26 October 2023 17:29

Functions

$f(x) = x^2 + x + 1$

dependent — range   independent — domain

$\sqrt{f(x)} = \sqrt{x^2-1}$

$\downarrow$

domain of $f(x) \geq 0$

$x^2 \geq 1 \rightarrow x \geq 1$ or $x \leq -1$

max(min) or min(max)

$f(x) = x-1 \qquad g(x) = 4-x$

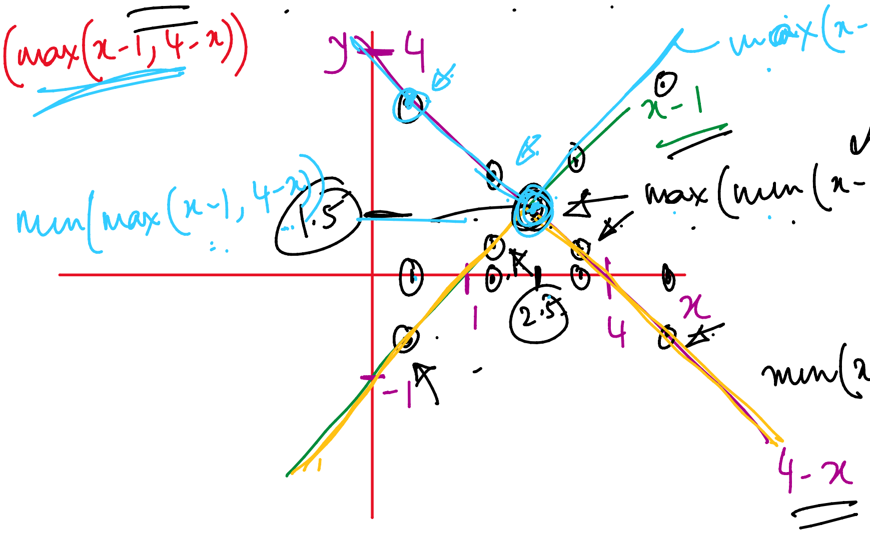
$h(x) = \max(\min(f(x), g(x))) = \max(\boxed{\min(x-1, 4-x)})$

$\phi(x) = \min(\max(x-1, 4-x))$

$\max(x-1, 4-x)$

$y = 4-x$

$y = x-1$



$4-x = x-1$

$5 = 2x$

$x = 2.5$

$\min(x-1, 4-x)$

1.5

$f(x) = \min(\max(\sqrt{x^2-1}, x+2))$

Step 1. $\sqrt{x^2-1} = x+2.$

$\cancel{x^2} - 1 = \cancel{x^2} + 4x + 4$

$4x = -5$

$x = -\frac{5}{4}$

Step 2 put x in $x+2$

$f(x) = -\frac{5}{4} + 2 = \frac{3}{4}$

$\boxed{\text{gint}} \rightarrow$ Greatest integer function

$y = [x] \rightarrow$ greatest integer less than or equal to x.

$x = 1$ $y = 1$

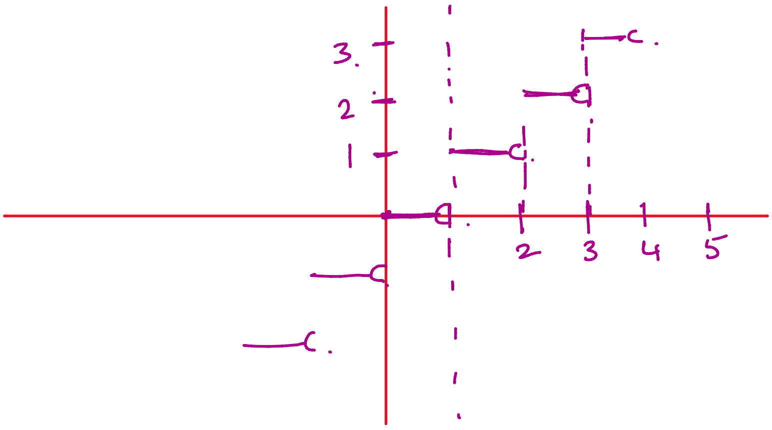
x .

$x = 1.32 \quad y = 1$

$x = 1 \quad y = 1$

$x = 2 \quad y = 2.$

$x = -2.5 \quad y = -3$



$[x]$ is discontinuous for all integer values of x.

find the no of points of discontinuity of $[x^2-9]$ in the domain $-4 < x < 4$

$0 < x^2 < 16$

$0-9 < x^2-9 < 16-9$

$-9 < x^2-9 < 7$

-8 to $-1 \rightarrow 8$

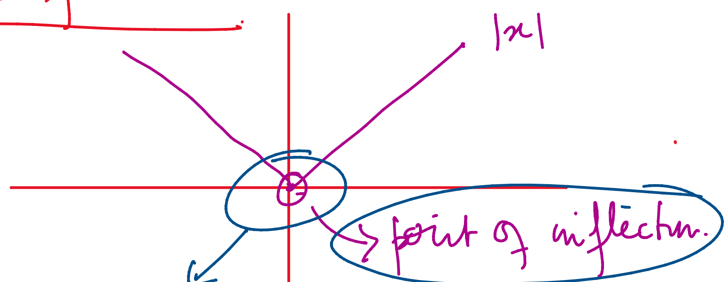
$0 \rightarrow 1$

$1-6 \rightarrow 6$

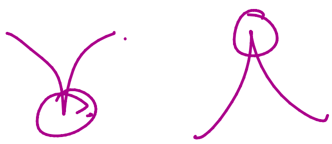
15 values.

modulus function

$y = |x|$



$|x|$ is continuous but not differentiable.



$y = |x^2-5x+6| \rightarrow$ domain? range?

$\downarrow$

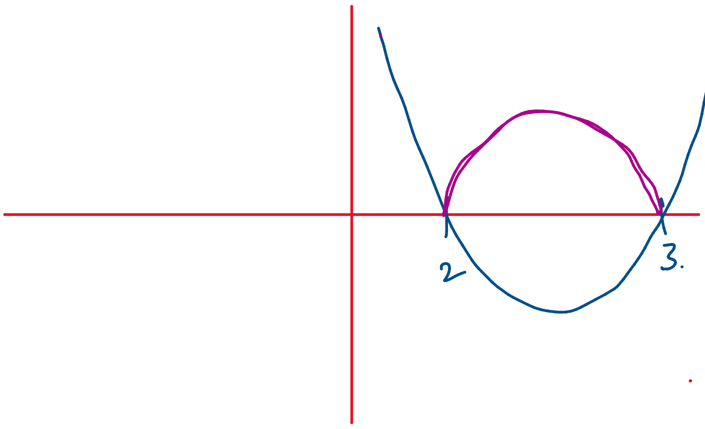
0 to $+\infty$

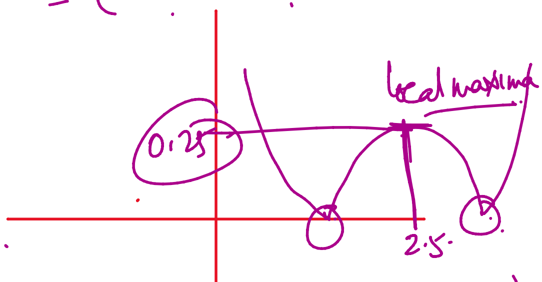
$y = x^2 - 5x + 6, \quad x^2 - 5x + 6 \geq 0$

$= -(x^2 - 5x + 6), \quad x^2 - 5x + 6 < 0$

$y = x^2 - 5x + 6 = (x-2)(x-3)$

$-(2.5-2)(2.5-3) = 0.25$





domain $(-\infty, +\infty)$

range $(0, +\infty)$

function of function (composite function)

$f(x) = x^2 + 4 \qquad g(x) = x - 2$

$y = f \circ g(x) = f[g(x)] = [g(x)]^2 + 4$

$= (x-2)^2 + 4$

$= x^2 - 4x + 8$

$\boxed{f(x) = x^2 + 2} \ \forall \ \boxed{x\,[-2, 2]}$

$g(x) = x - 2 \ \forall \ x\,[-2, 0)$

$y = f \circ g(x)$

$y = f[g(x)]$ for $g(x) \in [-2, 2]$

If $g(x) \neq [-2, 2]$ then $f \circ g(x)$ is undefined.

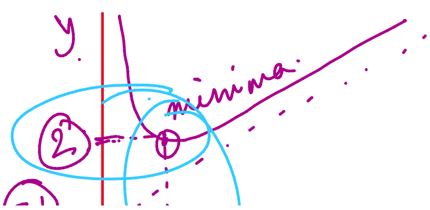
$-2 \leq x - 2 \leq 2$

$\boxed{0 \leq x \leq 4}$

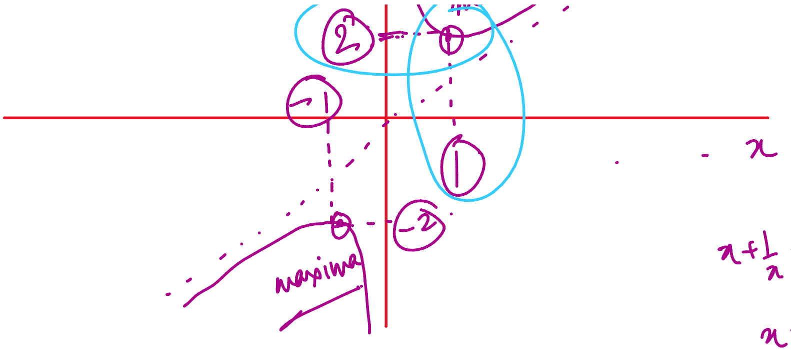
$-4 \leq x - 2 < 0$

Special function

$y = x + \frac{1}{x}$



maxima < minima



$$x + \frac{1}{x} \geq 2\sqrt{x \cdot 1/x}$$

$$x + \frac{1}{x} = 2$$

$$x = 1$$

$$x + \frac{1}{x} \geq 2$$

$$y = (x^2 + 2x + 3) + \frac{1}{(x^2+2x+3)}$$

find the minima

$$x^2 + 2x + 3 = 1 \qquad \boxed{x^2 + 2x + 2 = 0}$$

$$\varphi$$

$$D = b^2 - 4ac = -ve$$

HW.