Large Sample Tests

Testing for population proportion: (p)

To test: $H_0: p = p_0$ vs $H_{1A}: p > p_0$.

$H_{1B}: p < p_0$.

$H_{1C}: p \neq p_0$.

We have sample of 'n' obs and there are 'f' cases for 'TRUE'.

$\therefore$ Sample proportion $\hat{p} = \dfrac{f}{n}$

$X \sim Bin(n, p)$

$E(X) = np$

$Var(X) = np(1-p)$

$$E(\hat{p}) = E\left(\frac{f}{n}\right) = \frac{1}{n} \cdot E(f) = \frac{\cancel{n}p}{\cancel{n}} = p.$$

$$Var(\hat{p}) = Var\left(\frac{f}{n}\right) = \frac{1}{n^2} Var(f) = \frac{\cancel{n}(p)(1-p)}{n^{\cancel{2}}}$$

$$= \frac{p(1-p)}{n}$$

$$\therefore \boxed{\frac{\hat{p} - E(\hat{p})}{\sqrt{Var(\hat{p})}}} \sim N(0,1)$$

Test-statistic $$T = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \underset{H_0}{\overset{a}{\sim}} N(0,1)$$

Testing Rules:

$H_0: p = p_0$ vs $H_{1A}: p > p_0$.

We reject H_0 at $\alpha\%$ L.O.S if $T_{obs} > \tau_\alpha$

$H_0: p = p_0$ vs $H_{1B}: p < p_0$

We reject H_0 at $\alpha\%$ L.O.S if $T_{obs} < -\tau_\alpha$

$H_0: p = p_0$ vs $H_{1C}: p \neq p_0$.

We reject H_0 at $\alpha\%$ L.O.S if $|T$ ...

$H_0: p = p_0$ vs $H_{1c}: p \neq p_0$.

We reject H_0 at $\alpha\%$ l.o.s if $|T_{obs}| > \tau_{\alpha/2}$.

Testing for Equality of Population Proportions:

Suppose we two poplns with props p_1 & p_2.

To test:

$H_0: p_1 = p_2 \Rightarrow p_1 - p_2 = 0$ vs

$H_{1A}: p_1 - p_2 > 0$

$H_{1B}: p_1 - p_2 < 0$

$H_{1c}: p_1 - p_2 \neq 0$.

Consider sample of size n_1 & n_2 respectively from the 2 poplns with f_1 & f_2 number of successes / true events.

Sample proportions: $\hat{p}_1 = \frac{f_1}{n_1}$ & $\hat{p}_2 = \frac{f_2}{n_2}$

We will use: $(\hat{p}_1 - \hat{p}_2)$ to develop the test statistic.

$$E(\hat{p}_1 - \hat{p}_2) = E(\hat{p}_1) - E(\hat{p}_2)$$
$$= p_1 - p_2 = 0 \text{ [under } H_0]$$

$$Var(\hat{p}_1 - \hat{p}_2) = Var(\hat{p}_1) + Var(\hat{p}_2)$$
$$= \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$

Under H_0, $p_1 = p_2 = p$ (say)

$$Var(\hat{p}_1 - \hat{p}_2) = p(1-p)\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}$$

$$\therefore \frac{(\hat{p}_1 - \hat{p}_2) - E(\hat{p}_1 - \hat{p}_2)}{\sqrt{Var(\hat{p}_1 - \hat{p}_2)}} \overset{a}{\underset{H_0}{\sim}} N(0,1)$$

$\sqrt{var(\hat{p}_1 - \hat{p}_2)}$

$$\Rightarrow \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{p(1-p)\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}}} \underset{H_0}{\overset{a}{\sim}} N(0,1)$$

↳ unknown.

We will estimate p by $\hat{p} = \frac{f_1 + f_2}{n_1 + n_2}$

Test-statistic $T = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}(1-\hat{p})\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}}} \underset{H_0}{\overset{a}{\sim}} N(0,1)$

HW Testing criteria: