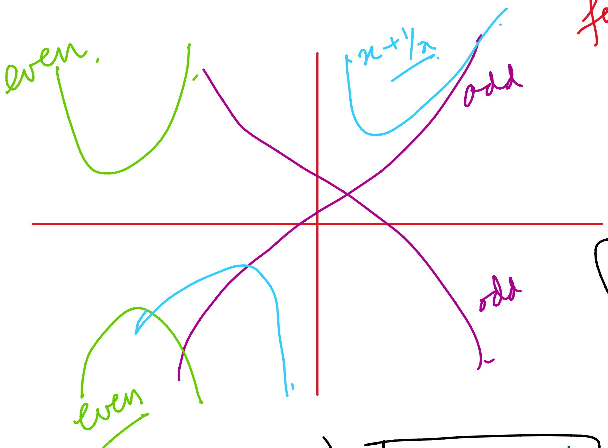
25 November 2023 17:01

Typical function questions

$f(x)+g(x)=2x$ $\forall$ $f(x)$ is even and $g(x)$ is an odd

find $f(x)$ and $g(x)$



$f(-x)=f(x)$ even → both.

$g(-x)=-g(x)$ odd → $\boxed{\text{increasing / decreasing}}$

⇓

$g'(x)>0$ or $g'(x)<0$.

even

$\boxed{f(x)=h(x)=x+\frac{1}{x}}=x+x^{-1}$

$h'(x)=1-\frac{1}{x^2}$

$=\frac{x^2-1}{x^2}$

$=\frac{(x+1)(x-1)}{x^2}$ +ve.

+ve $x>1,\ x<-1$

-ve $-1<x<1$

$2x=\left(x+\frac{1}{x}\right)+\left(x-\frac{1}{x}\right)$

even $f(x)$ odd $g(x)$

$\boxed{g(x)=x-\frac{1}{x}}$ odd.

$g'(x)=1+\frac{1}{x^2}>0.$

$f(x)+g(x)=\boxed{\sin x}$

↓ even ↓ odd.

↓ $g'(x)>0$ or $g'(x)<0$.

$\sin x+\frac{1}{\sin x}$. $\sin x-\frac{1}{\sin x}$.

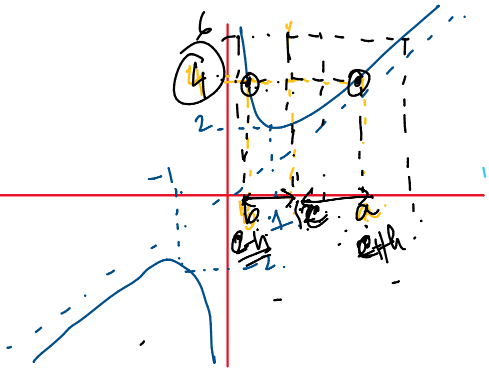
$f(x)=\frac{1}{2}\left[\sin x+\frac{1}{\sin x}\right]$

$g(x)=\frac{1}{2}\left[\sin x-\frac{1}{\sin x}\right]$

even $f(x)=x+\frac{1}{x}$.

$f(-x)=-x-\frac{1}{x}=-\left(x+\frac{1}{x}\right)=-f(x)$

$f(-x)=-f(x)$ → odd



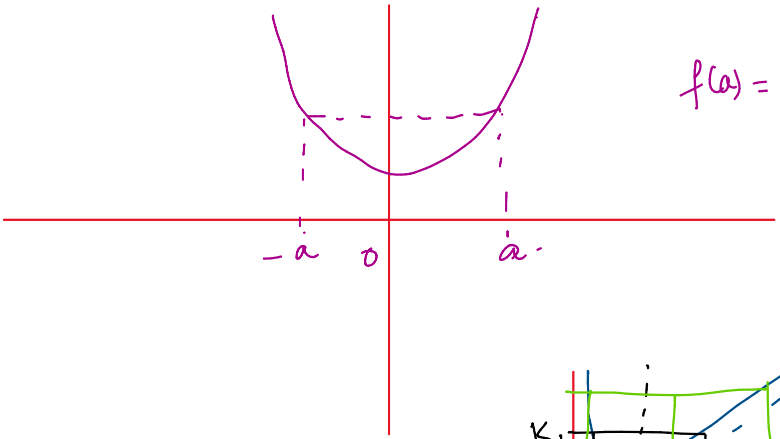
$f(-x)=f(x)$ ✓

$f(a)=f(b)=4$

$a=c+h$

$b=c-h$.

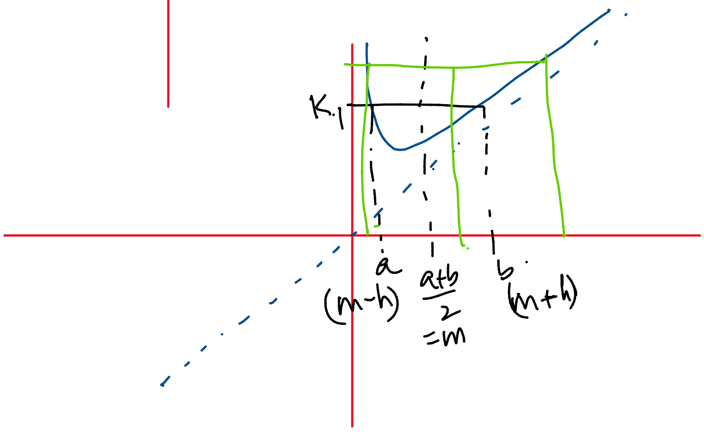
$c=\frac{a+b}{2}$



$f(a) = f(-a) \to$ Parabola

$\downarrow$

axis of symmetry is constant



$f(a) = f(b) = K_1$

$\Downarrow$

axis of symmetry $= \frac{a+b}{2}$

$y = x$ $\quad$ $y = x^2$

even

① $f(-x) = f(x)$

② $f'(x) > 0$ or < 0 depending on the value of x.

odd

$f(-x) = -f(x)$

Bijective (one-one & onto)

$\Downarrow$

$f'(x) > 0$ or $f'(x) < 0$.

$y = x^2$.

$y = x^{2n}$ parabola

$y = x$

$y = x^{2n+1}$ st line

$y + c = (x+\alpha)^2$.

$y = x^2 + x \to$ even

$f(-x) \neq f(x)$

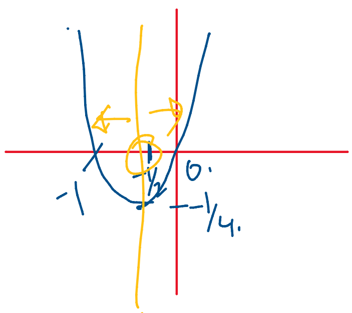
$\Downarrow$

assumption is that the axis of symmetry is $x = 0$ (constant)

$y = x(x+1)$

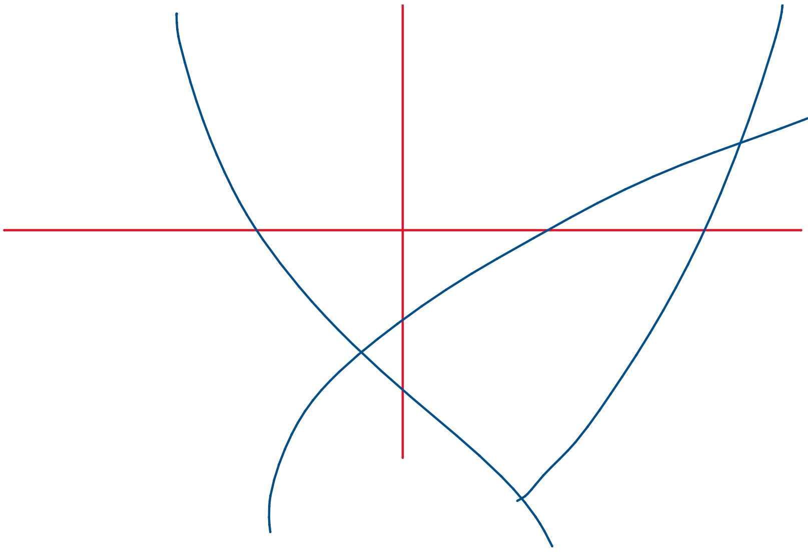
$$y = x^2 + 2 \cdot \tfrac{1}{2} \cdot x + \left(\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2$$

$$= \underbrace{\left(x+\tfrac{1}{2}\right)^2}_{\geq 0} - \left(\tfrac{1}{2}\right)^2$$



$f[(x+1/2)] = f(x+\tfrac{1}{2})$

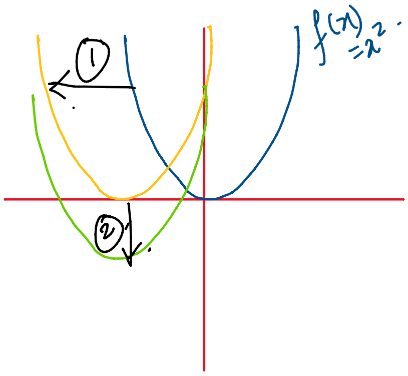
odd



odd

$g(x) = \left(x+\frac{1}{2}\right)^2 - \frac{1}{4}$

$f(x) = x^2$



$f(x) = x^2$.

$g(x) = A f\left[\frac{1}{B}(x+c)\right] + D$.

transformation of $f(x)$ to $g(x)$.

$A \rightarrow$ vertical dilation factor

$B \rightarrow$ horizontal " "

$C \rightarrow$ " shift $= +\frac{1}{2} \rightarrow$ left shift ①

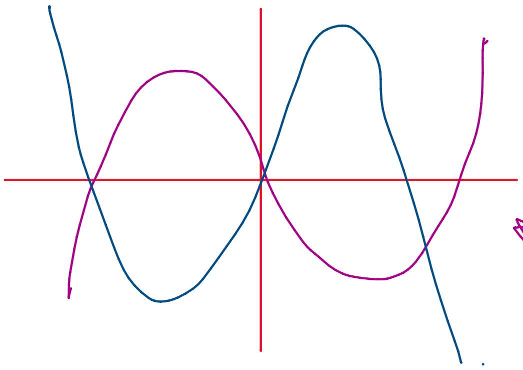
$D \rightarrow$ vertical shift $= -\frac{1}{4} \rightarrow$ downward shift ②

$f(-x) = f(x) \rightarrow$ y axis is the axis of symmetry ✗

$f[-(x+\alpha)] = f(x+\alpha)$ ✓ $\rightarrow$ $x = -\alpha$ is the axis of symmetry.

$\Downarrow$

applicable when $f(x)$ is a polynomial of degree $2n$.



$f(x) = a_0 x^{2n} + a_1 x^{2n-1} + \cdots + a_{2n}$.

$f(-x) = -f(x)$ ✗ $\rightarrow$ axis of symmetry $\Rightarrow$ y axis

$f[-(x+\alpha)] = -f[x+\alpha]$ ✓