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Matrix and Determinants

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 5 & 7 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}_{2\times 3}$$

$m \times n$

row columns.

3×2

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

Q 1

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 5 & 2 \\ 3 & 6 & 2 \end{bmatrix}_{3\times 3} \qquad B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & 1 \\ 2 & 5 & 1 \end{bmatrix}_{3\times 3}$$

a_{11} a_{12} a_{13} / a_{21} a_{22} a_{23} / a_{31} a_{32} a_{33}

$$A+B = \begin{bmatrix} 2+1 & 4+3 & 1+0 \\ 1+0 & 5+4 & 2+1 \\ 3+2 & 6+5 & 2+1 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 1 \\ 1 & 9 & 3 \\ 5 & 11 & 3 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 5 & 2 \\ 3 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & 1 \\ 2 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 4-3 & 1-0 \\ 1-0 & 5-4 & 2-1 \\ 3-2 & 6-5 & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Scalar Multiplication

multiply A-B matrix by 8

$$8[A-B] = 8\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 8 & 8 \\ 8 & 8 & 8 \\ 8 & 8 & 8 \end{bmatrix}$$

Vector Multiplication

$$A = \begin{bmatrix} 4 & 7 & 2 & 9 \end{bmatrix}_{1\times 4}$$

$$B = \begin{bmatrix} 12 \\ 1 \\ 5 \\ 6 \end{bmatrix}_{4\times 1}$$

What is the product AB

$$AB = \begin{bmatrix} 4 & 7 & 2 & 9 \end{bmatrix}_{1\times 4} \begin{bmatrix} 12 \\ 1 \\ 5 \\ 6 \end{bmatrix}_{4\times 1}$$

$$= \left[(4\times 12) + (7\times 1) + (2\times 5) + (9\times 6)\right]_{1\times 1}$$

$$= [119]_{1\times 1} = 119 \text{ (ans)}$$

$$BA = \begin{bmatrix} 12 \\ 1 \\ 5 \\ \end{bmatrix} \begin{bmatrix} 4 & 7 & 2 & 9 \end{bmatrix}_{1\times 4} = \begin{bmatrix} 48 & 84 & 24 & 108 \\ 4 & 7 & 2 & 9 \\ 20 & 35 & 10 & 45 \end{bmatrix}$$

$$BA = \begin{bmatrix} \vdots \\ 1 \\ 5 \\ 6 \end{bmatrix}_{4\times 1} \begin{bmatrix} 4 & 7 & 2 & 1 \end{bmatrix}_{1\times 4} = \begin{bmatrix} 4 & 7 & 2 & 1 \\ 20 & 35 & 10 & 45 \\ 24 & 42 & 12 & 54 \end{bmatrix}_{4\times 4}$$

Prove that $AB \neq BA$

Q2

$$A = \begin{bmatrix} 3 & 6 & 7 \\ 12 & 9 & 11 \end{bmatrix}_{2\times 3} \quad \text{and} \quad B = \begin{bmatrix} 6 & 12 \\ 5 & 10 \\ 13 & 2 \end{bmatrix}_{3\times 2}$$

Find AB?

$$AB = \begin{bmatrix} 3\times 6 + 6\times 5 + 7\times 3 & 3\times 12 + 6\times 10 + 7\times 2 \\ 12\times 6 + 9\times 5 + 11\times 13 & 12\times 12 + 9\times 10 + 11\times 2 \end{bmatrix}_{2\times 2}$$

1. Commutative Law: $A+B = B+A$
2. Associative Law: $(A+B)+C = A+(B+C)$
3. Distributive Law: $(XY)Z = X(YZ)$

$$A = \begin{bmatrix} 7 & 5 \\ 1 & 3 \\ \ldots & \ldots \end{bmatrix} \quad B = \begin{bmatrix} 4 & 9 & 10 \\ 2 & 6 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 \\ 6 \\ 7 \end{bmatrix}$$

Q

Q

$$A = \begin{bmatrix} 1 & 3 \\ 8 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 6 & 5 \end{bmatrix} \quad \begin{bmatrix} 7 \end{bmatrix}$$

Prove the Distributive Law

$$\boxed{\text{that is } (AB)C = A(BC)}$$

Soln

$$AB = \begin{bmatrix} 7 & 5 \\ 1 & 3 \\ 8 & 6 \end{bmatrix} \begin{bmatrix} 4 & 9 & 10 \\ 2 & 6 & 5 \end{bmatrix} = \begin{bmatrix} (7\times4+5\times2) & (7\times9+5\times6) & (7\times10+5\times5) \\ (1\times4+3\times2) & (1\times9+3\times6) & (1\times10+3\times5) \\ (8\times4+6\times2) & (8\times9+6\times6) & (8\times10+6\times5) \end{bmatrix}$$

$$= \begin{bmatrix} 38 & 93 & 95 \\ 10 & 27 & 25 \\ 44 & 108 & 110 \end{bmatrix}$$

Then

$$(AB)C = \begin{bmatrix} 38 & 93 & 95 \\ 10 & 27 & 25 \\ 44 & 108 & 110 \end{bmatrix}_{3\times3} \begin{bmatrix} 2 \\ 6 \\ 7 \end{bmatrix}_{3\times1} = \begin{bmatrix} 38\times2+93\times6+95\times7 \\ 10\times2+27\times6+25\times7 \\ 44\times2+108\times6+110\times7 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 1299 \\ 357 \\ 1506 \end{bmatrix}_{3\times1}$$

$$A(BC) =$$

Identity matrix, $I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Identity matrix, $I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Null matrix, $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solving simultaneous equation using matrix method:

$$3x + 4y + 5z = 1 \quad \text{— (1)}$$
$$2x + y + 3z = 2 \quad \text{— (2)}$$
$$x + y + z = 5 \quad \text{— (3)}$$

Writing (1) (2) and (3) in matrix form:

$$\underbrace{\begin{bmatrix} 3 & 4 & 5 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}}_{C}$$

$$AX = C$$

$$X = A^{-1} \cdot C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1} = \begin{bmatrix} 3 & 4 & 5 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}_{3\times3} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}_{3\times1}$$

$_{3\times 1}$ $_{3\times 3}$ $_{3\times 1}$

or,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix}$$

We will learn $A^{-1} = ?$

How can we find x, y, z using A^{-1}.