{\partial z}{\partial x}$$

$$z_x = f_x g_w - f_w g_x$$

9 For what real values of x and y, does the integral $\int_x^y (6-t-t^2)dt$ attain its maximum?

(a) x = -3, y = 2 (b) x = 2, y = 3 (c) x = -2, y = 2 (d) x = -3, y = 4

$1+3+2=0$

$$I = \int_x^y (6-t-t^2)dt$$

$$\frac{\partial I}{\partial t} = 0 = \int_x^y (-1-2t)dt = 0$$

$$(-1-t^2)\Big|_x^y = 0$$

$$-y-y^2+x+x^2=0$$

$$(x-y)(1+x+y)=0$$

$x=y$ or $1+x+y=0$

f_x f_y f_{xx} f_{yy} f_{yx}

minimum

10 The function $f(x, y)=3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$ has a saddle point at

(a) (0, 0) (b) (0, 2) (c) (1, 1) (d) (-2, 1)

$f_x = 6xy - 6x = 0$

$f_y = 3x^2 + 12y^2 - 24y = 0$

$f_x = 6xy - 6x = 0$
$f_y = 3x^2 + 3y^2 - 2\cdot 4y = 0$
$f_{xx} = 6y - 6$
$f_{yy} = 24y - 24$
$f_{xy} = 6x$

$1 < x < 3$
$2 < x < 12$
$0 < x < 9.5$

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(0,0) (0,2) (-2,1)

	r	t	$rt - s^2$	
(0,0)	< 0	< 0	> 0	max
(0,2)	> 0	> 0	> 0	mini
(-2,1)	0	0	< 0	A saddle pnt

11

32\. *Let f be a real valued function defined by* $f(x, y) = 2\ \ell n\left(x^2 y^2 e^{\frac{y}{x}}\right)$, $x > 0, y > 0$.

Then, the value of $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ *at any point (x,y), where x>0, y>0, is* ______________